Problems for Chapter 6

For Section 6.2:

Prob. 6.2.1 Consider the configuration described in Prob. 2.3.3. In the MQS approximation and at low frequencies the configuration can be represented by an inductance in series with a resistance. Because the current is distributed, and in fact essentially uniform and x-directed, how should the inductance be computed?

(a) One method uses the field in the zero frequency limit to determine the magnetic energy density, and hence by integration the total stored energy. This is then equated to $\frac{1}{2}Li^2$ to obtain L. Use this method to find L and show that it is 1/3 of the value for electrodes without the conducting material but shorted at z = 0.

(b) Now, consider an alternative approach which considers the fields as quasistatic with respect to the magnetic diffusion time $\mu\sigma\ell^2$. In terms of the driving current, find the zero order fields as if they were static. Then, from Eq. 6.2.7 find the first order fields that result from time variations of the zero order field. Evaluate the voltage at the terminals and show that it has the form taken for a series inductance and resistance.

For Section 6.3:

Prob. 6.3.1 Show that Eq. (b) of Table 6.3.1 describes the rotating cylindrical shell shown in that table.

Prob. 6.3.2 Show that Eq. (c) of Table 6.3.1 describes the translating cylindrical shell shown in that table.

Prob. 6.3.3 Show that Eqs. (d) and (e) of Table 6.3.1 describe the rotating spherical shell shown in that table.

Prob. 6.3.4 If a sheet is of extremely high permeability, the normal flux density B_n is not continuous. Consider the sheets of Table 6.3.1 in the limit of zero conductivity but with a very high permeability and show that boundary conditions are

$$\vec{n} \times [\![\vec{H}]\!] = 0 \; ; \qquad \Delta\mu(\nabla_\Sigma \cdot \vec{H}) + [\![B_n]\!] = 0$$

These boundary conditions are appropriate if wavelengths in the plane of the sheet are long compared to the sheet thickness. Thus the boundary condition can be used to represent a thin region that would otherwise be represented by the flux-potential transfer relations of Sec. 2.16. To see this connection, show that for a planar sheet, the above boundary condition can be written as

$$\Delta\mu k^2 \tilde{\psi} + [\![\tilde{B}_x]\!] = 0$$

Take the long-wave limit of the transfer relations from Table 2.16.1 to obtain this same result.

Prob. 6.3.5 In the boundary conditions of Table 6.3.1 representing a thin conducting sheet, B_n is continuous while the tangential $\vec{H}$ is not. By contrast, for the condition found in Prob. 6.3.4 for a highly permeable sheet, B_n is discontinuous and tangential $\vec{H}$ is continuous. What boundary conditions should be used if the sheet is both highly permeable and conducting? To answer this question it is necessary to give the fields in the sheet some dependence on the normal coordinate. Consider the planar sheet and assume that the fields within take the form

$$B_x = B_x^b + \frac{x}{\Delta}(B_x^a - B_x^b) \; ; \qquad H_y = H_y^b + \frac{x}{\Delta}(H_y^a - H_y^b)$$

Define $\langle A \rangle = (A^a + A^b)/2$ and show that the boundary conditions are

$$\mu\Delta\nabla_\Sigma \cdot \langle \vec{H} \rangle + [\![B_n]\!] = 0$$

and Eq. (a) of Table 6.3.1 with $B_x \to \langle B_x \rangle$.

For Section 6.4:

Prob. 6.4.1 A type of tachometer employing a permanent magnet is shown in Fig. P6.4.1a. In the developed model, Fig. P6.4.1b, the magnetized material moves to the right with velocity U so that the magnetization is the given function of (y,t). M_o is a given constant. The thickness, a, of the conducting sheet is small compared to the skin depth. Find the time average force per unit y-z area acting on the conducting sheet in the y direction. How would you design the device so that the induced force is proportional to U?

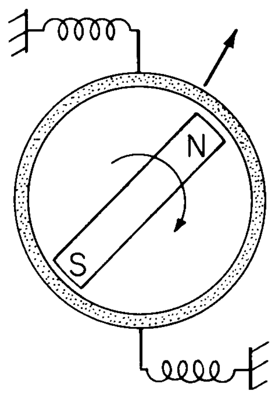

Fig. P6.4.1a

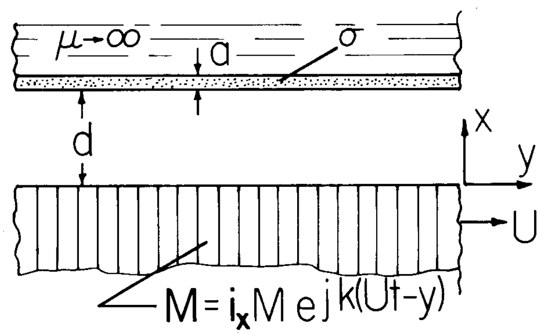

Fig. P6.4.1b

Prob. 6.4.2 Use the electrical terminal relations derived from the model, Eq. 6.4.17, to show that the equivalent circuit of Fig. 6.4.3 is valid.

Prob. 6.4.3 For the developed induction motor model shown in Fig. 6.4.1b, the time average force in the direction of motion is calculated. In certain applications, such as the magnetic levitation of vehicles (see Fig. 6.9.2), the lift force is also of importance. Find the time average lift force on the stator, $\langle f_x \rangle_t$, with two phase excitation. With single phase excitation, sketch this time average lift force as a function of S_m and explain in physical terms the asymptotic behavior.

Prob. 6.4.4 The cross section of a rotating induction machine is shown in Fig. 6.4.1a. The stator inner radius is (a), while the rotor has radius (b) and angular velocity Ω. The windings on the stator have p poles and two phases, as in the planar model developed in the section. For two phase excitation, find the time average torque on the rotor, an expression analogous to Eq. 6.4.11. Define θ as the clockwise angle from the vertical axis in Fig. 6.4.1a.

Prob. 6.4.5 For the rotating machine described in Prob. 6.4.4, find the two phase electrical terminal relations analogous to Eq. 6.4.17. Determine the parameters in the equivalent circuit, Fig. 6.4.3.

Prob. 6.4.6 This problem is intended to illustrate the application of the boundary conditions for a thin sheet that is both conducting and highly permeable, as in Prob. 6.3.5. In the plane x=0 there is a surface current density $\vec{K}_f = \vec{i}_z \text{Re}\, \hat{K}_o \exp j(\omega t - ky)$. The region x < 0 is infinitely permeable. In the plane x=d, a sheet of thickness Δ, permeability μ and conductivity σ moves in the y direction with velocity U. This sheet can shield the magnetic field from the region x > d either by virtue of its conductivity or its magnetizability. Find the magnetic potential just above the sheet ($x=d^+$). Consider $\mu \to \mu_o$ and show that for $\mu_o\sigma\Delta(\omega-kU)/k$ large, the field is excluded from the region x>d. Similarly, take $\sigma \to 0$ and show that if $k\Delta(\mu/\mu_o) >> 1$, shielding is obtained. Show that the effect of the permeability is to reduce the effectiveness of conduction shielding. In qualitative physical terms, why is there this conflict between the two types of shielding?

Prob. 6.4.7 A linear induction machine has the configuration of Fig. 6.4.1. However, the stator winding has a finite length ℓ in the y direction. Thus the stator surface current is

$$K_z^s = [u_{-1}(y) - u_{-1}(y-\ell)]\ \text{Re}\ \hat{K}_o \exp j(\omega t - \beta y)$$

Thus, the "stator" might be attached to a vehicle (such as that shown in Fig. 6.9.2) and the conducting sheet and magnetic backing might be the "rail." Using the approach of Sec. 5.17, show that the time average force exerted on the rail is

$$\langle f_z \rangle_t = \frac{\mu_o |\hat{K}_o|^2 w}{\pi} \int_{-\infty}^{+\infty} \frac{S_m \sin^2 [\frac{(k-\beta)\ell}{2}] dk}{(k-\beta)^2 \sinh^2 kd (1 + S_m^2 \coth^2 kd)}$$

where $S_m = \mu_o \sigma_s (\omega - kU)/k$.

Prob. 6.4.8 The induction machine rotor is a useful model for understanding phenomena observed if liquid metals are stressed by a-c magnetic fields. Motions of the liquid result from a competition of viscous and inertial forces with those from the magnetic field. Instability can result from the effect of the motion on the field. To illustrate, consider the single phase excitation of the configuration shown in Fig. 6.4.1. The "air gap" is filled with a liquid having viscosity η. Under the assumption that the flow in the gap resulting from the relative motion of the rotor and stator is fully developed and laminar, the viscous stress acting to retard the motion of the rotor is given by Eq. 7.13.1. As the magnetic field intensity $\hat{H}_o \equiv N_a\hat{i}_a$ is raised, there is a threshold at which the rotor spontaneously moves in one direction or the other. Write the condition for this instability in terms of the dimensionless numbers kd, R_M (product of frequency and magnetic diffusion time) and $\omega\tau_{MV}$ ($\tau_{MV} \equiv \eta/\mu_o H_o^2$, the magneto-viscous time as defined in Sec. 8.6).

For Section 6.5:

Prob. 6.5.1 Carry through the steps of Eqs. 6.5.8 - 6.5.10 leading to the transfer relations for rotating cylinders. Check relations (c) and (d) of Table 6.5.1.

Prob. 6.5.2 Carry through the steps beginning with Eq. 6.5.13 and leading to the transfer relations (e) and (f) of Table 6.5.1.

For Section 6.6:

Prob. 6.6.1 The rotor of an induction motor has finite thickness. Dimensions are defined in Fig. P6.6.1. The stator windings have p poles and two phases, the circular analogue of the windings for the developed model of Sec. 6.4. Hence the stator surface current distribution is the circular analogue of Eq. 6.4.1. Find the time average torque on the rotor.

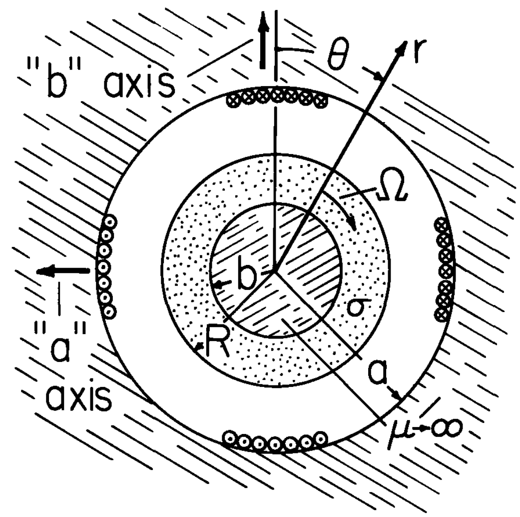

Fig. P6.6.1

Prob. 6.6.2 An induction machine is used to propel a circular cylindrical conductor in the longitudinal direction z. The "stator" consists of circumferential windings at the radius (a) surrounded by an infinitely permeable magnetic material in the region r > a. The material being propelled is coaxial with this structure and is of radius R, conductivity σ and permeability μ. Thus, there is an annular air gap of thickness a-R. The conducting rod has a velocity U in the z direction.

(a) The stator windings are in a three phase configuration driven by the three phase currents (i_a, i_b, i_c). Thus the surface current on the stator structure is

$$K_\theta = \text{Re}[\hat{i}_a e^{j\omega t}N_a \cos(kz) + \hat{i}_b e^{j\omega t}N_b \cos(kz-\frac{2\pi}{3}) + \hat{i}_c e^{j\omega t}N_c \cos(kz-\frac{4\pi}{3})]$$

Represent this driving surface current in the form

$$K_\theta = \text{Re}[\hat{K}_+^s e^{j(\omega t-kz)} + \hat{K}_-^s e^{j(\omega t+kz)}]$$

and identify $\hat{K}_+^s$ and $\hat{K}_-^s$ in terms of the terminal currents, turns per unit length N_a, N_b, N_c, etc.

(b) Find the time average longitudinal force $\langle f_z\rangle_t$ acting on a length ℓ of the rod.

Prob. 6.6.3 A linear induction machine has the configuration of Fig. 6.6.1, except that the stator surface current spans a limited length ℓ in the y direction. The driving current is

$$K_z^s = [u_{-1}(y)-u_{-1}(y-\ell)]\ \text{Re}\hat{K}_o \exp j(\omega t-\beta y)$$

Use the approach illustrated in Sec. 5.17 to show that the total force on the conducting slab and its highly permeable backing is

Prob. 6.6.3 (continued)

$$\langle f_y \rangle_t = -\frac{w\mu_o|K_o|^2}{\pi}\,\mathrm{Re}\int_{-\infty}^{+\infty}\frac{j\sin[\frac{(k-\beta)\ell}{2}]dk}{(k-\beta)^2\sinh^2 kd[\frac{k}{\gamma}\frac{\mu}{\mu_o}\coth\gamma a+\coth kd]}$$

where $a\gamma = \sqrt{(ak)^2 + j\,S_M}$, $S_M \equiv \mu\sigma a^2(\omega-kU)$

For Section 6.7:

Prob. 6.7.1 The conducting layer of Fig. 6.7.1 represents the only lossy element in a linear induction machine. Arrangement of air gaps and magnetic materials is arbitrary. Special cases are the configurations of Fig. 6.4.1 and 6.6.1. Stator windings impose a pure traveling wave having phase velocity ω/k in the y direction. With P_m and P_d defined as the time average mechanical power output and electrical dissipation, respectively, the electrical power input is $P_m + P_d$. Show that the efficiency, $E_{ff} \equiv P_m/(P_m+P_d)$, is $U/(\omega/k)$. Define the "slip" by $s \equiv [\omega/k)-U)]/(\omega/k)$, and show that $E_{ff} = 1-s$.

Prob. 6.7.2 In terms of the same variables as used to express the time average force (Eq. 6.6.10), determine the time average electrical dissipation for the induction machine of Fig. 6.6.1.

For Section 6.8:

Prob. 6.8.1 A high frequency magnetic field is used to raise a liquid metal against gravity, as shown in Fig. P6.8.1. The skin depth is short compared to other dimensions of interest. Express the magnetic surface force density acting on the interface at the right in terms of the power dissipated in the liquid. What is the height ξ as a function of the power dissipated? (See Section 7.8 for the modicum of fluid statics needed here.)

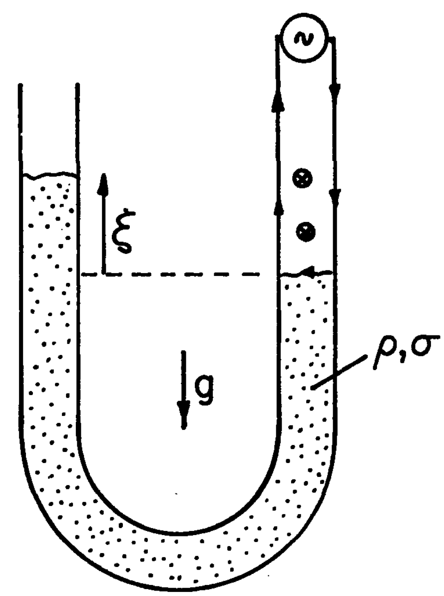

Fig. P6.8.1

For Section 6.9:

Prob. 6.9.1 Carry out the similarity transformation converting Eq. 6.9.3 to Eq. 6.9.7.

Prob. 6.9.2 A container holds a layer of liquid metal having depth b and length ℓ, as shown in Fig. P6.9.2. The system extends far enough in the z direction that it can be regarded as two-dimensional. At a distance h(y) above the interface is a bus-bar. Alternating current passes through this bar in the z direction and is returned through the liquid metal in the opposite z direction. Because the skin depth in both conductors is short compared to h(y) and b, magnetic flux is essentially ducted between the bus and the liquid metal, as sketched. The field throughout the air gap therefore has the same temporal phase. In the spirit of a quasi-one-dimensional model, in the air gap $\vec{H}$ has the zero order dependence $H_y = H_o a/h$, where $H_o = \mathrm{Re}\,\hat{H}_o\exp(j\omega t)$ is the field intensity at the left where y = 0. The slope of the bus, dh/dy, at y = 0 is given as S.

(a) Find H_y in the skin region of the liquid using the boundary layer model, Eq. 6.9.1. Assume that the fluid velocity has a negligible effect.

(b) Use the divergence law, Eq. 6.9.2, to approximate the normal flux density at the interface.

(c) Find the time average magnetic shearing surface force density acting over the thin skin layer.

(d) Show that if this quantity is to be independent of y, the bus geometry must be $h = a[1 - 2S(y/a)]^{-\frac{1}{2}}$.

(e) Show that this uniform surface force density is

$$\langle T_y \rangle_t = \frac{\mu_o}{4a}|\hat{H}_o|^2\, S\,\delta$$

Prob. 6.9.3 For the configuration described in Prob. 6.9.2, find the total power dissipation in the lower metal.

Prob. 6.9.4 For the configuration considered in this section, the magnetic structure has a total length L. As a function of time and y, compute the power dissipation in the conductor. What is the total power dissipation?

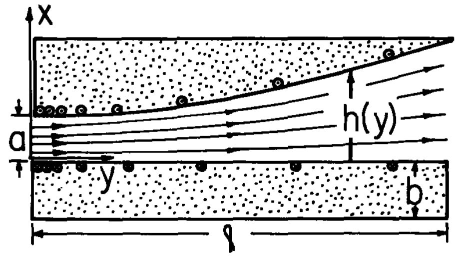

Fig. P6.9.2

For Section 6.10:

Fig. 6.10.1 A uniformly conducting slab of thickness 2a and permeability μ moves in the z direction with velocity U. To either side of the slab are air gaps of thickness d backed by infinitely permeable materials. Thus, half of the system is like that of Fig. 6.6.1 for $x > 0$, with $x=0$ a plane of symmetry. Because of the symmetry, temporal modes can be divided into those that are even and odd in H_y. Show that the odd modes are represented by Eq. 6.10.6. Find the analogous expression for the even modes, representing the graphical solution by a sketch similar to that of Fig. 6.10.2.

Prob. 6.10.2 A uniformly conducting circular cylindrical shell has outer radius a and inner radius b and spins about the z axis with angular velocity Ω. The regions outside and inside the shell are filled by infinitely permeable material. The system is long in the z direction compared to the outer radius a. However, the distance a-b is not small compared to the outer radius a.

(a) Find eigenfrequency equations from which the frequencies of the temporal modes can be determined. (The expression can be factored into two somewhat simpler expressions that define two classes of modes.)

(b) Define as a parameter the ratio b/a, and $\gamma a \equiv \sqrt{j\mu\sigma a^2(\omega-\Omega)}$ as another parameter representing the frequency. Describe how you would solve for the eigenfrequencies.

Prob. 6.10.3 A spherical shell has radius R and spins about the z axis with angular velocity Ω. It has a surface conductivity σ_s and is filled with an insulating material having permeability μ.

(a) Starting with the boundary condition, Eq. (d) of Table 6.3.1, find the temporal modes.

(b) Find the decay time resulting if a uniform external field directed along the z axis is suddenly turned off.

(c) What is the frequency of the temporal transient if a uniform field perpendicular to the z axis is suddenly turned off?

Prob. 6.10.4 For the configuration described in Prob. 6.6.2, the excitation is suddenly turned on or off. The resulting transient is initiated with the same k as imposed by the excitation.

(a) Find the transcendental equation that determined the eigenfrequencies of the temporal modes.

(b) Outline a procedure for numerically determining the eigenfrequencies. (Hint: Is it plausible that an infinite number of roots exist where the frequency measured in the frame of reference of the rod is purely imaginary?)

Prob. 6.10.5 In a configuration that generalizes that of Fig. 6.6.1, the entire region $0 < x < a+d$ is filled by a nonuniform conductor having conductivity $\sigma(x)$ and velocity $\vec{v}=U(x)\vec{i}_y$. Note that the uniformly conducting material partially filling the air gap and suffering rigid-body motion is a special case. Start with Eq. 6.2.6, keeping the x dependence of σ and U so that the expression is valid over the entire range of x. Show that the amplitudes $\hat{A}_n$ of the vector potential modes satisfy an orthogonality condition which is Eq. 6.10.12 with $\hat{J}_n \rightarrow \sqrt{\sigma(x)}\,\hat{A}_n$.