PROBLEMS

3.1. A simple plunger-type solenoid for the operation of relays, valves, etc., is represented in Fig. 3P.1. Assume that it is a conservative system and that its electrical equation of state is

$$\lambda = \frac{L_0 i}{1 + x/a}.$$

(a) Find the force that must be *applied to* the plunger to hold it in equilibrium at a displacement x and with a current i.

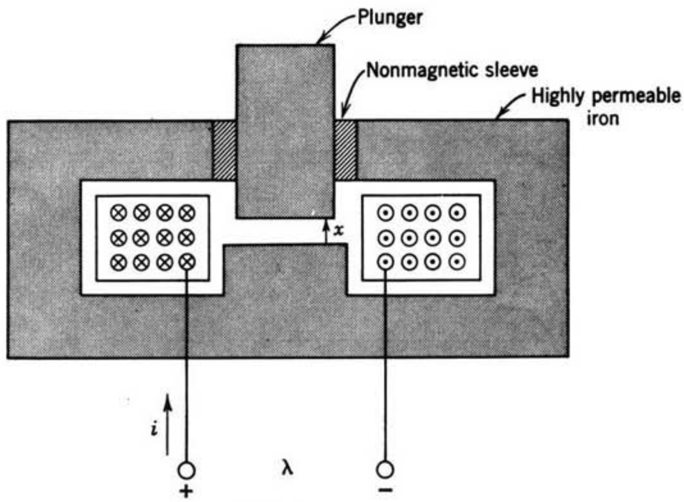

Fig. 3P.1

(b) Make a labeled sketch of the force of part (a) as a function of x with constant i.
(c) Make a labeled sketch of the force of part (a) as a function of x with constant λ.

3.2. An electrically linear electric field system with two electrical terminal pairs is illustrated in Fig. 3P.2. The system has the electrical equations of state $v_1 = S_{11}q_1 + S_{12}q_2$ and $v_2 = S_{21}q_1 + S_{22}q_2$. (See Example 3.1.1 for a physical case of this type.)

(a) Calculate the energy input to the system over each of the three paths A, B, and C in the q_1-q_2 plane illustrated in Fig. 3P.2*b*.
(b) What is the relation between coefficients S_{12} and S_{21} to make these three values of energy the same?
(c) Derive the result of (b) by assuming that the system is conservative and applying reciprocity.

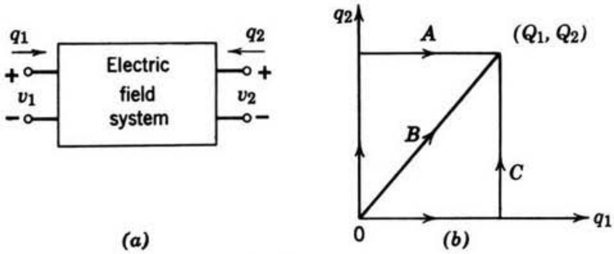

Fig. 3P.2

3.3. A slab of dielectric slides between plane parallel electrodes as shown. The dielectric obeys the constitutive law $\mathbf{D} = \alpha(\mathbf{E}\cdot\mathbf{E})\mathbf{E} + \epsilon_0\mathbf{E}$, where ϵ_0 is the permittivity of free space and α is a constant. Find the force of electrical origin on the slab. Your answer should take the form $f^e = f^e(v, x)$.

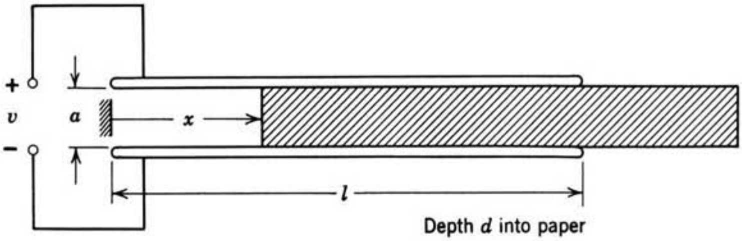

Fig. 3P.3

3.4. A magnetic circuit, including a movable plunger, is shown in Fig. 3P.4. The circuit is excited by an N-turn coil and consists of a perfectly permeable yoke and plunger with a variable air gap $x(t)$ and a fixed nonmagnetic gap d. The system, with the cross section shown, has a width w into the paper. The following parts lead to a mathematical formulation of the equations of motion for the mass M, given the excitation $I(t)$.

(a) Find the terminal relation for the flux $\lambda(i, x)$ linked by the electrical terminal pair. Ignore fringing in the nonmagnetic gaps. Note that the coil links the flux through the magnetic material N times.
(b) Find the energy $W_m(\lambda, x)$ stored in the electromechanical coupling. This should be done by making use of part (a).
(c) Use the energy function $W_m(\lambda, x)$ to compute the force of electrical origin f^e acting on the plunger.
(d) Write an electrical (circuit) equation of motion involving λ and x as the only dependent variables and $I(t)$ as a driving function.
(e) Write the mechanical equation of motion for the mass. This differential equation should have λ and x as the only dependent variables, hence taken with the result of (d) should constitute a mathematical formulation appropriate for analyzing the system dynamics.

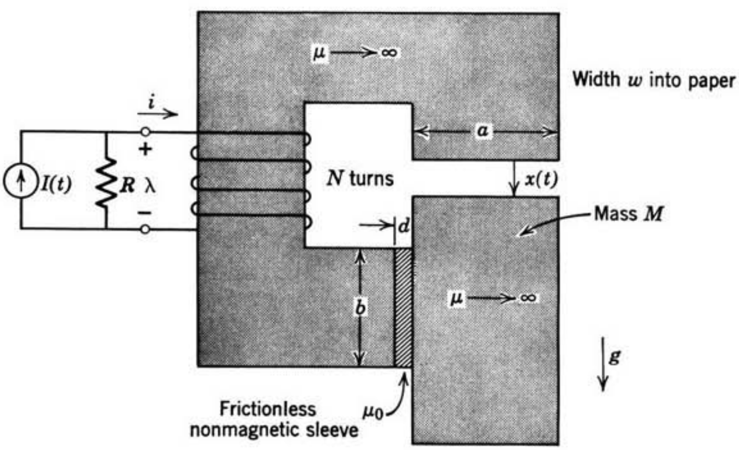

Fig. 3P.4

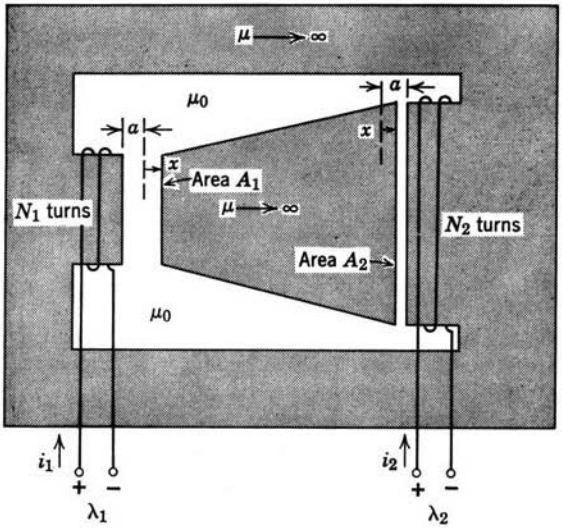

Fig. 3P.5

3.5. A magnetic circuit with a movable element is shown in Fig. 3P.5. With this element centered, the air gaps have the same length (a). Displacements from this centered position are denoted by x.

(a) Find the electrical terminal relations $\lambda_1(i_1, i_2, x)$ and $\lambda_2(i_1, i_2, x)$ in terms of the parameters defined in the figure.
(b) Compute the coenergy $W'_m(i_1, i_2, x)$ stored in the electromechanical coupling.

3.6. An electrically nonlinear magnetic field coupling network illustrated in Fig. 3P.6 has the equations of state

$$i = I_0\left[\frac{\lambda/\lambda_0 + (\lambda/\lambda_0)^3}{1 + x/a}\right], \qquad f^e = \frac{I_0}{a}\left[\frac{\frac{1}{2}\lambda^2/\lambda_0 + \frac{1}{4}\lambda^4/\lambda_0{}^3}{(1 + x/a)^2}\right],$$

where I_0, λ_0, and a are positive constants.

(a) Prove that this system is conservative.
(b) Evaluate the stored energy at the point λ_1, x_1 in variable space.

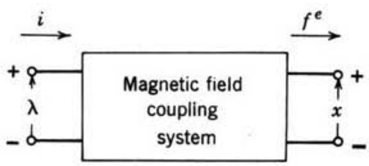

Fig. 3P.6

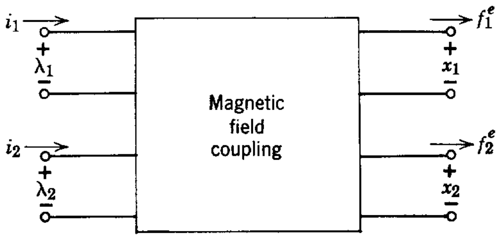

Fig. 3P.7

3.7. The electrical terminal variables of the electromechanical coupling network shown in Fig. 3P.7 are known to be $\lambda_1 = ax_1 i_1^3 + bx_1x_2i_2$ and $\lambda_2 = bx_1x_2i_1 + cx_2i_2^3$, where a, b, and c are constants. What is the coenergy $W_m'(i_1, i_2, x_1, x_2)$ stored in the coupling network?

3.8. A schematic diagram of a rotating machine with a superconducting rotor (moment of inertia J) is shown in Fig. 3P.8. Tests have shown that $\lambda_1 = i_1L_1 + i_2L_m \cos\theta$ and $\lambda_2 = i_1L_m \cos\theta + i_2L_2$, where $\theta(t)$ is the angular deflection of the shaft to which coil (2) is attached. The machine is placed in operation as follows:

(a) With the (2) terminals open circuit and the shaft at $\theta = 0$, $I(t)$ is raised to I_0.
(b) Terminals (2) are shorted to conserve the flux λ_2 regardless of $\theta(t)$ or $i_1(t)$.
(c) $I(t)$ is now made a given driving function.

Write the equation of motion for the shaft. Your answer should be one equation involving only $\theta(t)$ as an unknown. Damping can be ignored.

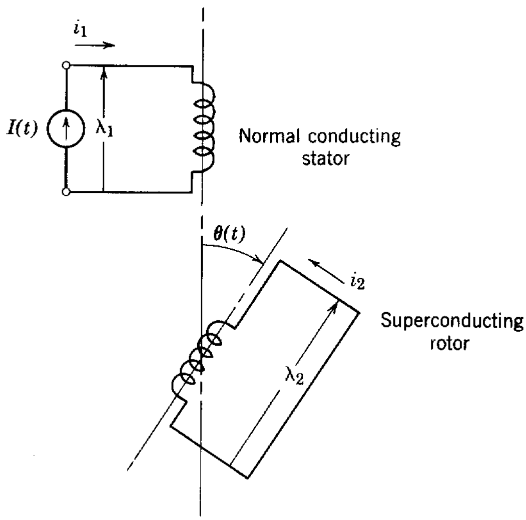

Fig. 3P.8

3.9. The electric terminal variables of the electromechanical coupling system shown in Fig. 3P.9 are known to be $\lambda_1 = ax_1^2i_1^3 + bx_2^2x_1i_2$ and $\lambda_2 = bx_2^2x_1i_1 + cx_2^2i_2^3$, where a, b, and c are constants.

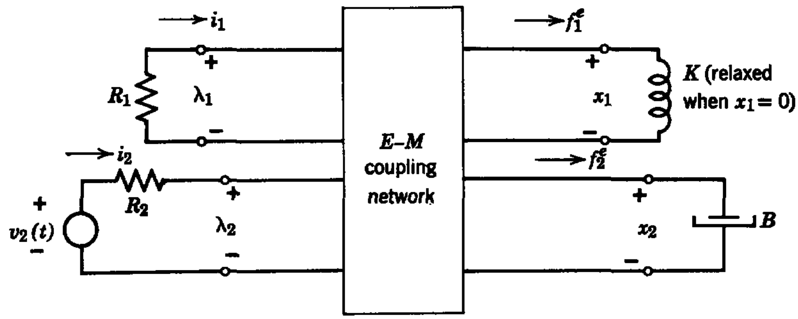

Fig. 3P.9

(a) What is the coenergy $W_m'(i_1, i_2, x_1, x_2)$ stored in the coupling network?
(b) Find the forces f_1^e and f_2^e.
(c) Write the *complete* set of equations for the system with the terminal constraints shown.

3.10. The following equations of state describe the conservative, magnetic field coupling system of Fig. 3P.10 for the ranges of variables of interest ($i_1 > 0, i_2 > 0$). $\lambda_1 = L_0 i_1 + A i_1 i_2^2 x$ and $\lambda_2 = A i_1^2 i_2 x + L_0 i_2$, where L_0 and A are positive constants.

(a) Find the force applied by the coupling system on the external mechanical circuit as a function of i_1, i_2, and x.
(b) Write the *complete* set of differential equations for the system by using i_1, i_2, and x as dependent variables.

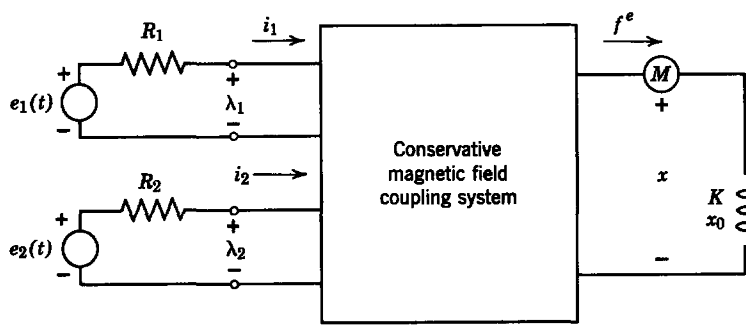

Fig. 3P.10

3.11. Two coils are free to rotate as shown in Fig. 3P.11. Each coil has a moment of inertia J. Measurements have shown that $\lambda_1 = L_1 i_1 + M i_2 \cos\theta \cos\psi$ and $\lambda_2 = M i_1 \cos\theta \cos\psi + L_2 i_2$, where L_1, L_2, and M are constants. Because the system is electrically linear, we know that the coenergy $W_m'(i_1, i_2, \psi, \theta)$ is given by $W_m' = \frac{1}{2}L_1 i_1^2 + M \cos\theta \cos\psi i_1 i_2 + \frac{1}{2}L_2 i_2^2$. The coils are driven by the external circuits, where I_1 and I_2 are known functions of time

(a) What are the torques of electrical origin T_1^e and T_2^e that the electrical system exerts on the coils?
(b) Write the *complete* equations of motion that define $\theta(t)$ and $\psi(t)$.

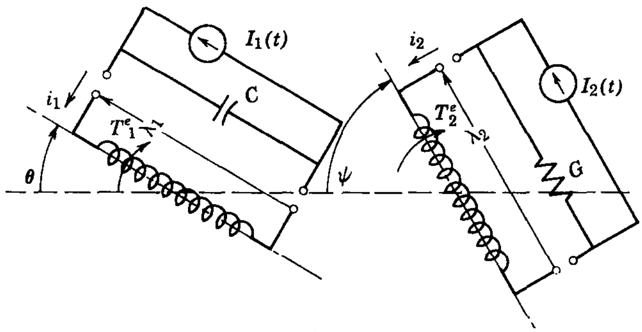

Fig. 3P.11

3.12. A magnetic field system has three electrical terminal pairs and two mechanical terminal pairs as shown in Fig. 3P.12. The system is electrically linear and may be described by the relations $\lambda_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$, $\lambda_2 = L_{21}i_1 + L_{22}i_2 + L_{23}i_3$, and $\lambda_3 = L_{31}i_1 + L_{32}i_2 + L_{33}i_3$. Each of the inductances L_{ij} $(i = 1, 2, 3; j = 1, 2, 3)$ may be functions of the mechanical variables x_1 and x_2. *Prove* that if the system is conservative, $L_{12} = L_{21}$, $L_{13} = L_{31}$, and $L_{23} = L_{32}$. To do this recall that for a conservative system the energy (or coenergy) does not depend on the path of integration but only on the end point.

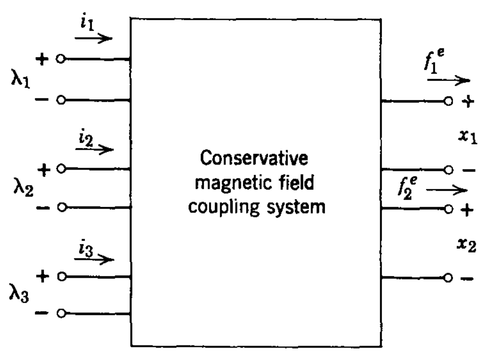

Fig. 3P.12

3.13. Electrostatic voltmeters are often constructed as shown in Fig. 3P.13*a*. N pairs of pie-shaped plates form the stator and rotor of a variable capacitor (Fig. 3P.13*a* shows six pairs of rotor plates and six pairs of stator plates). The rotor plates are attached to a conducting shaft that is free to rotate through an angle θ. In the electrostatic voltmeter a pointer is attached to this shaft so that the deflection θ is indicated on a calibrated scale (not shown).

(a) Determine $q(v, \theta)$, where q is the charge on the stator and v is the voltage applied between the rotor and stator. The device is constructed so that fringing fields can be ignored and the area of the plates is large compared with the cross section of the shaft. In addition, it is operated in a region of θ in which the plates overlap but not completely.

(b) Find the torque of electrical origin on the rotor.

(c) The shaft is attached to a torsional spring which has the deflection θ when

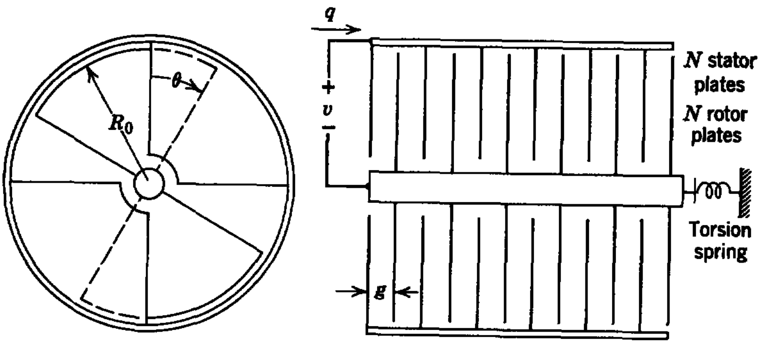

Fig. 3P.13a

subjected to a torque T_m, where θ and T_m are related by $T_m = K(\theta - \alpha)$. The shaft has a moment of inertia J and is subject to a damping torque $B\,d\theta/dt$. Write the torque equation for the shaft.

(d) The circuit of Fig. 3P.13*b* is attached to the terminals. Write the electrical equation for the system. [The results of parts (c) and (d) should constitute two equations in two unknowns.]

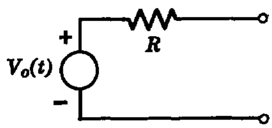

Fig. 3P.13*b*

(e) A "zero adjust" knob on the electrostatic voltmeter is used to set α in such a way that a pointer attached to the shaft indicates 0 when $\theta = \alpha$. A constant voltage $v = V_0$ is attached to the terminals. What is the static deflection of the pointer $(\theta - \alpha)$ as a function of V_0?

3.14. A fixed cylindrical capacitor of length L is made of a solid perfectly conducting inner rod of radius a which is concentric with a perfectly conducting outer shell of radius b. An annular half cylinder (inner radius a, outer radius b) of dielectric with permittivity ϵ and length L is free to move along the axis of the capacitor as shown in Fig. 3P.14 (ignore fringing).

(a) Find the charge on the outer cylinder $q = q(v, x)$, where v is the voltage between the inner and outer conductors and x is the displacement of the half cylinder of dielectric (assume $L > x > 0$).

(b) Write the conservation of power for this system in terms of the terminal voltage and current, the electric energy stored, the force of electric origin, and the velocity of the dielectric.

(c) Find the electric energy stored in terms of q and x.

(d) Find the electric coenergy in terms of v and x.

(e) Find the force of electric origin exerted by the fields on the dielectric.

Suppose one end of the dielectric is attached to a spring of constant K, which is relaxed when $x = l$.

(f) Write the differential equation of motion for the dielectric, assuming that it has mass M and slides without friction.

(g) If a constant voltage V_0 is established between the conductors, find x.

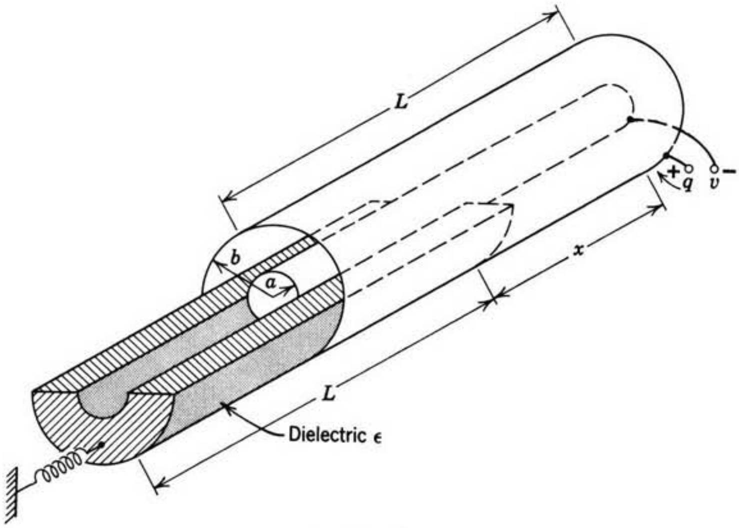

Fig. 3P.14

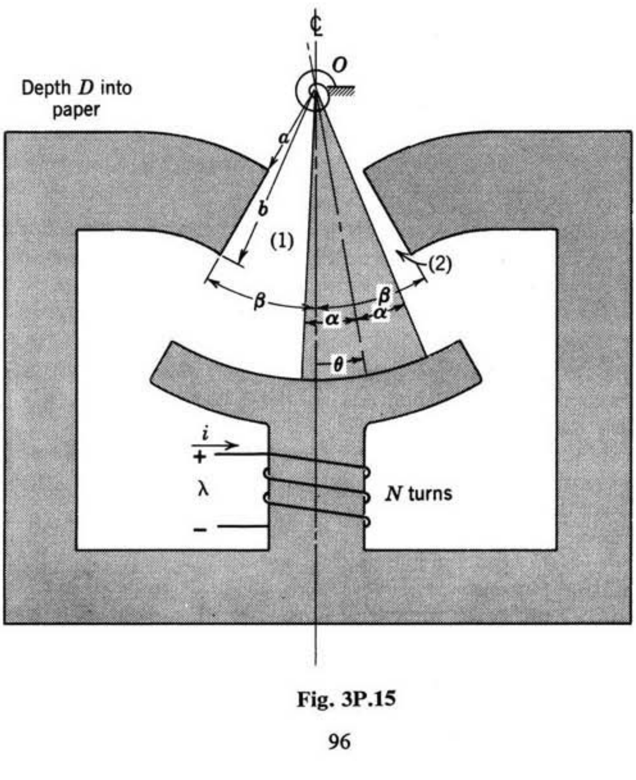

Fig. 3P.15

3.15. A magnetic transducer is shown in Fig. 3P.15. A wedge-shaped infinitely permeable piece of metal is free to rotate through the angle θ $(-(\beta - \alpha) < \theta < \beta - \alpha)$. The angle θ is the deflection of the wedge center line from the center line of the device. A magnetic field is produced in regions (1) and (2) by means of the infinitely permeable yoke and the N-turn winding.

(a) Find λ (i, θ). (You may assume that the fringing fields at the radii $r = a$ and $r = b$ from the origin O are of negligible importance.)

(b) Compute the magnetic coenergy stored in the electromechanical coupling $W'_m(i, \theta)$.

(c) Use the conservation of energy to find the torque T^e exerted by the magnetic field on the wedge.

(d) The wedge has a moment of inertia J about O and is constrained by a torsion spring that exerts the torque $T_m = K\theta$. Write the equation of motion for the wedge, assuming that i is a given function of time.

(e) If $i = I_0 =$ constant, show that the wedge can be in static equilibrium at $\theta = 0$.

3.16. A plane electrode is free to move into the region between plane-parallel electrodes, as shown in Fig. 3P.16. The outer electrodes are at the same potential, whereas the inner electrode is at a potential determined by the constant voltage source V_0 in series with the output of an amplifier driven by a signal proportional to the displacement of the movable electrode itself. Hence the voltage of the inner electrode relative to that of the outer electrodes is $v = -V_0 + Ax$, where A is a given feedback gain. Find the force of electrical origin $f^e(x)$. (Note that this force is only a function of position, since the voltage is a known function of x.)

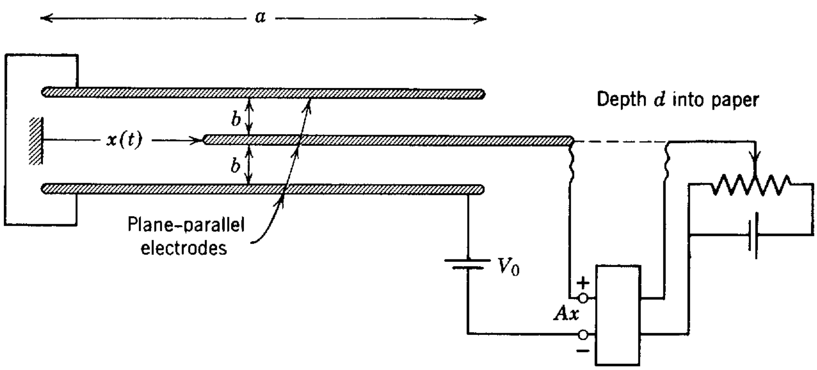

Fig. 3P.16

3.17. In Fig. 3P.17 we have a slab of magnetic material positioned between three pole faces. The nonlinear magnetic material is such that the constituent relation is given by $\mathbf{B} = \alpha(\mathbf{H} \cdot \mathbf{H})\mathbf{H} + \mu_0\mathbf{H}$, where α is a known constant.

(a) Show that

$$\lambda_1 = L_0\left(1 + \frac{d}{g}\right)i_1 + L_0\beta\left(1 - \frac{x}{l}\right)i_1^3 + L_0\frac{d}{g}i_2.$$

and

$$\lambda_2 = L_0\frac{d}{g}i_1 + L_0\beta\left(\frac{x - g}{l}\right)i_2^3 + L_0\left(1 + \frac{d}{g}\right)i_2,$$

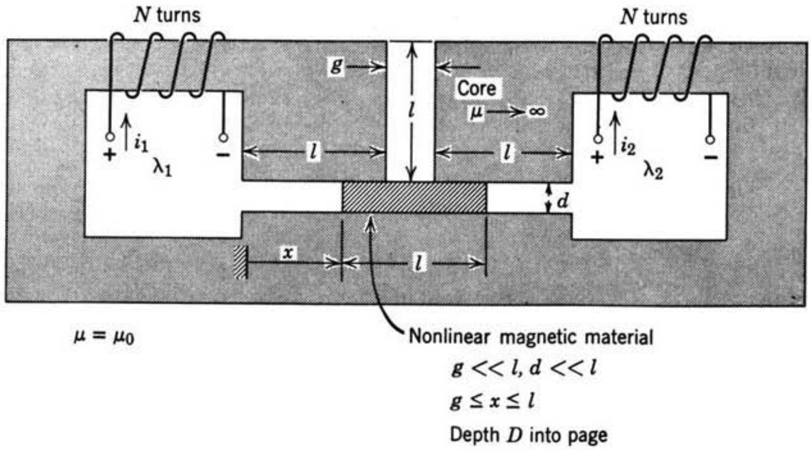

Fig. 3P.17

where

$$L_0 = \frac{\mu_0 N^2 l D}{d},$$

$$\beta = \frac{\alpha}{\mu_0}\left(\frac{N}{d}\right)^2,$$

and

$$g \le x \le l$$

(b) Determine an expression for the magnetic coenergy $W_m' = W_m'(i_1, i_2, x)$.

(c) What is the force of magnetic origin f^e acting on the nonlinear magnetic material?

3.18. A slab of dielectric material is positioned between three perfectly conducting plates shown in Fig. 3P.18. The dielectric is such that the displacement vector **D** is related to the electric field **E** through the relation $\mathbf{D} = \alpha(\mathbf{E}\cdot\mathbf{E})\mathbf{E} + \epsilon_0\mathbf{E}$, where α is a known positive constant. The slab and adjacent plates have a width (into paper) w.

(a) With the slab at the position x, find the electrical terminal relations. Ignore fringing fields and assume that the slab is always well within the plates

$$q_r = q_r(v_r, v_l, x) \text{ and } q_l = q_l(v_r, v_l, x).$$

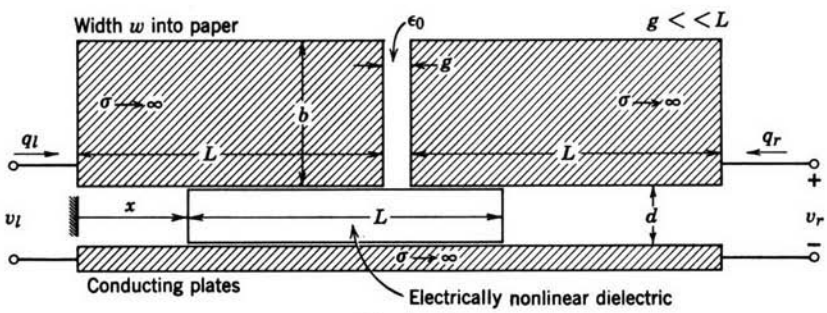

Fig. 3P.18

(b) Find the electrical coenergy $W_e'(v_r, v_l, x)$ stored between the plates.
(c) What is the force of electrical origin f^e on the slab of dielectric?

3.19. A perfectly conducting plate of length $2\alpha R$ and depth D is attached to the end of a conducting bar that rotates about the axis O, shown in Fig. 3P.19. A pair of conducting electrodes forms half cylinders, coaxial with the axis O. The gap $\Delta \ll R$. We ignore fringing fields in the present analysis.

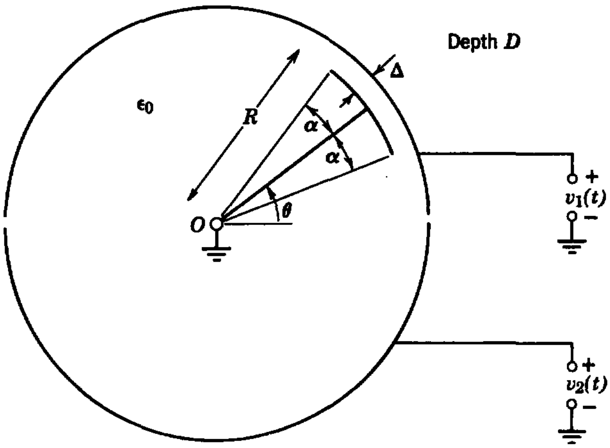

Fig. 3P.19

(a) Make a dimensioned plot of the coenergy $W_e'(\theta, v_1, v_2)$ as a function of θ.
(b) Make a dimensioned plot of the torque exerted by the electric fields on the rotor.
(c) In terms of this torque, write a differential equation for $\theta(t)$. You may assume that the rotor has an inertia J but is not impeded by a viscous damping force.

3.20. A parallel-plate capacitor has its bottom plate fixed and its top plate free to move vertically under the influence of the externally applied mechanical force f. A slab of the dielectric of thickness d is between the plates shown in Fig. 3P.20*a*. With plate area A and displacement x, the electrical terminal relation (neglecting fringing fields; see Example 2.1.4) is

$$q(v, x) = \frac{\epsilon A}{d(1 + \epsilon x/\epsilon_0 d)} v.$$

(a) The capacitor is charged to a value of charge $q = Q$ and the terminals are open-circuited. Calculate, sketch, and label the externally applied force $f(Q, x)$ necessary to hold the plate in equilibrium and the terminal voltage $v(Q, x)$ as functions of x for the range $0 < x < 2(\epsilon_0/\epsilon)d$.
(b) A battery of constant voltage V is connected to the terminals of the capacitor. Calculate, sketch, and label the externally applied force $f(V, x)$ necessary to hold the plate in equilibrium and the charge $q(V, x)$ as functions of x for the range $0 < x < 2(\epsilon_0/\epsilon)d$.
(c) By the use of suitable electrical and mechanical sources the system of Fig. 3P.20*a* is made to traverse the closed cycle in q-x plane shown in Fig. 3P.20*b* in the direction indicated by the arrows. Calculate the energy converted per cycle and specify whether the conversion is from electrical to mechanical or vice versa.

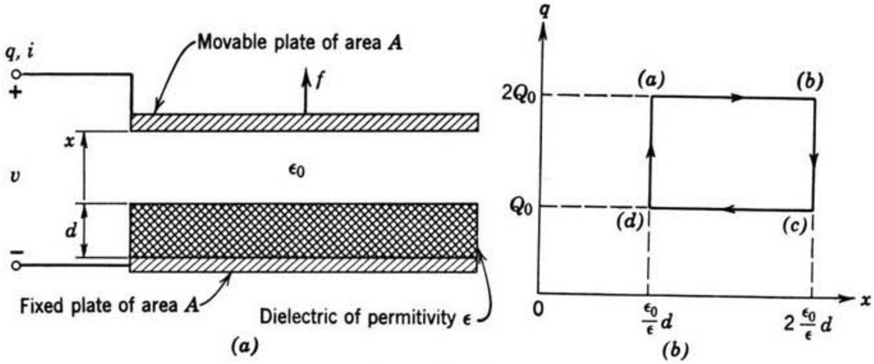

Fig. 3P.20

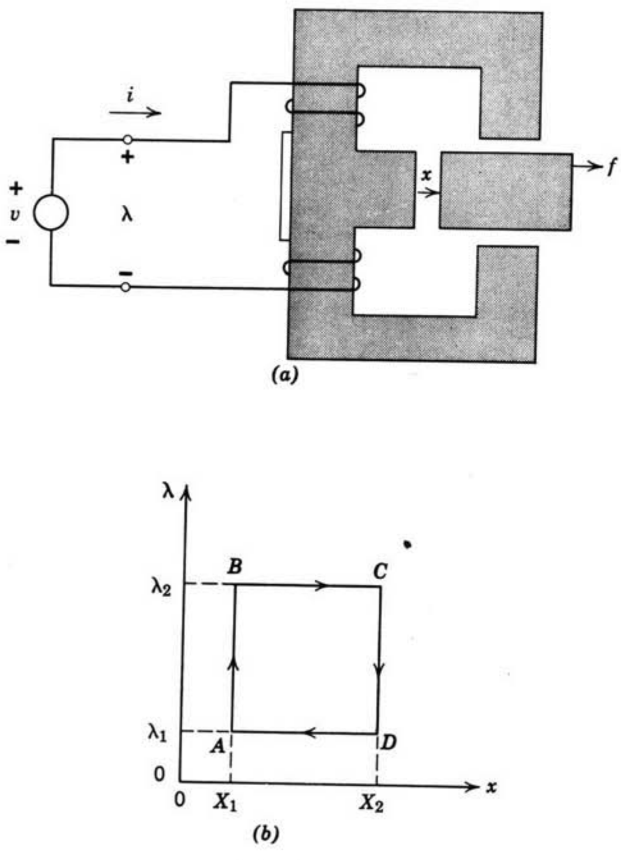

Fig. 3P.21

3.21. The magnetic field transducer illustrated schematically in Fig. 3P.21*a* has a movable plunger that is constrained to move only in the x-direction. The coupling field is conservative and electrically linear and has the electrical equation of state

$$\lambda = \frac{L_0 i}{1 + x/a}$$

where L_0 and a are positive constants (see Example 2.1.1).

(a) For any flux linkage λ and position x find the external force f (see Fig. 3P.21*a*), which must be applied to hold the plunger in static equilibrium.

We now constrain the electrical terminal pair with a voltage source v and the movable plunger by a position source x in such a way that the system slowly traverses the closed cycle in the λ-x plane illustrated in Fig. 3P.21*b*.

(b) Sketch and label current i as a function of flux linkage λ for the closed cycle of Fig. 3P.21*b*.

(c) Sketch and label the force f applied by the position source as a function of x for closed cycle of Fig. 3P.21*b*.

(d) Find the energy converted between electrical and mechanical forms for one traversal of the cycle of Fig. 3P.21*b*. Specify the direction of flow.

3.22. The system shown in Fig. 3P.22 consists of two thin perfectly conducting plates, one of which is free to move. The movable plate slides on a perfectly conducting plane. It has been proposed that energy could be converted from mechanical to electrical form by the following scheme:

The process is started by holding the plate at $x = X_b$ with the switch in position (1). An external mechanical system moves the plate to $x = X_a$ and holds it there. S is then put in

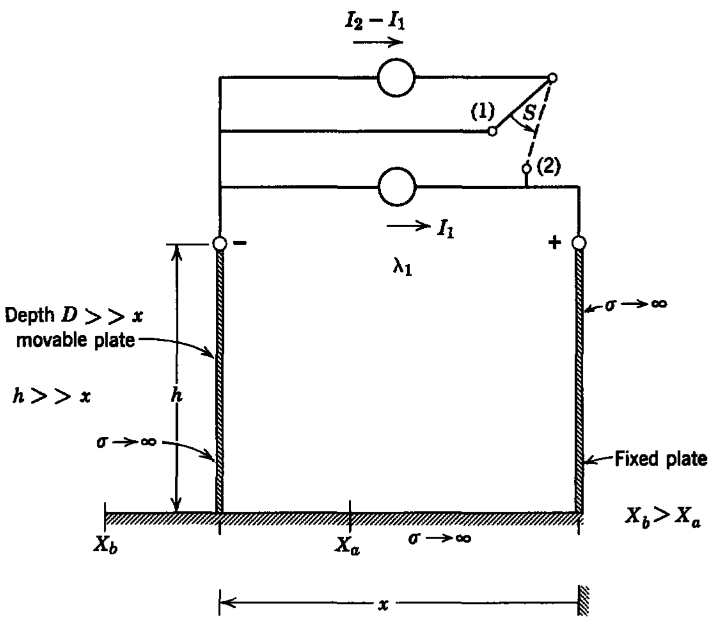

Fig. 3P.22

position (2) and the mechanical system moves the plate back to $x = X_b$ and holds it in place. S is then reset in position (1) and the cycle is repeated several times.

(a) To convert energy *from mechanical to electrical* form during each cycle, how must I_1 and I_2 be related?

(b) Sketch the path of operation in the i-x plane under the conditions of part (a) and compute the amount of energy converted from mechanical to electrical form during one cycle.

(c) Sketch carefully the path of operation for one cycle in the λ-x plane under the conditions of part (a). Compute the amount of energy converted from mechanical to electrical form along each part of the path in the λ-x plane.