NETWORK STRUCTURES FOR FIR SYSTEMS AND PARAMETER QUANTIZATION EFFECTS IN DIGITAL FILTER STRUCTURES

Solution 13.1

(i) Since H(z) has only real zeros we will use only first-order sections in the cascade form. Then Figure S13.1-1 represents one possible ordering for these sections.

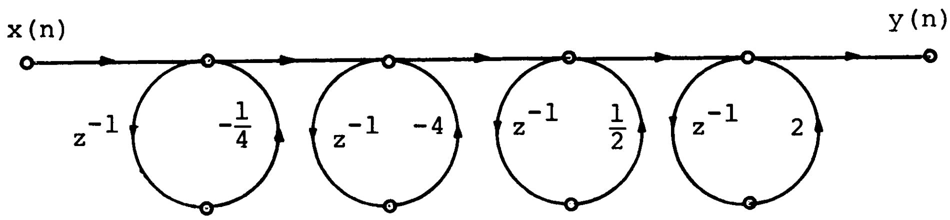

Figure S13.1-1

(ii) For the direct form we first express H(z) as

$$H(z) = 1 - \frac{7}{4}z^{-1} - \frac{69}{8} z^{-2} - \frac{7}{4} z^{-3} + z^{-4}$$

The direct-form structure is then as shown in Figure S13.1-2

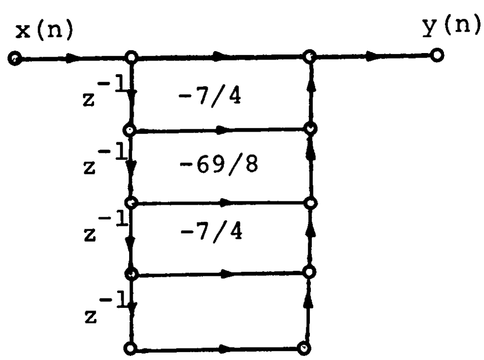

Figure 13.1-2

(iii) Since the unit-sample response is symmetrical the filter is in fact linear phase. The linear phase form is as shown in Figure S13.1-3.

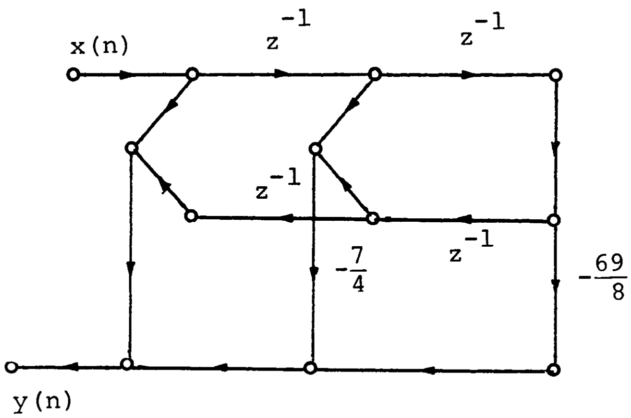

Figure S13.1-3

(iv) The coefficients in the frequency-sampling structure are the samples of the frequency response equally spaced in frequency. To evaluate these frequency samples it is convenient to rewrite H(z) in the form

$$H(z) = z^{-2}\left[z^2 - \frac{7}{4}z - \frac{69}{8} - \frac{7}{4}z^{-1} + z^{-2}\right]$$

or

$$H(z) = z^{-2}\left[(z^2+z^{-2}) - \frac{7}{4}(z+z^{-1}) - \frac{69}{8}\right].$$

Then

$$\tilde{H}(0) = H(z)\Big|_{z=1} = -\frac{81}{8}$$

$$\tilde{H}(1) = H(z)\Big|_{z=e^{j\frac{2\pi}{5}}} = e^{-j\frac{4\pi}{5}}\left(2\cos\frac{4\pi}{5} - \frac{7}{2}\cos\frac{2\pi}{5} - \frac{69}{8}\right)$$

$$\tilde{H}(2) = H(z)\Big|_{z=e^{j\frac{4\pi}{5}}} = e^{-j\frac{8\pi}{5}}\left[2\cos\frac{8\pi}{5} - \frac{7}{2}\cos\frac{4\pi}{5} - \frac{69}{8}\right]$$

Because of the conjugate symmetry of the Fourier transform,

$$\tilde{H}(3) = \tilde{H}^*(5-3) = \tilde{H}^*(2) = e^{j\frac{8\pi}{5}}\left[2\cos\frac{8\pi}{5} - \frac{7}{2}\cos\frac{4\pi}{5} - \frac{69}{8}\right]$$

$$\tilde{H}(4) = \tilde{H}^*(5-4) = \tilde{H}^*(1) = e^{j\frac{4\pi}{5}}\left[2\cos\frac{4\pi}{5} - \frac{7}{2}\cos\frac{2\pi}{5} - \frac{69}{8}\right] .$$

The frequency sampling structure, in the form of chalkboard (b) lecture 13, is then as shown below:

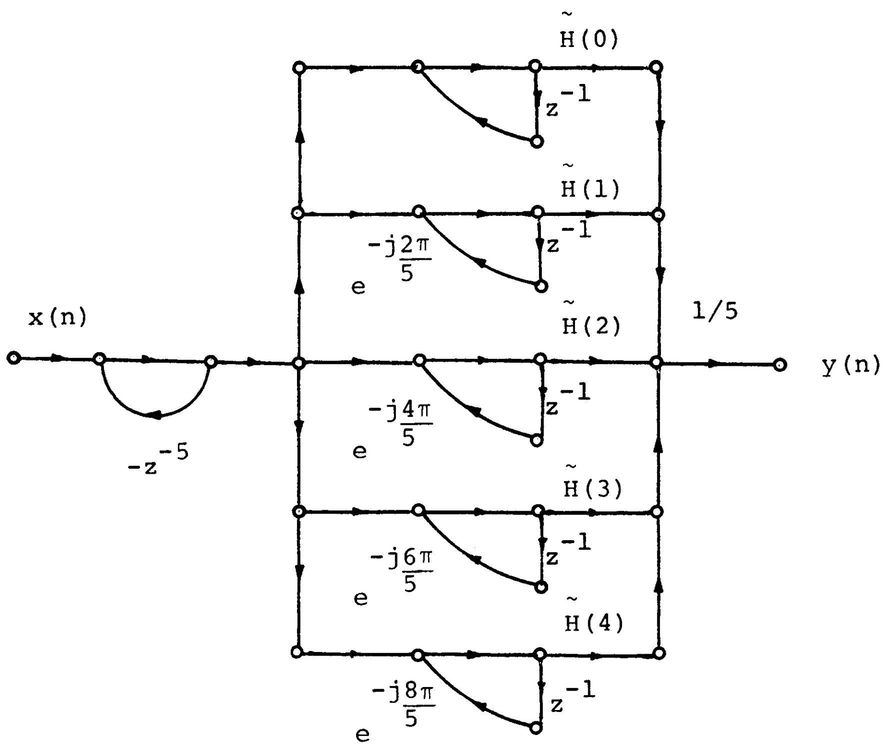

Figure S13.1-4

In this form the sections involve complex coefficients. By utilizing the conjugate symmetry of the frequency samples the network can be rearranged in terms of second-order sections with real coefficients.

Specifically the transfer function corresponding to the recursive part of the network of Figure S13.1-4 is given by

$$G(z) = \frac{\tilde{H}(0)}{1-z^{-1}} + \left[\frac{\tilde{H}(1)}{1-e^{-j\frac{2\pi}{5}}z^{-1}} + \frac{\tilde{H}(4)}{1-e^{-j\frac{8\pi}{5}}z^{-1}}\right] + \left[\frac{\tilde{H}(2)}{1-e^{-j\frac{4\pi}{5}}} + \frac{\tilde{H}(3)}{1-e^{-j\frac{6\pi}{5}}}\right]$$

The terms paired in brackets are complex conjugates, i.e.,

$$\tilde{H}(1) = \tilde{H}^*(4) \text{ and } e^{-j\frac{8\pi}{5}} = e^{j2\pi}\, e^{-j\frac{8\pi}{5}} = e^{j\frac{2\pi}{5}}$$

$$\tilde{H}(2) = \tilde{H}^*(3) \text{ and } e^{-j\frac{6\pi}{5}} = e^{j\frac{4\pi}{5}} \quad .$$

Continuing these complex conjugate terms, G(z) can be expressed as

$$G(z) = \frac{\tilde{H}(0)}{1-z^{-1}} + \frac{2\mathrm{Re}[\tilde{H}(1)]-2z^{-1}\mathrm{Re}[\tilde{H}(1)e^{j\frac{2\pi}{5}}]}{1-2z^{-1}\cos\frac{2\pi}{5} + z^{-2}}$$

$$+ \frac{2\mathrm{Re}[\tilde{H}(2)]-2z^{-1}\mathrm{Re}[\tilde{H}(2)e^{j\frac{4\pi}{5}}]}{1-2z^{-1}\cos\frac{4\pi}{5} + z^{-2}}$$

leading to the structure shown in Figure S13.1-5.

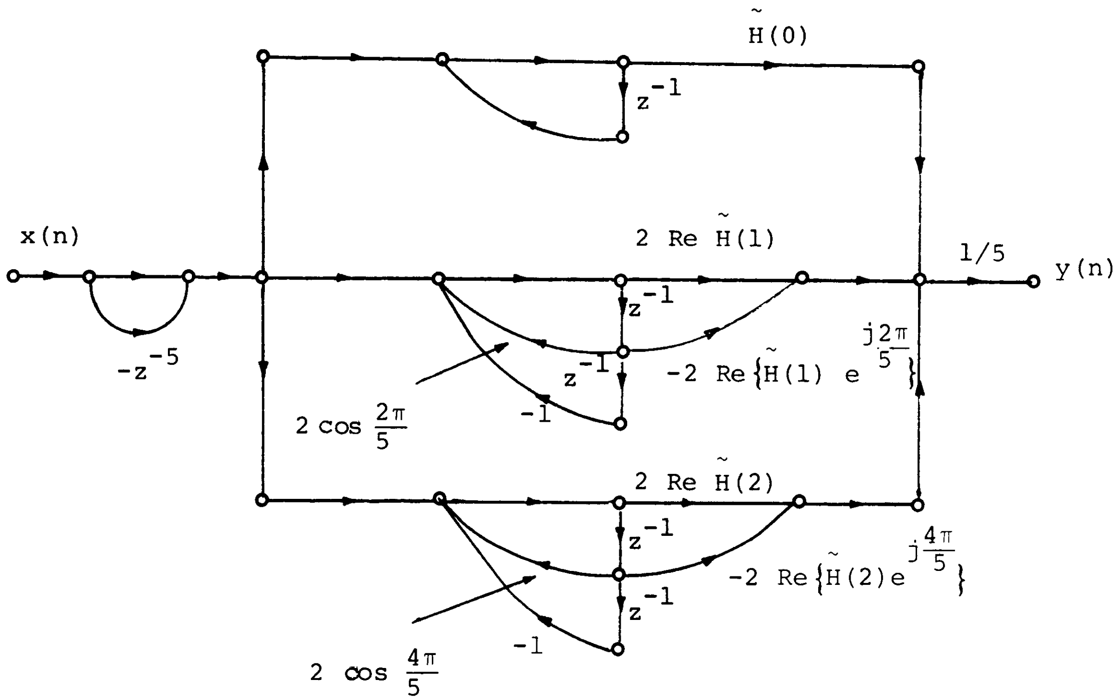

Figure S13.1-5

Solution 13.2

(a) The form of the desired transfer function is easily obtained by expressing h(n) in the form

$$h(n) = \frac{A}{2}\,[e^{j\phi}e^{j\omega_0 n}u(n) + e^{-j\phi}e^{-j\omega_0 n}u(n)]$$

Thus

$$H(z) = \frac{A}{2}\left[\frac{e^{j\phi}}{1-e^{j\omega_0}z^{-1}} + \frac{e^{-j\phi}}{1-e^{-j\omega_0}z^{-1}}\right] = A\left[\frac{\cos\phi - z^{-1}\cos(\omega_0-\phi)}{1-2z^{-1}\cos\omega_0+z^{-2}}\right]$$

Since the direct form filter does not contain any zeros, A and ϕ must be such that A $\cos\phi = 1$ and $\cos(\omega_0-\phi)=0$. Then for $\omega_0=\frac{\pi}{4}$ H(z) is

$$H(z) = \frac{1}{1-\sqrt{2}\,z^{-1} + z^{-2}}$$

Thus,

$$a_1 = \sqrt{2}$$

$$b_1 = -1$$

(b) The form of the transfer function for the coupled form network is (see Problem 11.3)

$$H_2(z) = b_2 z^{-1}/[1-2a_2 z^{-1} + (a_2^2+b_2^2)z^{-2}]$$

Thus to obtain a unit-sample response of the desired form, $A \cos\phi = 0$ and $A \cos(\omega_0-\phi)=-1$. The coefficients a_2 and b_2 are given by

$$a_2 = \cos\omega_0 = \frac{\sqrt{2}}{2}$$

$$b_2 = \sin\omega_0 = \frac{\sqrt{2}}{2}$$

(c) For the direct form filter since the coefficient b_1 is unity, it is not affected by quantization. For the coefficient a_1, the quantized value is

$$\hat{a}_1 = \frac{11}{8} = 1.375$$

The resulting transfer function is

$$H_1(z) = \frac{1}{1 - \frac{11}{8} z^{-1}+z^{-2}} \quad .$$

Since this is still in the form of the desired H(z), the unit-sample response will still be of the form of Eq. (13.2-1). With the quantized coefficient, $2 \cos\omega_0$ is now equal to 11/8 so that $\omega_0 \cong .26\pi$

For the coupled form, the quantized coefficients are given by

$\hat{a}_2 = \hat{b}_2 = \frac{5}{8} = 0.625$

Thus $H_2(z)$ with quantized coefficients becomes

$$H_2(z) = {}^{(5/8)z^{-1}}/[1 - \frac{5}{4}z^{-1} + \frac{25}{32}z^{-2}] \ .$$

In comparing this with the desired H(z) we note that since the coefficient of z^{-2} is not unity, the resulting unit-sample response will not be of the desired form. In particular, the unit-sample response will be of the form of a damped sinusoidal sequence, corresponding to the fact that the poles of the coupled form system with quantized coefficients have moved inside the unit circle. In contrast, the poles of the direct form system with quantized coefficients remain on the unit circle but are displaced in angle. These results are of course consistent with the differences in the quantization grids for the two structures, as illustrated in Figures 6.49 and 6.51 of the text.

Solution 13.3

(a) $$\tilde{H}(k) = \sum_{n=0}^{N-1} h(n)\ W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)\ W_N^{nk} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n)\ W_N^{(N-1-n)k}$$

which, because of the symmetry constraint becomes

$$\tilde{H}(k) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\ [W_N^{nk} + W_N^{(N-1-n)k}]$$

For $k = \frac{N}{2}$,

$$\tilde{H}(\tfrac{N}{2}) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\ [e^{-j\pi n} + e^{-j\pi(N-1-n)}]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)\ \underbrace{[e^{-j\pi n} - e^{j\pi n}]}_{0}$$

$$= 0\ .$$

(b) $$\tilde{H}(k) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\ [W_N^{nk} + W_N^{(N-1-n)k}]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)\ W_N^{(\frac{N-1}{2})k}\ [W_N^{k(n-\frac{N-1}{2})} + W_N^{-k(n-\frac{N-1}{2})}]$$

$$= W_N^{(\frac{N-1}{2})k} \sum_{n=0}^{\frac{N}{2}-1} 2h(n)\ \cos[\tfrac{2\pi k}{N}(n-\tfrac{N-1}{2})]\ .$$

The summation in the above expression is real. Let us assume for convenience that it is also positive. If it is not, then for those values of k for which it is negative, an additional phase of π will be added. Then, with this assumption,

$$e^{j\theta(k)} = W_N^{(\frac{N-1}{2})k}$$

or

$$\theta(k) = -\frac{\pi}{N}(N-1)k = -\pi k + \frac{\pi k}{N}$$

(c) $$H_k(z) = \frac{\cos(\frac{\pi k}{N} - \pi k) - z^{-1}\cos(\frac{-\pi k}{N} - \pi k)}{1-2z^{-1}\cos(\frac{2\pi k}{N}) + z^{-2}}$$

$$= \frac{(-1)^k \cos\frac{\pi k}{N}(1-z^{-1})}{1-2z^{-1}\cos(\frac{2\pi k}{N}) + z^{-2}} \quad .$$

Thus, from Equation (4.49) of the text.

$$H(z) = \frac{1-z^{-N}}{N}\left[\sum_{k=1}^{\frac{N}{2}-1} \frac{(-1)^k |\tilde{H}(k)| 2\cos(\pi k/N)(1-z^{-1})}{1-2z^{-1}\cos(2\pi k/N) + z^{-2}} + \frac{\tilde{H}(0)}{1-z^{-1}}\right] .$$

(d)

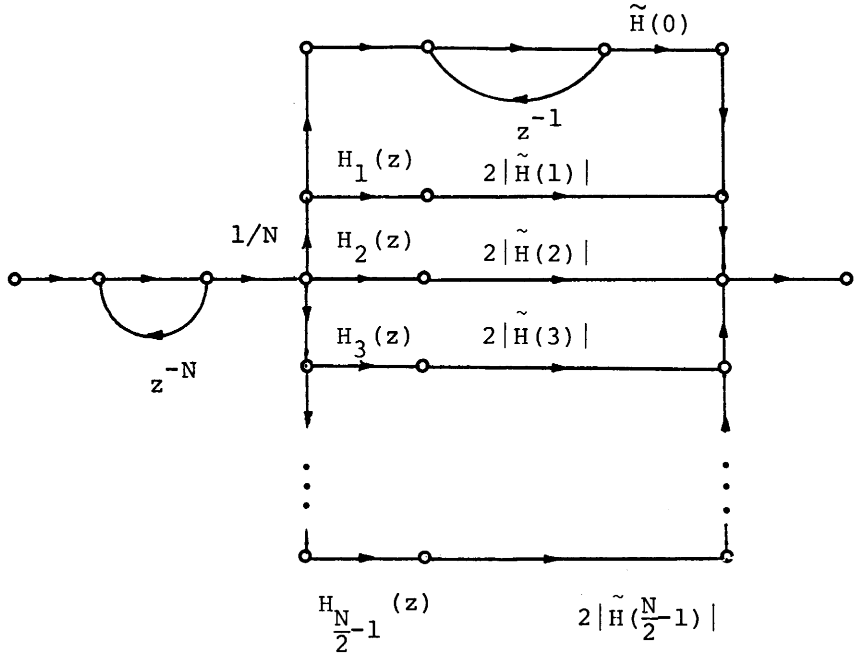

Figure S13.3-1

where the subnetworks $H_k(z)$ are of the form

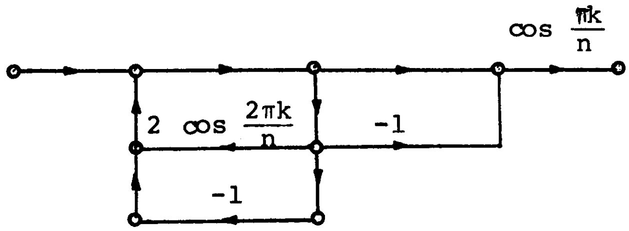

Figure S13.3-2