15

Discrete-Time Modulation

The modulation property is basically the same for continuous-time and discrete-time signals. The principal difference is that since for discrete-time signals the Fourier transform is a periodic function of frequency, the convolution of the spectra resulting from multiplication of the sequences is a periodic convolution rather than a linear convolution.

While the mathematics is very similar, the applications are somewhat different. In continuous time, modulation plays a major role in communications systems for transmission of signals over various types of channels. That application usually is inherently a continuous-time application. However, in many modern communication systems, signals may go through various stages and types of modulation as they move from one channel to another, and often this conversion from one modulation system to another is best implemented digitally. In their digital form the signals are discrete-time signals, and such transmodulation systems are based on modulation properties associated with discrete-time signals.

In addition to digital modulation systems, the concepts of discrete-time modulation (and, for that matter, continuous-time modulation also) are useful in the context of filtering, particularly when it is of interest to implement filters with a variable center frequency. It is often simpler in such situations to implement a fixed filter (either continuous time or discrete time) and through modulation shift the signal spectrum in relation to the fixed filter center frequency rather than shifting the filter center frequency in relation to the signal. For discrete-time signals, for example, from the modulation property it follows that multiplying a signal by $(-1)^n$ has the effect of interchanging the high and low frequencies. Consequently, by alternating the algebraic sign of the input signal, processing with a lowpass filter, and then alternating the algebraic sign of the output signal, a highpass filter can be implemented.

In discussing continuous-time modulation in Lecture 13 and discrete-time modulation in the first part of this lecture, the emphasis is on a carrier signal that is a complex exponential or sinusoidal signal. Another important and useful class of carrier signals is periodic pulse trains that are constant for some fraction of the period and zero for the remainder. In effect, then, either in continuous time or discrete time, modulation with such a pulse train consists of extracting time slices of the modulating signal. In data representation or transmission, this permits, for example, a type of multiplexing referred to as time division multiplexing since during the "off" part of the pulse train, time slices from signals in other channels can be inserted. Somewhat amazingly, the original modulating signal can be recovered exactly after pulse-amplitude modulation provided only that the fundamental frequency of the carrier pulse train is greater than twice the highest frequency in the modulating signal. The modulating signal can then theoretically be recovered exactly by filtering the pulse-amplitude-modulated signal with an ideal lowpass filter. Furthermore, this ability to exactly reconstruct the original signal is independent of the "duty cycle" of the carrier, i.e., it is theoretically possible no matter how narrow the "on" time of the pulse train is made. If for the continuous-time case, the "on" time of the carrier pulse train is made arbitrarily small, with the amplitude increasing proportionately, the carrier then corresponds to an impulse train. For discrete time, the pulse train with the smallest "on" time would likewise correspond to a periodic train of impulses or unit samples. In both cases, then, modulation with the impulse train carrier would correspond to sampling the modulating (input) signal. This leads to an extremely important concept, referred to as the *sampling theorem.* The sampling theorem states that a bandlimited signal can be exactly reconstructed from equally spaced time samples provided that the fundamental frequency of the sampler (i.e., the impulse train carrier) is greater than twice the highest frequency in the signal to be reconstructed. This fundamental and important result, to be explored further in Lecture 16, provides a major bridge between continuous-time and discrete-time signals and systems.

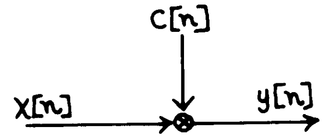
Suggested Reading

Section 7.5, Discrete-Time Amplitude Modulation, pages 473–479

Section 7.4, Pulse Amplitude Modulation and Time-Division Multiplexing, pages 469–473

MARKERBOARD 15.1

- Complex exponential Carrier
- Sinusoidal Carrier
- Synchronous modulation
- Asynchronous modulation
- Single Sideband



$$x[n]c[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\int_{2\pi} X(\theta)C(\Omega-\theta)\,d\theta$$

Complex exponential

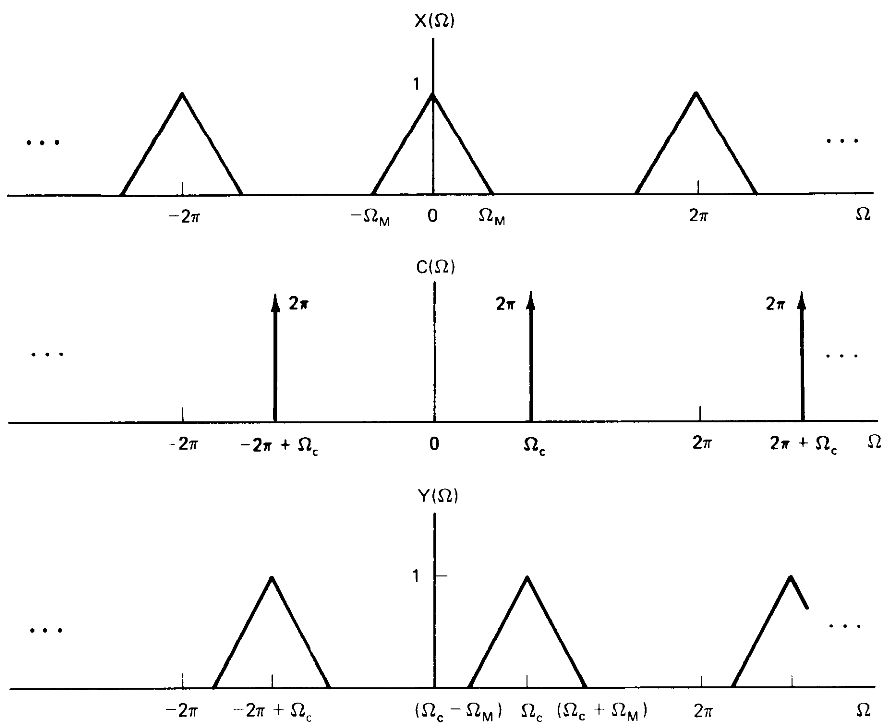
$$c[n]=e^{j(\Omega_c n+\theta_c)}$$

Sinusoidal Carrier

$$c[n]=\cos(\Omega_c n+\theta_c)$$

Pulse Carrier

$$x[n]\ c[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\int_{2\pi} X(\theta)\, C(\Omega-\theta)\, d\theta$$

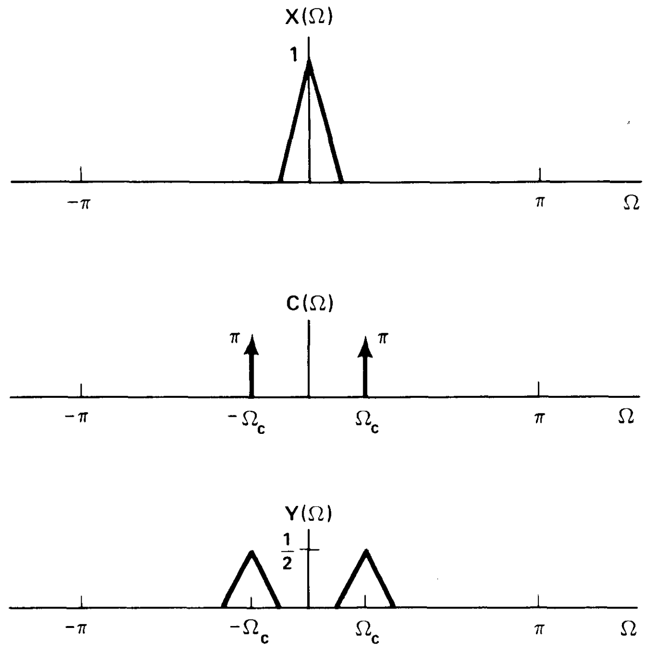


TRANSPARENCY 15.1
Spectra associated with discrete-time amplitude modulation with a complex exponential carrier.

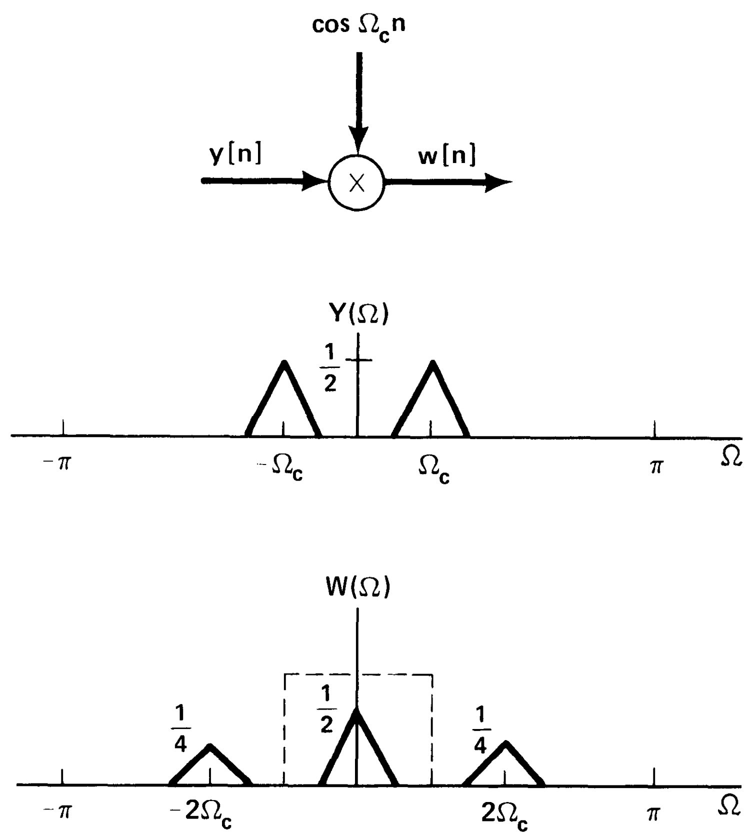
TRANSPARENCY 15.2
Spectra associated with discrete-time amplitude modulation with a sinusoidal carrier.

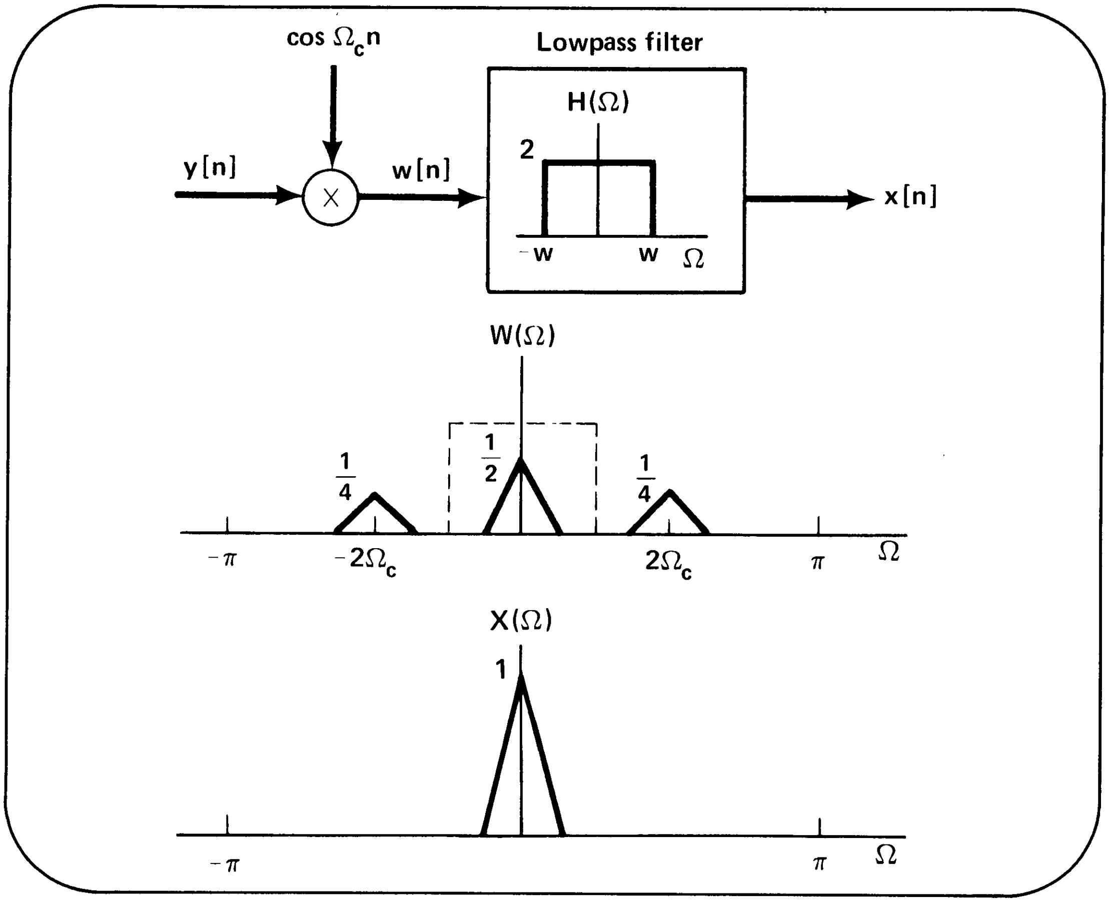
$$c[n] = \cos \Omega_c n$$

$$= \frac{1}{2} e^{j\Omega_c n} + \frac{1}{2} e^{-j\Omega_c n}$$

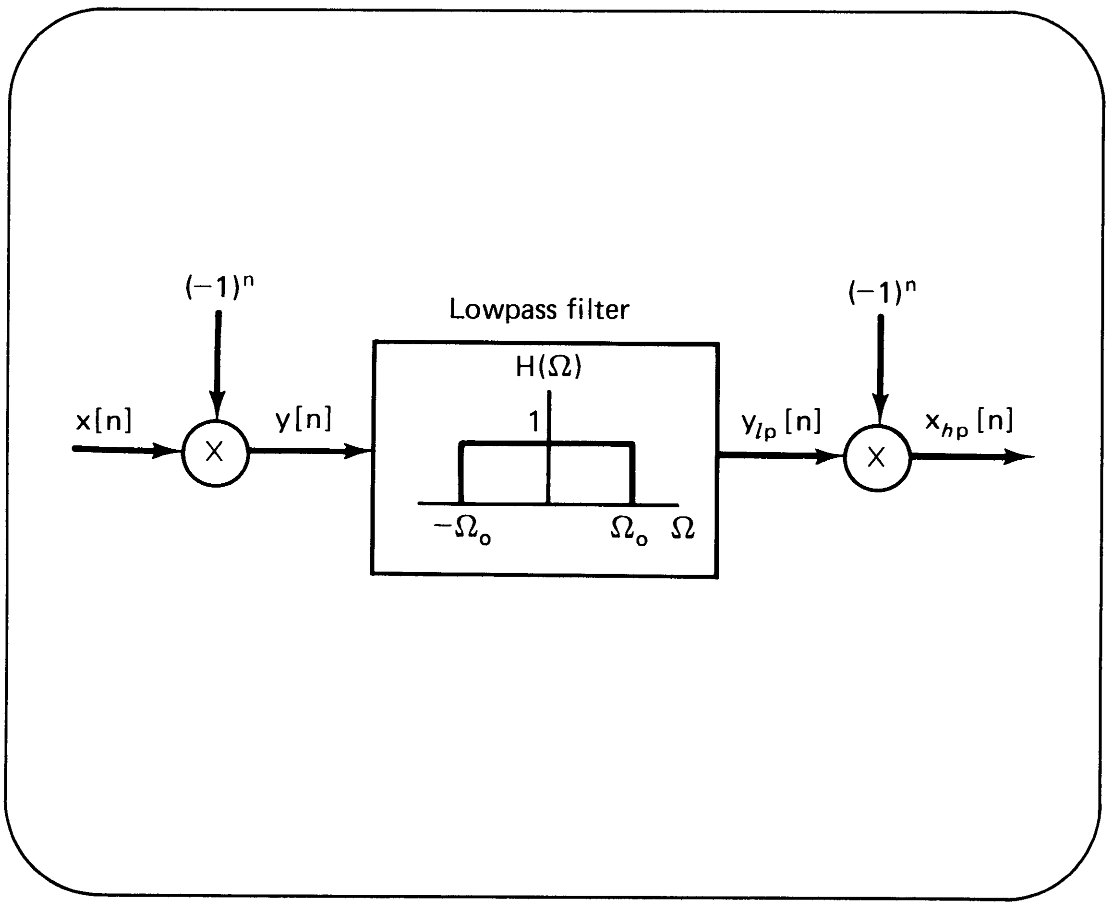


TRANSPARENCY 15.3
Spectra associated with demodulation of an amplitude-modulated signal with a sinusoidal carrier.



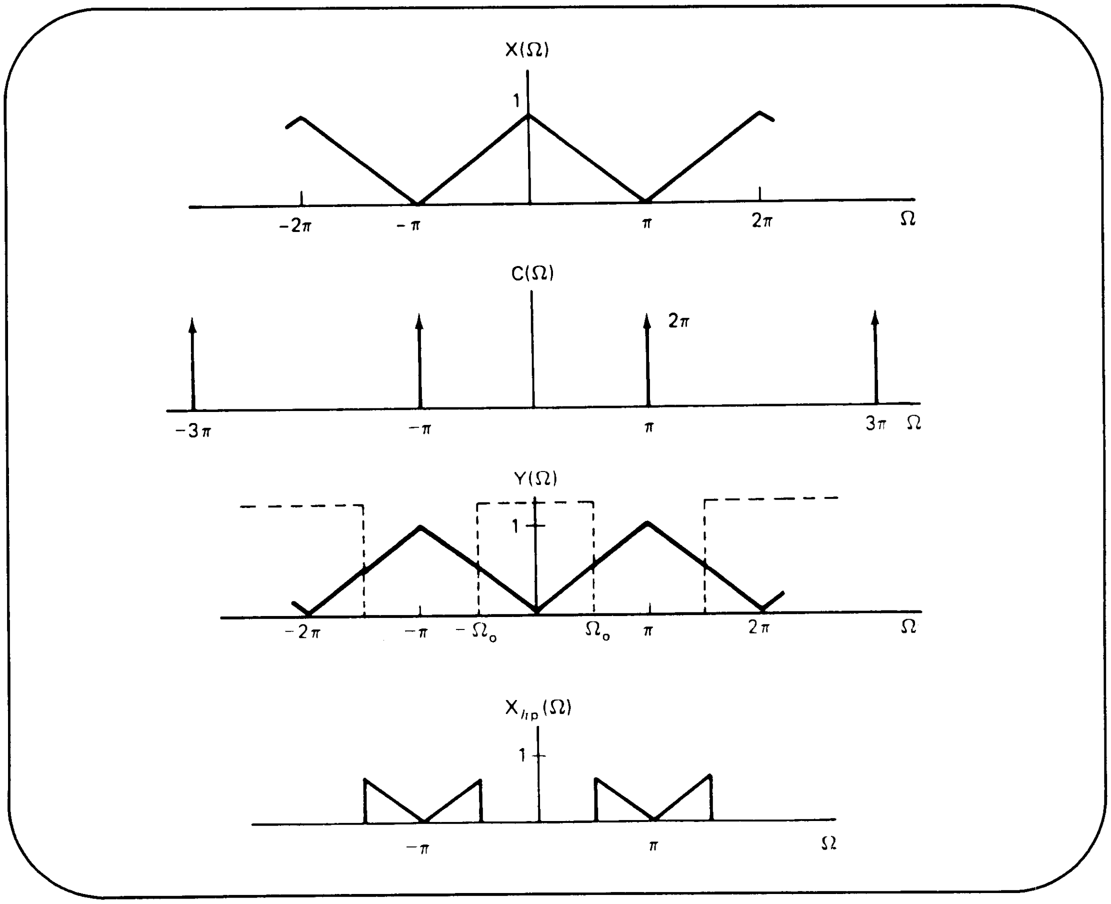


TRANSPARENCY 15.4
System and spectra for sinusoidal amplitude demodulation.

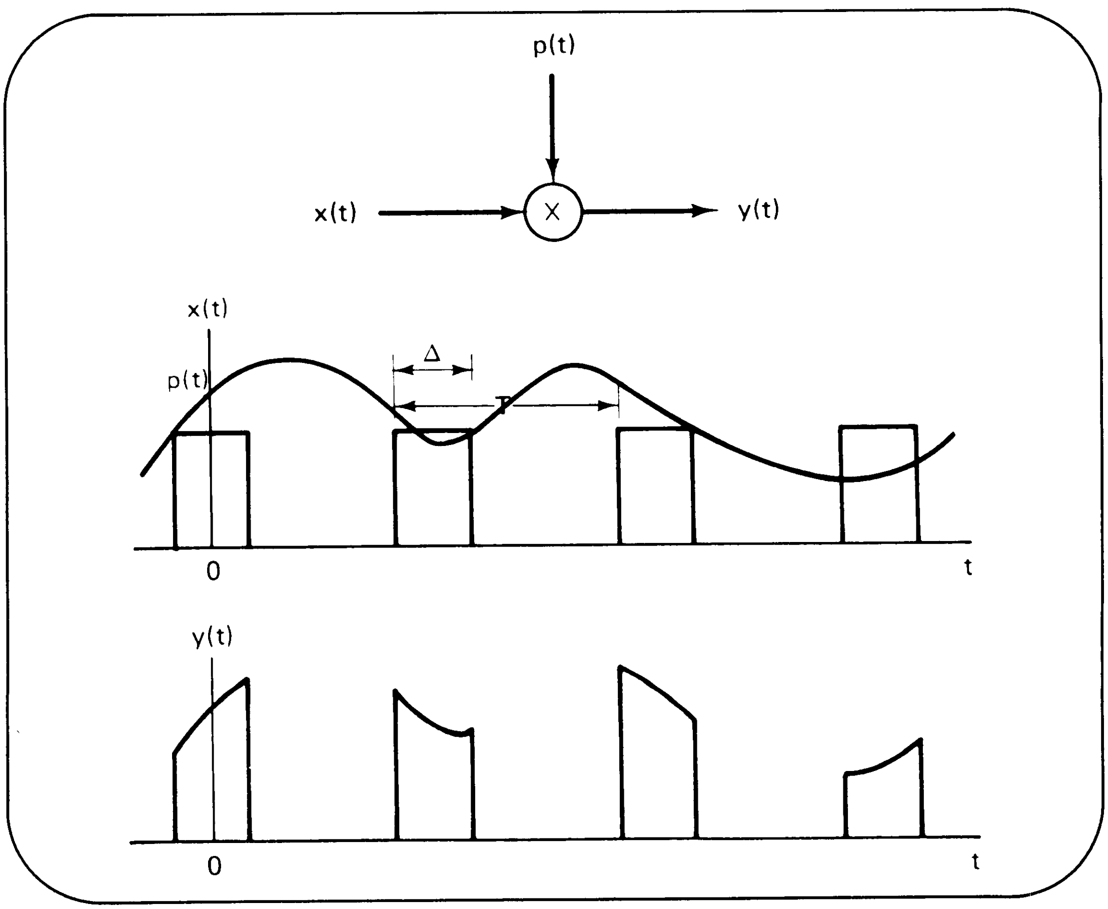


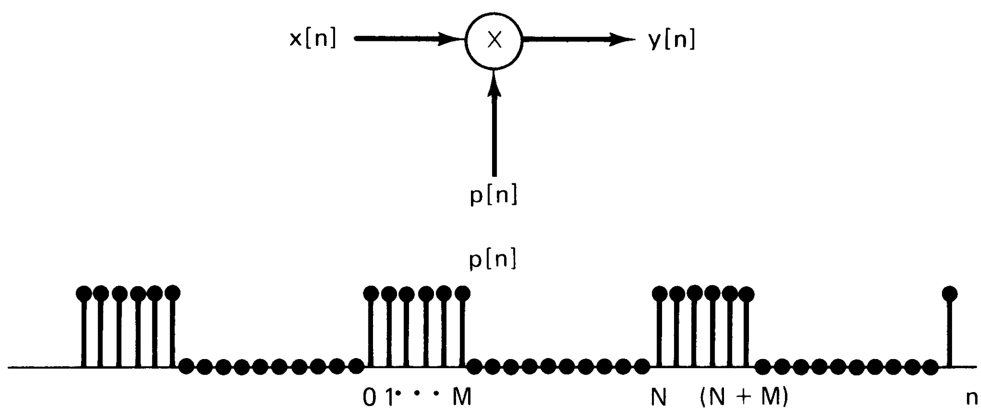
TRANSPARENCY 15.5
The use of amplitude modulation to implement highpass filtering with a lowpass filter.

TRANSPARENCY 15.6
Spectra associated with the use of modulation and demodulation to implement highpass filtering using a lowpass filter.

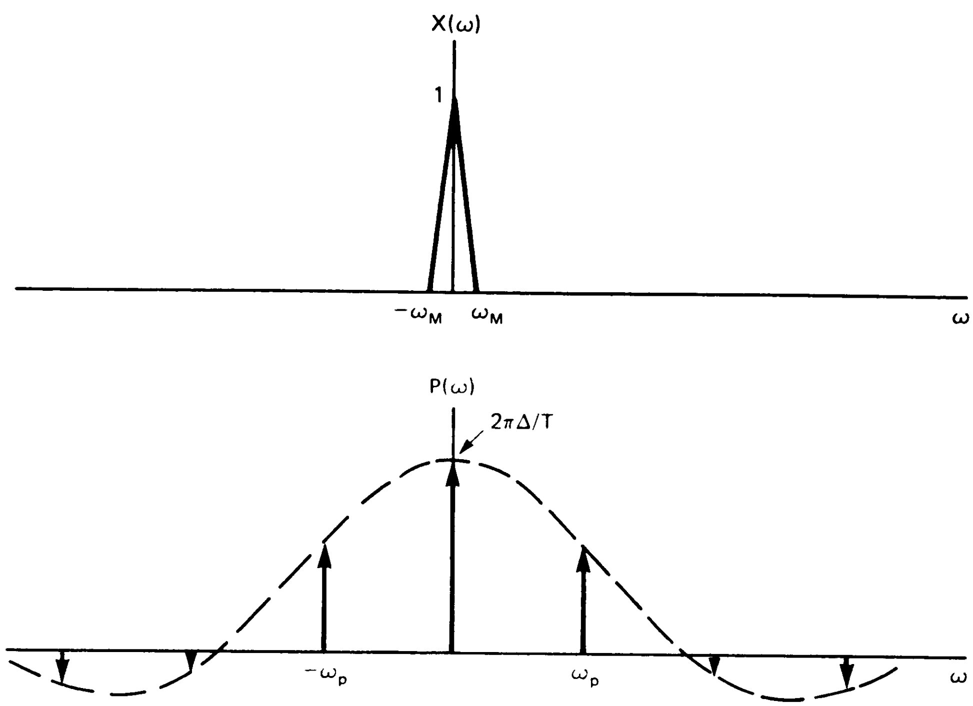


TRANSPARENCY 15.7
Continuous-time amplitude modulation with a pulse carrier.



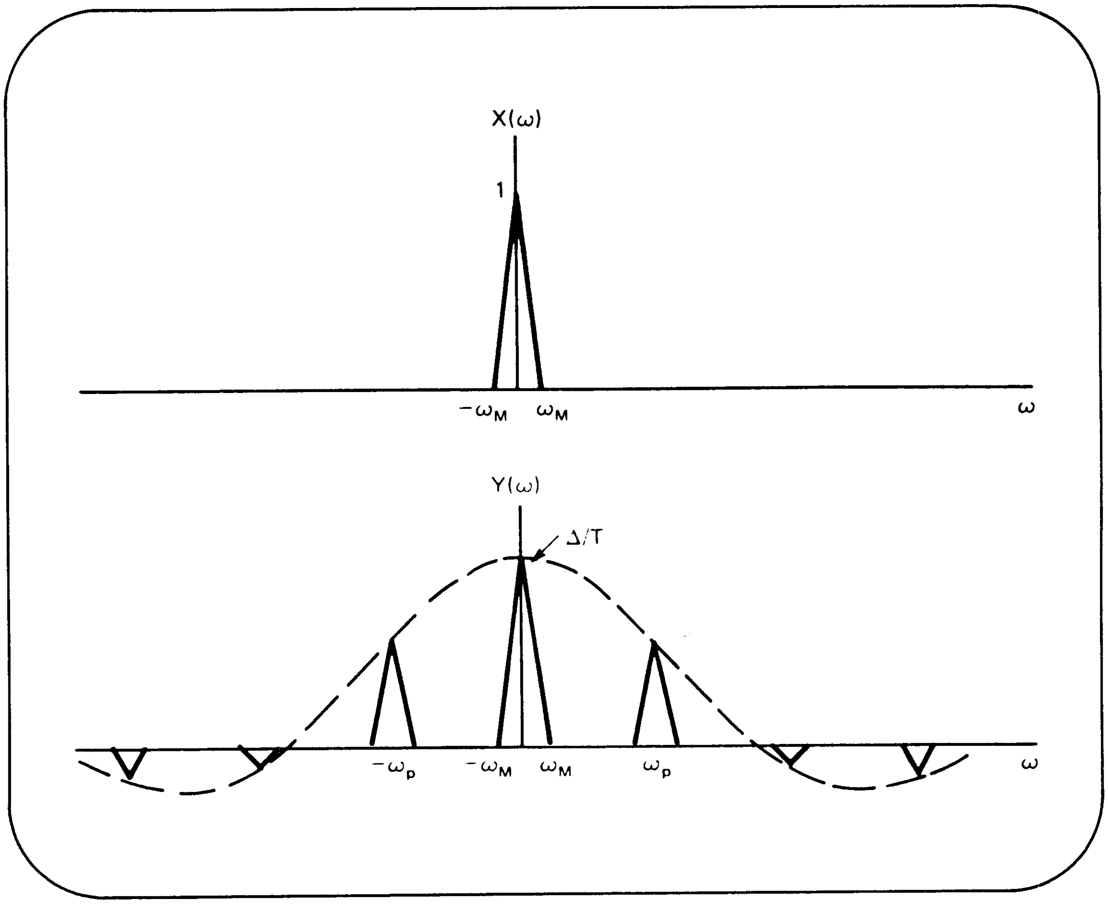


TRANSPARENCY 15.8
Discrete-time amplitude modulation with a pulse carrier.

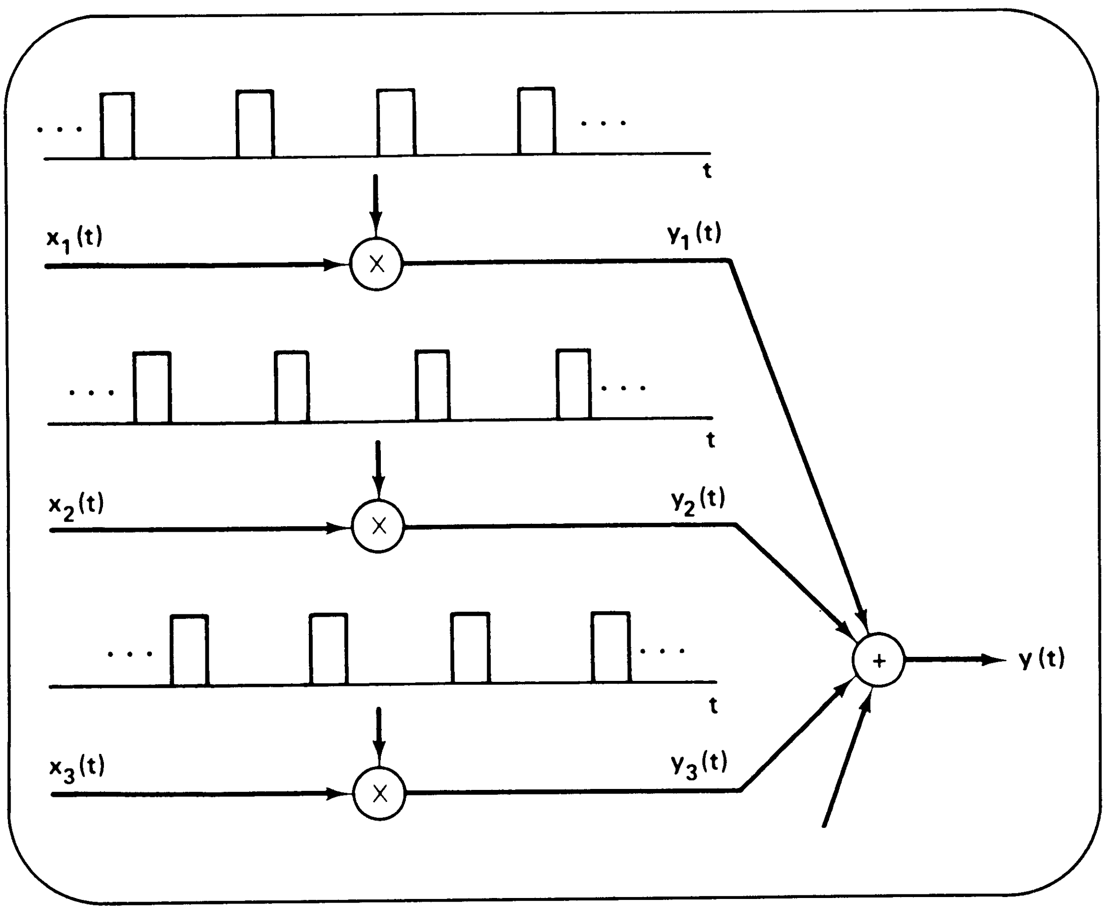


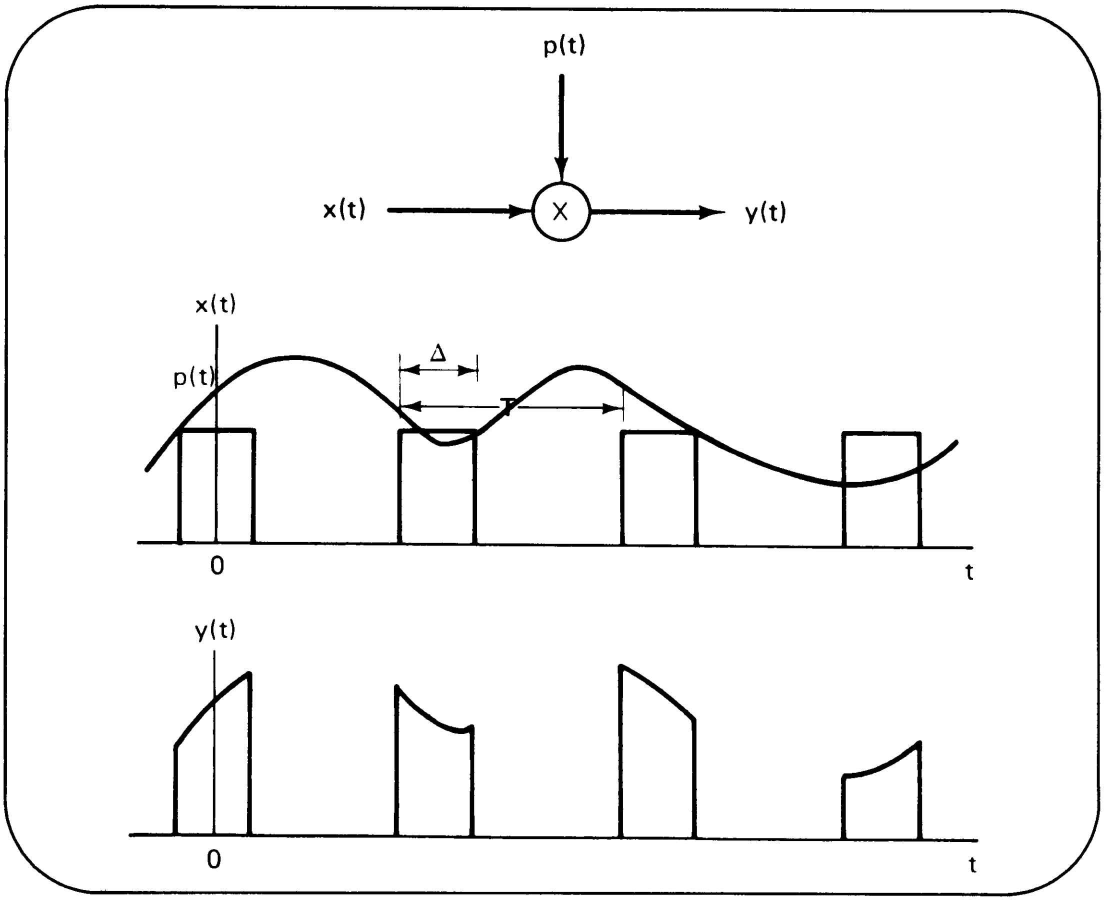
TRANSPARENCY 15.9
Transparencies 15.9 and 15.10 illustrate spectra associated with the system of Transparency 15.7. Shown here are the input spectrum and spectrum of a pulse carrier.

TRANSPARENCY
15.10
Input spectrum and output spectrum.

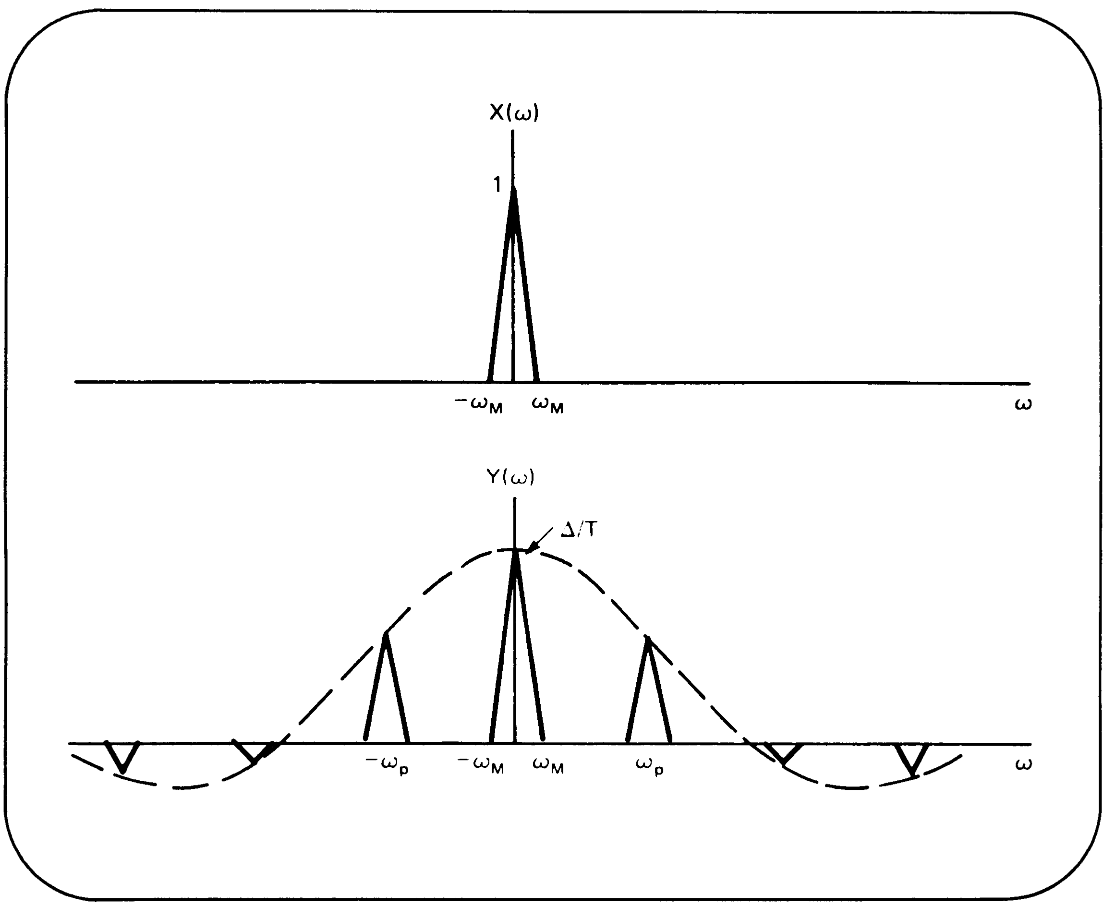


TRANSPARENCY
15.11
Time division multiplexing using amplitude modulation with a pulse carrier.



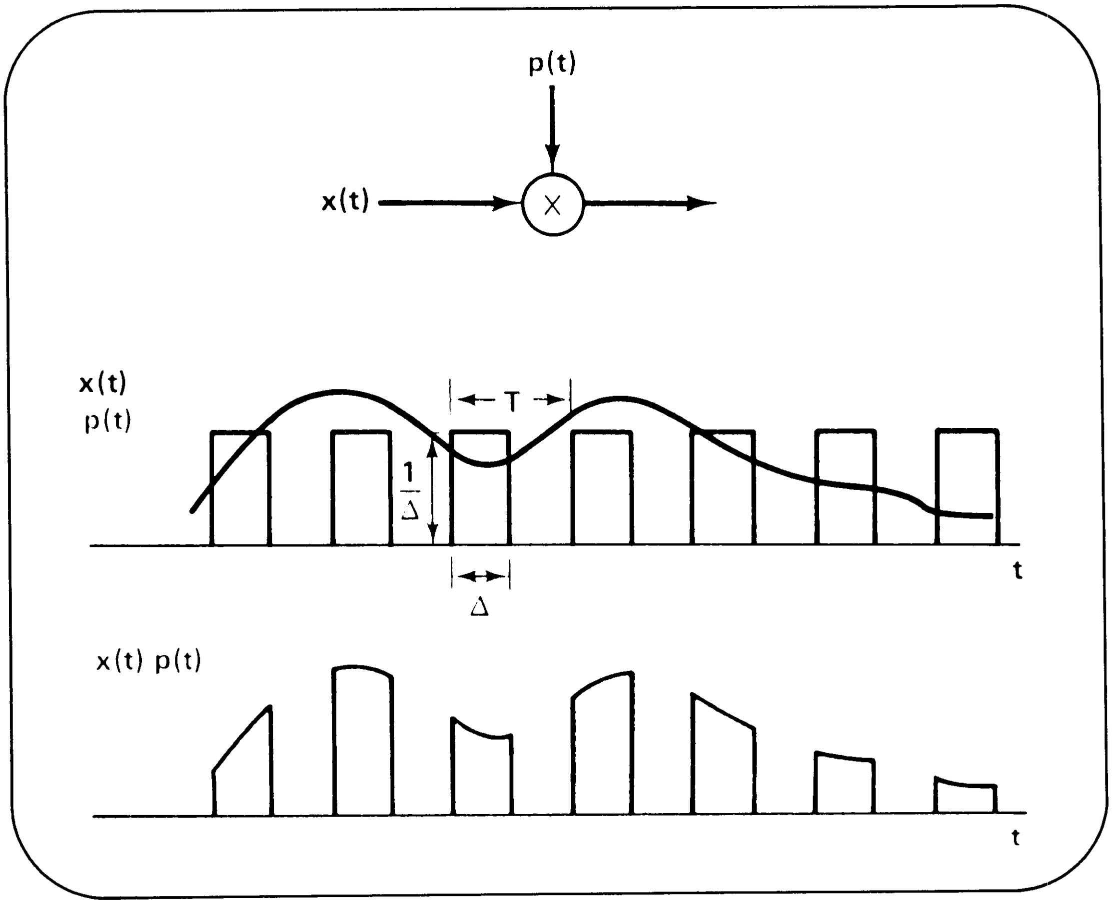


TRANSPARENCY 15.12
Time waveforms associated with amplitude modulation with a pulse carrier.

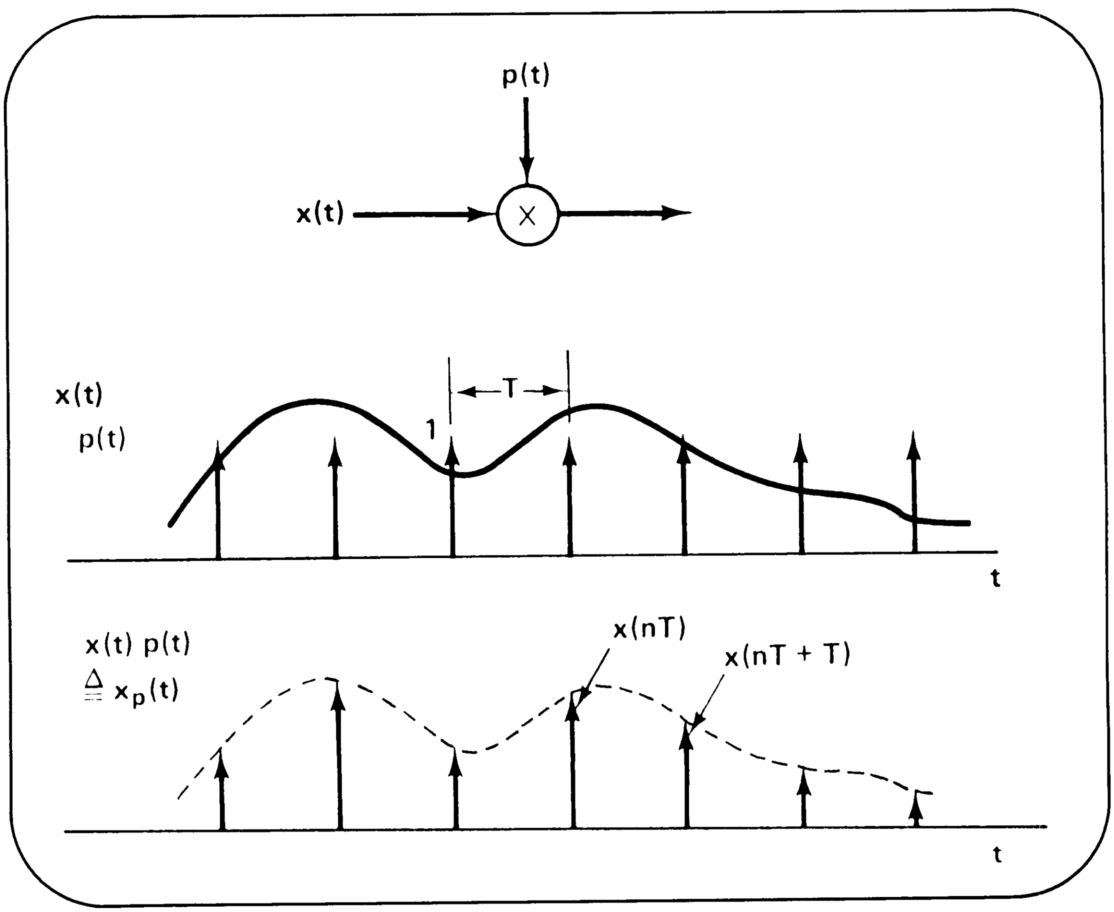


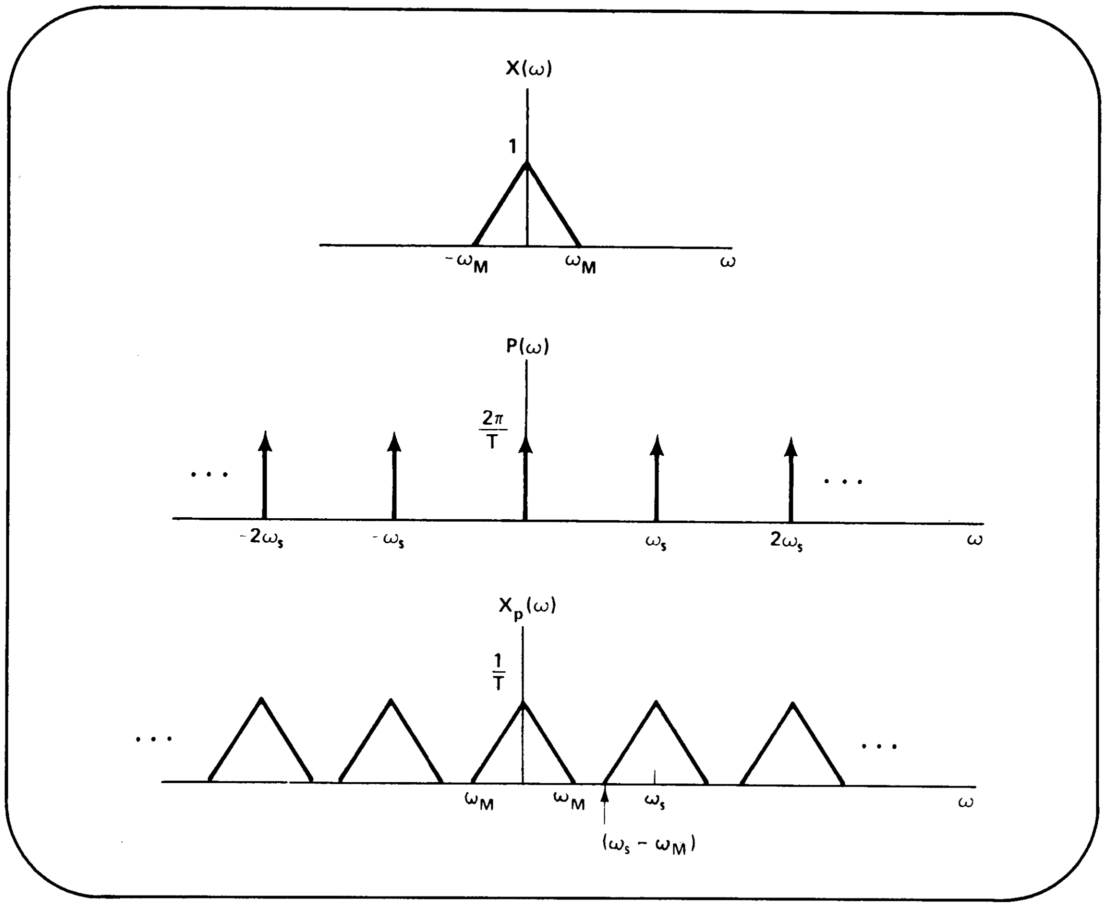
TRANSPARENCY 15.13
Spectra associated with amplitude modulation with a pulse carrier.

TRANSPARENCY 15.14
Amplitude modulation with a pulse carrier with the pulses chosen to have unit area.

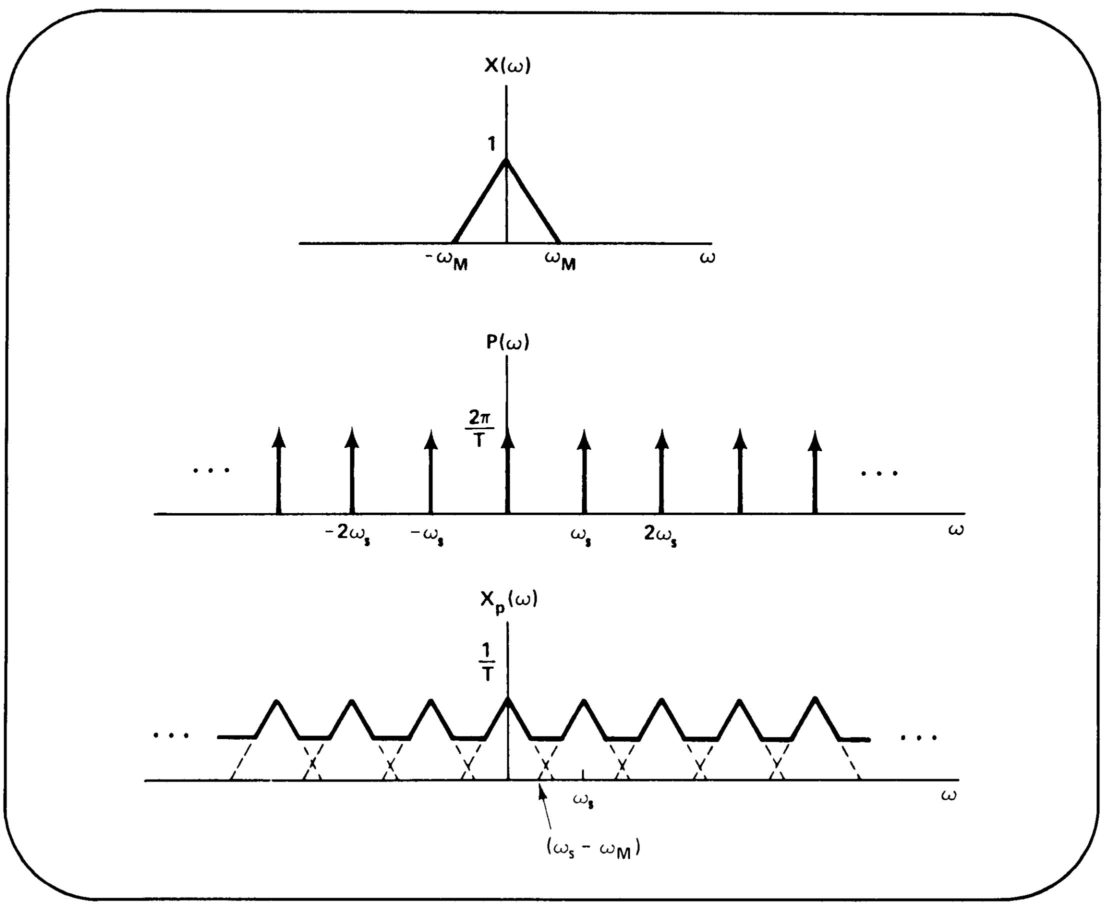


TRANSPARENCY 15.15
Amplitude modulation with a pulse carrier in the limit as the pulse width approaches zero and the pulse area remains unity. This corresponds to amplitude modulation with an impulse train carrier.

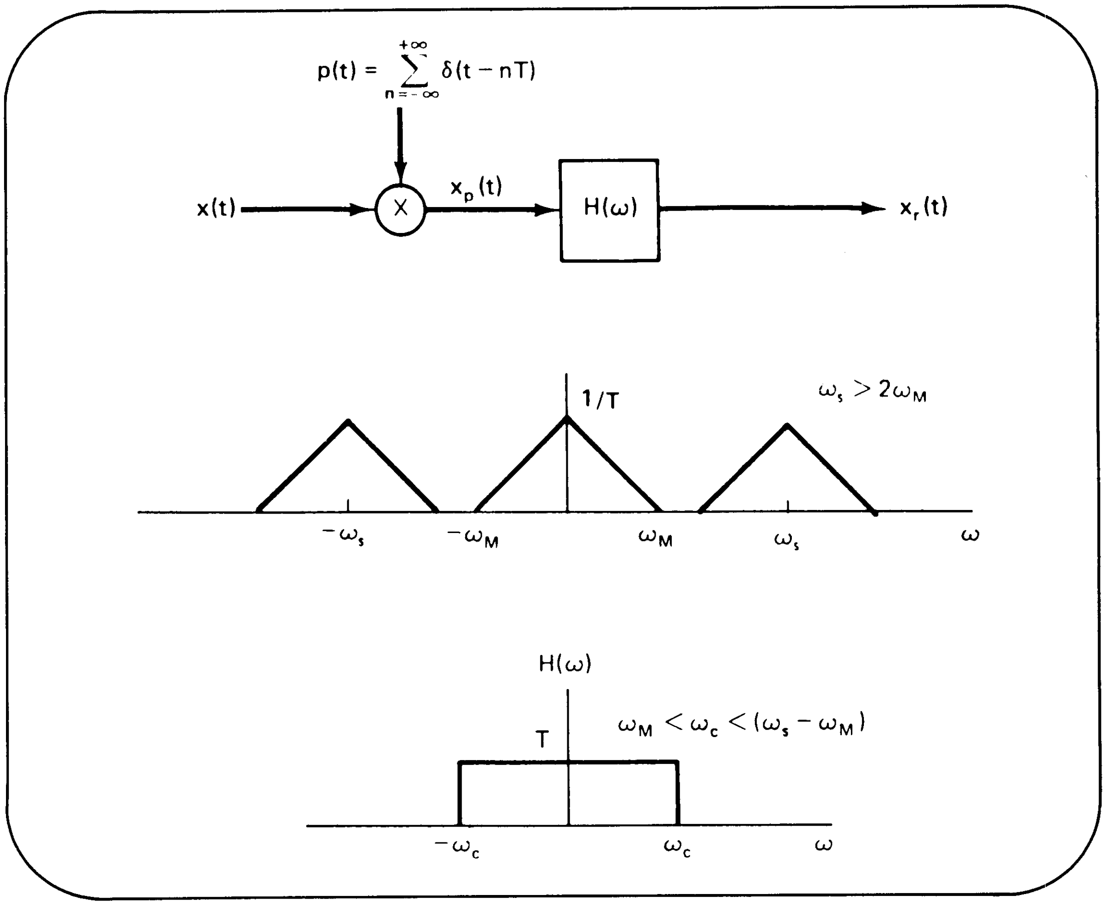




TRANSPARENCY 15.16
Transparencies 15.16 and 15.17 illustrate spectra associated with impulse train modulation. Here, the frequency of the modulating impulse train is chosen large enough so that the individual replications of the input spectrum do not overlap.



TRANSPARENCY 15.17
The carrier fundamental frequency is chosen such that the individual replications of the input spectrum overlap.



TRANSPARENCY 15.18
Modulation and demodulation with an impulse train carrier.

MARKERBOARD 15.2

Sampling Theorem

Given:

equally spaced samples of x(t) $\quad x(nT) \quad n = 0, \pm 1, \pm 2, \ldots$

x(t) bandlimited

$X(\omega) = 0 \quad |\omega| > \omega_M$

If $\frac{2\pi}{T} \triangleq \omega_s > 2\omega_M$

Then x(t) uniquely recoverable with a lowpass filter