PROFESSOR: Welcome back. And today we are going to look at a harder situation. At oscillations waves in the electromagnetic field. Why I say it's harder, for many reasons. First of all, so far we've always considered situations which we could either visualize or had some sensual way of getting a feel for what the physical situation is.

When it comes to the electromagnetic field, as you well know, we can't see it, sense it, at all. And the only way to describe it is, in fact, in terms of mathematics. So there isn't, first, a word-- a description by analogy with what we see around. Secondly, it's more complicated. These are oscillations in three dimensions. And, as you well know, there both electric and magnetic fields. Overall, it is just much more difficult situation.

So first of all, I start by a mathematical description of this system. Because, as I say, there is no other way we know of discussing it. And the mathematical description of the electromagnetic field, as you all know, are the so-called Maxwell's equations. I've written here the four Maxwell's equations for vacuum. So this is what the electric and magnetic fields have to satisfy.

And I'm just reminding you that the definition of what electric and magnetic field-- the operational definition comes from the Lorentz force. Basically, this is just quickly to remind you, if I have a charge in vacuum and if it experiences a force, I know there is an electric field there. On the other hand, if it experiences a force when it's moving, then I know that there is a magnetic field. So this tells us that here, although we can't see it, there is an electromagnetic field.

If one looks at these equations and plays around with them, one find that the electromagnetic field actually satisfy wave equations. This is the wave equation for the-- three-dimensional wave equation for the electric field. And this is for the magnetic field, where c is the phase velocity as always, and in the case of electromagnetism c is given by that. That's the speed of light, or the speed of electromagnetic waves.

Now, so what this tells us, is that in vacuum, you can have excitations, oscillations of the electromagnetic and magnetic fields, which propagate. And we have all of the wave phenomena we've learned for other systems. The thing to keep in mind is that whatever the solution of the system is, whatever is propagating, it must satisfy all of these equations.

Not every situation has to satisfy this. This is a subset of the infinite possibilities that are allowed by Maxwell's equations. OK. So now, instead of doing solutions to some specific situations with a specific boundary condition, et cetera, since it's already much more difficult, all I will do today is see how we can identify solutions of these equations. What kind of waves they correspond to. Or vice versa, if you want to describe in terms of mathematics some particular wave, how do we do that? That is the kind of problems I will be discussing today.

So, let me come to the first problem. And probably using the word problem is a misnomer. The description. I'll consider first progressive wave solutions of these equations. Suppose we know that there is an electric field, which is a propagating electric field, sinusoidal. All right? I assure you, this does not contradict Maxwell's equations. You can try it. All right? It's not complete, as you'll see in a moment.

The question here is, if you have an electric field like that, can we describe as well as possible in words, what kind of a wave this corresponds to? And secondly, answer the question if this is a real electromagnetic wave in the vacuum, what must be the corresponding magnetic field? By itself, this equation does not satisfy all the Maxwell's equations. You need a corresponding magnetic field. So, let's look at that.

First of all, we know that any function, which is a function of x plus or minus vt describes a progressive wave. It satisfies the classical wave equation, you can try it and see. If these two terms-- the x and the t terms-- are of opposite sign, then this describes a progressive wave, which goes in the plus-x direction. If they are the same sign, then it goes in the opposite direction. And again I say, plot any function like this, and see what happens as you change t. The shape of the function will not change. But it will move either to the left or to the right as you change the time.

So we immediately see, since this is a cosine of this, which is of this form, if I divide by a, this is x minus b over a. And I could take the a outside that. So this is a progressive wave. These two have opposite signs. It's a function of x and t. So this is a progressive wave, which is moving or progressing in the x direction. They're opposite signs, so in the plus-x direction. So immediately I know that this is a progressive wave. It is a sinusoidal one. Well, this is a cosine function, right? It's a sinusoidal wave.

Now we know if I divide by a, I get minus-B over a t. So it becomes of this form. So the phase velocity of this wave will be B over a. And this we call normally, by the letter c, that's the phase velocity of electromagnetic waves in vacuum. Or commonly known as speed of light. That identifies it so far, as best as we can in words. This is, as I say, a progressive sinusoidal electric field moving in the plus-x direction.

What are the a's and b's? By how much must you change x so that the wave gets the same amplitude as where you started? And the answer of that is that, of course, a must be 2 pi over lambda. because then if x changes by lambda, your cosine changes by 2 pi. So a in that equation must be 2 pi over lambda. That quantity is normally given the symbol k, it's called the wave number.

Similarly, if I look at the time turn, b must be equal to 2 pi divided by the period. Because if t changes by the period, then that cosine-- the angle of that cosine, the phase of that function-- changes by 2 pi. And you're back where you started. So b must be 2 pi over t. So this tells you for that particular wave what the a must be, what the b is. 2 pi over t is, of course, the same as 2 pi times the frequency, which we normally call the angular frequency, omega. So a is k, and b is omega.

Next. I said that any solution that is real of the electromagnetic field must satisfy Maxwell's equations. So the same must be true of this. If this is the wave of the electron, electric field, there must be associated with it a magnetic field such that all of Maxwell's equations are satisfied. In particular, if we take this one-- Faraday's law-- we know that the rate of change of the magnetic field must be equal to minus the curl of the electric field.

This you can look up in books of mathematics. If you look at all the components, the way I always remember it, it is the determinant where here you have the unit x direction yz. This is dx, dy, dz. And here is the x component of electric field, y component, and z component.

For our particular electric field, I only have the z component. And it's only a function of x. So most of the terms of this expansion are 0, except the one-- the rate of change of with x of Ez. And that will be in the y direction. So this db dt must be equal to that if that is a solution the Maxwell's equations. If I take the x derivative of E up there. I end up-- and you could almost do it in your head-- db dt is minus this quantity.

But if this is the rate of change of t, I can integrate this. And if I integrated it, B must be equal to-- the a comes from here, a minus a, a over-- sorry. The b comes from here, I misspoke. That comes out, and the integral of sine gives you cosine, so that must be satisfied. But since we integrated B, there will be a constant of integration. So if I add to this any constant B, this will still satisfy this equation.

All of this is telling me is that if I have that electric field-- propagating electric field-- I must simultaneously have this propagating magnetic field. And on top of that, I can have any constant magnetic field.

It means that is a more general situation where this electric field and these magnetic fields can exist with any constant B. I'll just call it 0. It's not an interesting part of this, it's not a propagating field. And so we end up that if you have that electric field propagating, and in with this magnetic field, then that system satisfies all Maxwell's equations. Both the E and B will satisfy these wave equations. Try it for yourself, and you'll see.

So the answer to this is, what this is, this is a polarized-- plane-polarized electromagnetic wave, where we identified the wavelength, the frequency, it's propagating in the x direction. And the electric field is polarized in the z direction. One of the things we will learn from this so we don't have to repeat over and over again when we're looking at different formulae, which describe ways to help us to identify , them is-- notice that what we have found was that the electric and the magnetic fields are perpendicular to each other. The electric field in the z direction, the magnetic in the y direction. But the sinusoidal part and the phase velocity and everything else-- wavelength, frequency-- are exactly the same and phase. This is completely in general.

If you have a progressive electromagnetic wave in vacuum, you find that the only way it can exist if you have simultaneously an electric and a magnetic field propagating. They are always at right angles to each other. This is the electric field, this will be the magnetic field. If it's propagating in that direction. It's always from e to b in a clockwise rotation, if they're propagating in that direction.

So I drew a general sketch here. This is true for any progressive wave, electromagnetic progressive wave. And you have the electric field, magnetic field perpendicular to it, and the two propagate in that direction, given by this vector equation. Furthermore, if they satisfy Maxwell's equation the ratio of E to B, the magnitude, is equal to c. This is completely general. It is worth remembering when we're analyzing different situations.

So that I went slowly through this, but that is one example where we see this mathematical description of something which we can recognize what it is, and which is a solution to Maxwell's equations in vacuum. What actually happens in the physical situation depends, as always, on all the boundary conditions, the initial conditions, et cetera. This doesn't address all those questions. All this says is this is one of the infinite possible solutions of Maxwell's equation. In other words, for electromagnetic fields corresponding to the plane wave propagating in one direction.

Let's take a harder example. The question is the following. Can we now do the opposite? Not someone tells us the equation. Can we actually describe in mathematical forms a electromagnetic wave whose properties we know what we want and would like to write it mathematically. And I took a slightly harder one, so I said we would like to describe both the electric and the magnetic fields, which describes a monochromatic electromagnetic wave-- monochromatic means a single frequency, single wavelength-- with wavelength lambda, which propagates now not along the x or y or z axes that makes life easy.

Let's say it goes at some angle. It goes at 45 degrees to the x-axis and y-axis. And the z is out of the board. So the wave-- we want the wave, which is propagating like this, where the wave front is-- let me come to it in a second-- where the vector perpendicular to the wave front is at 45 degrees to both the x-axis and y-axis.

We want it plane-polarized, meaning that the electric vector is always in a plane and it's linearly polarized so it's in the same direction in the x-y plane. So how can we translate that into mathematics? Well, we'll use some of the knowledge we've just gained before. First of all, we know from what I discussed about the electric and magnetic field being perpendicular to each other and perpendicular to the direction of propagation that if the propagation is in this direction, then we know that the plane in which the electric and magnetic fields find themselves are perpendicular to that.

Since this is propagating like this, the distance between the planes of equal phase will be lambda. That's the meaning of the wavelength. Once you've gone the distance of 1 lambda, the magnitude and direction is back to what it was before for both the electric and magnetic fields. So that's what it will look like. So the electric vector will be in this plane, but we are told furthermore it's in the xy, so it will be in this direction. If it's like this, and in this plane, so this must be the direction of the electric vector. So let's give it a magnitude E-zero.

And what is this unit vector? Well, clearly that is in the x direction. It has a component like this, and in the y direction, it has a component like that. The magnitude of the components is the same, because of the 45 degrees. But for the x, it'll be negative, and for the y, positive. So the unit vector in the direction of the electric vector will be minus x-hat over root-2 plus y-hat over root-2. This is a unit vector, you can check for yourself. If you take square this, square that, take the square root, you get 1. So this is a unit vector in this direction where we wanted it. So if I write this as the amplitude and the direction of the electric field, I do have a field which is linearly polarized always in the same direction.

We'll put a sine or cosine there, because we're talking about a monochromatic electromagnetic wave with the wavelengths, so it's a sinusoidal function. Where I put the sine or cosine or any other phase just determines where time equals 0. So let's put sine. It's going to be propagating in this direction, plus k, so these two will have opposite sign. This will be the frequency-- angular frequency-- of oscillations of this. And here we must describe a plane. Because along this plane, the phase has to be the same. That's what we mean by wavefront. Vectorially, how do we describe a plane?

Well, we will have the plane which is perpendicular to k if we take k dot product of the vector r. r is the vector. Here is the vector r, from the origin to a point on the plane which I want to describe. So this is k dot r. So this now, we'll have k, which is the wave number, and this whole thing is called the k vector, will have a magnitude which is 2 pi over lambda. Same as in the other problem. But now it's pointing in this direction, which again, by analogy, how we calculated that is the unit vector x over root-2 plus unit vector y over root-2. So this is k. r is nothing I want to describe this point. I have x in the x direction, y in the y direction, z in the z direction. So that describes any point on that plane. If I take the dot product between them, I will get then a wave which is moving the the k direction. And this describes the position on the wavefront.

So putting it all together, this electric field at every point of x, y, and t will have a magnitude is E-zero times this direction, the direction of polarization of the electric field, times sine. This is now telling me it's propagating in this direction. And with angular frequency omega. So that describes the electric part of this wave. How about the magnetic one?

Well, we could do the same as before. The magnetic part is determined by this, because all Maxwell's equations have to be satisfied, including Faraday's law. But I told you, so it saves me doing it over and over again, we've learned once and for all, for a progressive wave the e and b are perpendicular to each other, and the ratio between them is c. So since I know what E is, the magnitude of the magnetic field is E-zero over c. It'll be at right angles to this direction and to the propagation, and therefore it will be out of the board. So that from E-cross-B, the vectors are in the k direction. So the b will be out of the board, which is easier this time. That's in the z direction.

And it will be, as I said, exactly in phase in time and space with the electric field. The two are coupled together. So that now describes it entirely. So this is, in fact, the answer to our question. It describes an electric or magnetic field which is monochromatic. It's an electromagnetic wave. It has wavelength lambda. It propagates at 45 degrees to x and y axes, and is plane-polarized. e is always in the same direction and in the xy plane. So this is the answer.

See, notice. In the past when we were doing problems, we focus more on things like what is the wave equation for this string? Or for a pipe with a gas in it? Or a transmission line, et cetera. Here, even guessing what solutions we're interested in, what kind of solution, it's already hard or even to describe the wave we're interested in. So this, for the other situations, this would have taken a few minutes. Here it needs a fair amount of analysis. And it takes much longer.

Let me take one more case. The last case I'm going to exhibit is the following. Again the issue will be, there's this particular wave we want to produce. We know what we want, and we want to know how to describe it mathematically. So once again, we want to find a solution of our Maxwell's equations, which have the following property that correspond to a circularly polarized electromagnetic wave which is propagating in y direction. And it just says "any." so any, any circularly polarized electromagnetic wave which is propagating in the minus y direction.

First of all, what we mean by circularly polarized wave? A circularly polarized wave is that, if I took a snapshot, if I could, at a given instant of time, one would find that the electric vector along the propagation direction is rotating like this on the spiral. If that wave is moving towards you, what you would see in any plane, a rotating electric field. And associated with a magnetic field at right angles to it. It doesn't tell us whether we want a left-handed or a right-handed rotated field. So just arbitrarily take one.

And by the way, if ever you're interested in the left- and right-handed and figuring out which is which? It's a mess. Different communities use different definitions, what they mean by right- and left-handed. So I won't try to confuse you more than that. So here we want any wave, which corresponds to circular polarization, and is moving in the minus y direction. So if it's moving in plus or minus y direction, we know that the electric field will be in the xz plane at every instant of time. If it's circularly polarized, we know that the magnitude of the electric field at all locations of x, y, and z at all times will be the same. It does not change. It's a constant.

Now so how do we create such a thing? Well, if we stop and think for a second, if we superimpose two solutions-- suppose we have one solution, which is a plane-polarized electromagnetic wave going towards you, and I superimpose on that another one which is out of phase with it and at 90 degrees, then at every location in space, I'll have two components. If I make those components change, but in such a way that the vector addition of the two gives me a unit vector, a constant vector, I will have achieved what I wanted to do. So here is a equation which satisfies everything I've said.

Let's consider an electromagnetic wave which is the same in all x and all z positions. The only variable is in the y direction. If I write that as the superposition of an electric field which is in the x direction, and propagating as a sine-- it's a sinusoidal wave-- and I add to it a cosine, which is at right angles.

Furthermore I'll use the other information. It's going in the minus y direction. So I'll make these two opposite sign-- sorry, I make them the same sign, it is in minus-y direction. If it was in the plus-y, they would have opposite signs. If it's minus-y, this would have to be the same. So this is a sinusoidal wave moving in the minus-y direction. It'll have the wave number k, this is 2 pi over lambda. And this is 2 pi, the frequency or 2 pi over the period. Omega over k has to be c, the speed of electromagnetic waves.

If I add to this, the resultant electric vector everywhere in space has a magnitude E-zero. I can check it. The magnitude of E is the square root of the x component of this squared plus the z component of this squared. So it's E-zero, the x component squared-- the sine squared of this. The z component is the cosine, so the squared is that. For all values of x, y, and z at all times, if I add these and take the square root, I get 1. And so this is E-zero. So this propagating wave does satisfy my requirement that everywhere is magnitude E-zero. It is a propagating wave. Each one of these are propagating with the speed of light in the direction of y.

I'm sorry, forgive me, can't copy from one line to the next. This is plus, this is plus. All right. It's moving in the minus-y direction. The way I had it, it was going in the plus-y direction. I corrected it. This is in the minus-y direction. All right? And this is what was required. OK.

So this mathematical description of the electric vector, how it's propagating. And now we want to know what the magnetic one is doing. Well, again, we could go back and make sure that Maxwell's equations are completely satisfied. And you'll find that here, in order for Faraday's law to hold, I have to have also a changing magnetic field. But instead of doing that, I'll make use of what we learned by the previous examples.

We know that this is a superposition of two progressive waves. Each one of these is a solution of Maxwell's wave. I don't need both of them. I only needed both to get a circularly polarized wave. Each one of these has to satisfy Maxwell's equation. So associated with each of these components, I must have a magnetic field which satisfies the requirement that there is an electric vector and magnetic vector at right angle to each other moving together in the direction of propagation in phase and in time. So for each one of these, I will find the corresponding magnetic field, the magnitude will be E-zero over c, because we know that the ratio of the electric field to the magnetic field is always equal to c in vacuum.

It's at right angles. This was in the x direction, this is in the z direction. And in this case, then add this one. Here, this was plus-z and this is minus-x. And you can draw yourself a little picture to make sure you get everything right. Let me just talk about, say, this one. The second component. What I have in the [INAUDIBLE], this is moving there in minus-y. This component is in the z direction, so it's over here, coming out of the board. If it's in this direction, moving down here, then the b must be in that direction. So it must be in this direction, which is minus-x, which is correct. So this is how I get this right. If I add these, I get the total magnetic field. This, now, describes one possible wave which satisfies this requirement. It's a circularly polarized electromagnetic wave propagating in the minus-y direction.

OK, so let me stop at these examples of progressive waves, and I'll move over to standing waves. So let's continue in a second, thank you.

So I've now erased the board, and I can continue talking about wave solutions to Maxwell's equations. But let's recap for a second. What we find is the following, that basically in vacuum at every location in space it's as if there was an oscillator. It can be displaced from equilibrium. It can be made to oscillate.

Displacement from equilibrium means there is an electric field there, or there is a magnetic field there. These can oscillate. They don't have to oscillate. So for example, you could have a static field, just an electric field constant in time everywhere in space. That means every location space is displaced from equilibrium. There could be a constant magnetic field instead, or both constant.

Imagine how complicated this is. At every location the direction of this displacement from equilibrium for the electric and magnetic fields, they are vectors. There are possibility of the electric field facing a different directions of the magnetic field. What we find is that whatever that combination is in space and time, that combination has to satisfy Maxwell's equations. That completely describes what happens in vacuum at every point in space and time.

Now there are in particular combinations of these displacements of oscillations in space and time, which satisfy the wave equation for the electric and magnetic fields. It's a tiny subset of total, but there are such. And we are considering now for that tiny subset what kind of solutions exist, how to describe them. And even there, we're limiting ourselves to a tiny subset of a tiny subset.

So far, I took the subset where this displacement from equilibrium of the electric and magnetic fields is a progressive wave. And what we found, in order to make sure that the Maxwell's equations are satisfied, you can't have any old electric field wave, or any old magnetic field wave. There's an interplay. There is, in reality, just one electromagnetic field, and that propagates.

We'll now go and look for other solutions of these equations. And very interesting solutions are standing waves. So let me take a concrete example and discuss it. So here is, you could call it a problem. Suppose that I have everywhere in space an electric field which consists of a standing wave. You can recognize this when we were talking about standing waves on strings, for example. Where you have the electric field always pointing in the x direction. It's oscillating at every point in space with the same frequency and phase, cosine omega t. It's oscillating with that angular frequency. And spatially, it not change in the x and y direction, but it does in the z direction. And that is a cosine like this.

So this is a standing wave of electric field. This by itself cannot be a solution. Is not a situation you can have in vacuum. It violates, by itself, Maxwell's equation. If you look at them, you find that in order for this to satisfy Maxwell's equation, the must be associated with it a magnetic field that looks like that. And so the question, the first thing is, show that if you have this, you must also have this present.

The second part is some more discussion about when you have these two present, when you have a standing wave in vacuum of electromagnetic waves, for example, then what is the energy density? You know, in an electric field or a magnetic field, if you have in space, if you take any value inside the volume, there will be energy.

And the energy per unit volume per cubic meter is the energy density. So we're going to calculate how much energy density there is in this standing wave.

And another quantity, which is for practical reasons very important, is when you have an electric and magnetic fields present, actually energy flows through that system. And the amount of energy per unit area that flows-- per unit area perpendicular to the direction of flow-- is called the Poynting vector. And by the way, the Poynting has nothing to do with a vector that points, it's to do with a gentleman by the name of Poynting, after which this was called.

So the second part of the problem is, once we found a standing wave that satisfies everything possible [INAUDIBLE] in vacuum, for this particular case what is the energy density, the magnetic and electric fields, and what's the Poynting vector? OK, so how do we do this?

We know what the electric field is doing, it's the standing wave. We know that it must satisfy all Maxwell's equations, in particular Faraday's law. As before, we can calculate the curl of the electric field. Now here, the electric field is only in the x direction. And it's a function of z. And so the curl of this, to be only just one component of that, and that is given by this quantity. So this is minus the curl of this E. And we know by Faraday's law that this must equal to the rate of change of the magnetic field at that place of x, y, and z.

Now I can integrate this equation, and find what B is at every point in space and every time. And that's easy enough. We just have to integrate that, which gives you the sine here, and the omega comes down, and you get this. Whenever you integrate, there is a constant. All it's telling us is that I can satisfy Maxwell's equations not only with an oscillating electric field present with an oscillating magnetic field, but I can always add a constant magnetic field throughout space. I could have also added a constant electric field. So there's an infinite number of solutions I can superimpose. I'm not interested in them. I am interested in the standing wave, the time-dependent part. So might as well make that 0.

And so we are essentially home. We have found that the magnetic field is also a standing wave. And this, by the way, we look at the problem, is what we were asked to prove. So we have proven the first part, that if this is the description of the standing wave of the electric field, then there must be corresponding a standing wave magnetic field.

So the two-- but notice, unlike in the case of progressive waves, where in the progressive waves, wherever you had an electric field, the magnetic field was at right angle to it and in magnitude proportional to the electric field and in phase with it, et cetera. Here, they're not. Here, the electric field, when this is cosine omega t, this is sine omega t. When this is cosine kz, this is sine kz. These two are out of phase with each other, both in time and in space. I've tried to sketch it here, it's not very good sketch, but anyway.

Suppose at some instant of time, if I look at these, at some instant of time, the electric vector-- the magnitude of it-- is represented by this curve. And it is in the x direction. So the electric vector is this, like this, and like that. If this is the maximum, it is, the magnetic field at that time will be 0, if I look at these equations. So there'll be no magnetic field. Over this distance in space, there will be the electric field up here, down here, and no magnetic field.

Later on, half a period later, what you find is that when this comes to 0-- it's a quarter period-- when this comes to 0, the electric field is 0, there will be a magnetic field at its maximum. But it will not be this shape. It will be, first of all, pointing in the y direction. This is in the x, it will be the y direction. It's maximum will be in the middle, well here it was always 0. And these two oscillate. It's a standing wave. The B does this, and the E does this, all in the same place. But both in space and time, the two are out of phase with each other. Completely different solution. And both progressive waves satisfy Maxwell's equations, and the standard waves.

So it's important to realize there is this difference, often it's easy to get confused about it. In a progressive wave, the electric and magnetic fields are right angle. And as if they were locked together, and they move forward like this. On the other hand, in a standing situation, they're still at right angle to the other. But when one is a maximum, the other's a minimum. When this one is-- They're out of phase with each other in both space and time.

So that's the first part. And the next part we were asked, now for this standing wave, imagine this could be inside your microwave oven. Inside the microwave oven, there is a standing wave. Unless they specially make it so it moves a little bit in space so you cook your meat everywhere. But then the cheapo microwave oven, you have a stationary standing wave. And suppose this is it.

At every place in space, there is an energy density which actually fluctuates, goes up and down in time and is different in every location. Let's calculate that. Well, as Professor Walter Lewin showed, the energy density in an electric field, whether it's changing with time or not, if I've got in space somewhere an electric field e, at that location, I have an energy density. The amount is 1 over epsilon-zero times the magnitude of the electric field squared. That is the energy density of an electric field. It is not a vector. This is E-squared, the square of the magnitude of the electric field energy is a scalar quantity. So not surprising, this is not a vector, it's a scalar quantity.

I can now immediately go over to what we know. We know the electric field, we know the magnetic field. So I can replace E-squared by what it is at every location. At every position z and every x, y. At all times. And this is the energy density. You can see it does oscillate, but there's always [INAUDIBLE].

How about the magnetic field? The magnetic field also has energy. If I take it anywhere, suppose you have a bar magnet, one of these pocket magnets, you hold it, and there's a magnetic field all around the magnet. Take any cubic meter of the volume, you'll find this amount of energy. It's 1 over 2 Mu 0 times the magnitude of the magnetic field squared. Again, I know what B is in for my standing, wave so I can calculate it, and I get this answer. So these are the two energy densities.

Now what one finds, if one does-- if you plot this, or thinks about it-- that in this standing wave, you find that that energy moves backwards and forwards. At any location in space, I can calculate how much energy is moving per second per square meter-- per unit area-- perpendicular to the direction of motion of that energy. And that is what is called the Poynting vector. If you think, for example, suppose you take an electromagnetic wave like light shining that the wall. It'll warm up to the wall, I mean there's heat being transmitted, there's energy comes over. At any instant of time, how much energy per unit area is hitting the wall? It will be equal to the Poynting vector at that instant of time. And the Poynting vector s is E-cross-B over Mu 0.

By the way, this applies to any electric and magnetic fields, not necessarily for progressive waves or standing waves, et cetera. It's something we want to think about and this is very surprising. Even if you have static electric and magnetic fields which are not parallel to each other, so that this is not 0, there is a flow of energy. It's something we want to think about.

But in our case, E and B are perpendicular to each other. The electric field everywhere was in the x direction, the magnetic in the y direction. And so they are right angles, so it's just the x component of E and the y component of B. Well, they're the only components that are there. So it's 1 over Mu 0. E x times B y in the z direction, so this if E and B are perpendicular to each other, z is perpendicular to both of those, which is in the z direction. If I calculate this for this, I get this equation. And I can rewrite it. And I find that the energy is some constant, goes in the z direction, and this looks like sine 2 omega t times sine 2 kz.

Going back to our diagram, what this looks like is that-- if you remember that E oscillates, it's a maximum here, maximum here, and it oscillates up and down, up and down, like this. B is a maximum in the middle, and that's going like this. The product of the two, it'll be 0 here, because B is always 0 here. It'll be here because E is always 0. And you cross B there for 0. So here, here, and here is going to be 0. And if you look at that function, its actually a function which has twice the frequency of the electric field oscillations or the magnetic field oscillations, and also half the wavelength. And you will find that the maximum is somewhere here.

So if you look at where the maximum transfer of energy is, it's at the quarter and 3/4 location. And so it's consistent with this picture. Energy is doing this in that situation. And so that answers what they were ask. This is the Poynting vector as a function of-- for all positions in space as a function of time. This is the energy, the electric and magnetic field, and we found the magnetic field corresponding to the electric field.

So this is another example of a possible solution to Maxwell's equations, this time corresponding to standing waves. As I mentioned before, I'm repeating myself , there are infinite possibilities of solutions of Maxwell's equations. So to cover them all makes no sense. What is important, that one gets a good understanding of the interesting situations. Interesting situations are some static solutions to, say, magnetic fields if you need special magnets. Or if you have a progressive wave, like light, or standing waves, like in the microwave, for example. And so I've taken two cases here. First progressive wave solution. And then standing wave solution. And from this, we will later go on to some other problems. Thank you.