

## Chain Rule for the Slope of $f(g(x))$

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### Chain Rule for the Slope of $f(g(x))$

$y = g(x)$   $z = f(y)$   $\rightarrow$  the chain is  $z = f(g(x))$   
 $y = x^5$   $z = y^4$   $\rightarrow$  the chain is  $z = (x^5)^4 = x^{20}$   
Average slope  $\frac{\Delta z}{\Delta x} = \left(\frac{\Delta z}{\Delta y}\right) \left(\frac{\Delta y}{\Delta x}\right)$  Just cancel  $\Delta y$   
Instant slope  $\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx} =$  **CHAIN RULE** (like cancelling  $dy$ )  
You **MUST** change  $y$  to  $g(x)$  in the final answer

Example of chain  $z = y^4 = (x^5)^4$   $\frac{dz}{dy} = 4y^3$   $\frac{dy}{dx} = 5x^4$

Chain rule  $\frac{dz}{dx} = \left(\frac{dz}{dy}\right) \left(\frac{dy}{dx}\right) = (4y^3)(5x^4) = 20y^3x^4$

**Replace  $y$  by  $x^5$  to get only  $x$**   $\frac{dz}{dx} = 20(x^5)^3x^4 = 20x^{19}$

CHECK  $z = (x^5)^4 = x^{20}$  does have  $\frac{dz}{dx} = 20x^{19}$

1. Find  $\frac{dz}{dx}$  for  $z = \cos(4x)$  Write  $y = 4x$  and  $z = \cos y$  so  $\frac{dz}{dx} =$

2. Find  $\frac{dz}{dx}$  for  $z = (1 + 4x)^2$  Write  $y = 1 + 4x$  and  $z = y^2$  so  $\frac{dz}{dx} =$

CHECK  $(1 + 4x)^2 = 1 + 8x + 16x^2$  so  $\frac{dz}{dx} =$

### Practice Questions

3. Find  $\frac{dh}{dx}$  for  $h(x) = (\sin 3x)(\cos 3x)$

Product rule first Then the Chain rule for each factor

$$\frac{dh}{dx} = (\sin 3x) \frac{d}{dx}(\cos 3x) + (\cos 3x) \frac{d}{dx}(\sin 3x)$$

$$= (\sin 3x)(\text{CHAIN}) + (\cos 3x)(\text{CHAIN}) = ?$$

Chain Rule for the Slope of  $f(g(x))$ 

4. Tough challenge: Find the **second derivative** of  $z(x) = f(g(x))$

$$\begin{array}{l} \text{FIRST} \\ \text{DERIV} \end{array} \quad \frac{dz}{dx} = \left(\frac{dz}{dy}\right) \left(\frac{dy}{dx}\right) \quad \begin{array}{l} \text{Function of } y(x) \\ \text{times function of } x \end{array}$$

$$\begin{array}{l} \text{PRODUCT} \\ \text{RULE} \end{array} \quad \frac{d^2z}{dx^2} = \left(\frac{dz}{dy}\right) \frac{d}{dx} \left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right) \frac{d}{dx} \left(\frac{dz}{dy}\right)$$

$$\begin{array}{l} \text{SECOND} \\ \text{DERIV} \end{array} \quad \left(\frac{dz}{dy}\right) \left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right) \left(\frac{d^2z}{dy^2}\right) \left(\frac{dy}{dx}\right) \quad \frac{dy}{dx} \text{ twice!}$$

$$\text{Check } y = x^5 \quad z = y^4 = x^{20} \quad \frac{dz}{dx} = 20x^{19} \quad \frac{d^2z}{dx^2} = 380x^{18}$$

$$\begin{array}{l} \text{SECOND} \\ \text{DERIV} \end{array} \quad (4y^3)(20x^3) + (5x^4)(12y^2)(5x^4) \quad 80 + 300 = 380 \text{ OK}$$

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Resource: Highlights of Calculus  
Gilbert Strang

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