

# Table of Contents

## Essay 1: Dimensional Analysis of Models and Data Sets: Similarity Solutions and Scaling Analysis

<b>1</b>	<b>About dimensional analysis</b>	<b>4</b>
1.1	The goal and the plan . . . . .	4
1.2	About this essay . . . . .	5
<b>2</b>	<b>Models of a simple pendulum</b>	<b>5</b>
2.1	A physical model . . . . .	6
2.2	A mathematical model . . . . .	6
2.3	Models generally . . . . .	8
<b>3</b>	<b>An informal dimensional analysis</b>	<b>9</b>
3.1	Invariance to a change of units . . . . .	9
3.2	Natural units . . . . .	11
3.3	Extra and omitted variables . . . . .	12
<b>4</b>	<b>A basis set of nondimensional variables</b>	<b>13</b>
4.1	The mathematical problem . . . . .	13
4.2	The null space . . . . .	15
4.3	A basis set for the simple, inviscid pendulum . . . . .	16
<b>5</b>	<b>The viscous pendulum</b>	<b>18</b>
5.1	A physical model of the viscous pendulum . . . . .	19
5.2	Drag on a moving sphere . . . . .	20
5.2.1	Zero order solution . . . . .	21
5.2.2	The other nondimensional variables: remarks on the Reynolds number . . . . .	22
5.3	A numerical simulation . . . . .	23
5.4	An approximate model of the decay rate . . . . .	25
<b>6</b>	<b>A similarity solution for diffusion in one dimension</b>	<b>27</b>
6.1	Honing the physical model . . . . .	28
6.2	A similarity solution . . . . .	28
<b>7</b>	<b>Scaling analysis</b>	<b>31</b>
7.1	A nonlinear projectile problem . . . . .	31
7.2	Small parameter $\rightarrow$ small term? . . . . .	34
7.3	Scaling the dependent variable . . . . .	36
7.4	Approximate and iterated solutions . . . . .	37
<b>8</b>	<b>Summary and closing remarks</b>	<b>39</b>

## Essay 2: The Coriolis force

<b>1</b>	<b>Large-scale flows of the atmosphere and ocean.</b>	<b>4</b>
1.1	Classical mechanics observed from a rotating Earth . . . . .	8
1.2	The goal and the plan of this essay . . . . .	11
1.3	About this essay . . . . .	13
<b>2</b>	<b>Part I: Rotating reference frames and the Coriolis force.</b>	<b>14</b>
2.1	Kinematics of a linearly accelerating reference frame . . . . .	15
2.2	Kinematics of a rotating reference frame . . . . .	17
2.2.1	Transforming the position, velocity and acceleration vectors . . . . .	17
2.2.2	Stationary $\Rightarrow$ Inertial; Rotating $\Rightarrow$ Earth-Attached . . . . .	24
2.2.3	Remarks on the transformed equation of motion . . . . .	26
<b>3</b>	<b>Inertial and noninertial descriptions of elementary motions.</b>	<b>27</b>
3.1	Switching sides . . . . .	28
3.2	To get a feel for the Coriolis force . . . . .	30
3.2.1	Zero relative velocity . . . . .	31
3.2.2	With relative velocity . . . . .	32
3.3	An elementary projectile problem . . . . .	34
3.3.1	From the inertial frame . . . . .	34
3.3.2	From the rotating frame . . . . .	34
3.4	Appendix to Section 3: Circular motion and polar coordinates. . . . .	37
<b>4</b>	<b>A reference frame attached to the rotating Earth.</b>	<b>38</b>
4.1	Cancellation of the centrifugal force . . . . .	39
4.1.1	Earth's (slightly chubby) figure . . . . .	39
4.1.2	Vertical and level in an accelerating reference frame . . . . .	41
4.1.3	The equation of motion for an Earth-attached frame . . . . .	41
4.2	Coriolis force on motions in a thin, spherical shell . . . . .	42
4.3	Why do we insist on the rotating frame equations? . . . . .	44
4.3.1	Inertial oscillations from an inertial frame . . . . .	45
4.3.2	Inertial oscillations from the rotating frame . . . . .	47
<b>5</b>	<b>A dense parcel on a slope.</b>	<b>49</b>
5.1	Inertial and geostrophic motion . . . . .	54

1	<i>LARGE-SCALE FLOWS OF THE ATMOSPHERE AND OCEAN.</i>	4
5.2	Energy budget . . . . .	56
<b>6</b>	<b>Part II: Geostrophic adjustment and potential vorticity.</b>	<b>57</b>
6.1	The shallow water model . . . . .	58
6.2	Solving and diagnosing the shallow water system . . . . .	60
6.2.1	Energy balance . . . . .	61
6.2.2	Potential vorticity balance . . . . .	61
6.3	Linearized shallow water equations . . . . .	65
<b>7</b>	<b>Models of the Coriolis parameter.</b>	<b>65</b>
7.1	Case 1, $f = 0$ , non rotating . . . . .	66
7.2	Case 2, $f = \text{constant}$ , an $f$ -plane, . . . . .	70
7.3	Case 3, $f = f_0 + \beta y$ , a $\beta$ -plane, . . . . .	77
7.3.1	Beta-plane phenomena . . . . .	78
7.3.2	Rossby waves; low frequency waves on a beta plane . . . . .	82
7.3.3	Modes of potential vorticity conservation . . . . .	86
7.3.4	Some of the varieties of Rossby waves . . . . .	87
<b>8</b>	<b>Summary of the essay.</b>	<b>90</b>
<b>9</b>	<b>Supplementary material.</b>	<b>92</b>
9.1	Matlab and Fortran source code . . . . .	92
9.2	Additional animations . . . . .	93

## Essay 3: Lagrangian and Eulerian Representations of Fluid Flow: Kinematics and the Equations of Motion

<b>1</b>	<b>The challenge of fluid mechanics is mainly the kinematics of fluid flow.</b>	<b>4</b>
1.1	Physical properties of materials; what distinguishes fluids from solids? . . . . .	5
1.1.1	The response to pressure — in linear deformation liquids are not very different from solids . . . . .	6
1.1.2	The response to shear stress — solids deform and fluids flow . . . . .	9
1.2	A first look at the kinematics of fluid flow . . . . .	13
1.3	Two ways to observe fluid flow and the Fundamental Principle of Kinematics . . . . .	14
1.4	The goal and the plan of this essay; Lagrangian to Eulerian and back again . . . . .	17
<b>2</b>	<b>The Lagrangian (or material) coordinate system.</b>	<b>19</b>
2.1	The joy of Lagrangian measurement . . . . .	21
2.2	Transforming a Lagrangian velocity into an Eulerian velocity . . . . .	23
2.3	The Lagrangian equations of motion in one dimension . . . . .	24
2.3.1	Mass conservation; mass is neither lost or created by fluid flow . . . . .	24
2.3.2	Momentum conservation; $F = Ma$ in a one dimensional fluid flow . . . . .	28
2.3.3	The one-dimensional Lagrangian equations reduce to an exact wave equation . . . . .	30
2.4	The agony of the three-dimensional Lagrangian equations . . . . .	31
<b>3</b>	<b>The Eulerian (or field) coordinate system.</b>	<b>33</b>
3.1	Transforming an Eulerian velocity field to Lagrangian trajectories . . . . .	34
3.2	Transforming time derivatives from Lagrangian to Eulerian systems; the material derivative . . . . .	35
3.3	Transforming integrals and their time derivatives; the Reynolds Transport Theorem . . . . .	38
3.4	The Eulerian equations of motion . . . . .	41
3.4.1	Mass conservation represented in field coordinates . . . . .	41
3.4.2	The flux form of the Eulerian equations; the effect of fluid flow on properties at a fixed position . . . . .	44
3.4.3	Momentum conservation represented in field coordinates . . . . .	46
3.4.4	Fluid mechanics requires a stress tensor (which is not as difficult as it first seems) . . . . .	47
3.4.5	Energy conservation; the First Law of Thermodynamics applied to a fluid . . . . .	53
3.5	A few remarks on the Eulerian equations . . . . .	54
<b>4</b>	<b>Depictions of fluid flows represented in field coordinates.</b>	<b>55</b>
4.1	Trajectories (or pathlines) are important Lagrangian properties . . . . .	55
4.2	Streaklines are a snapshot of parcels having a common origin . . . . .	58
4.3	Streamlines are parallel to an instantaneous flow field . . . . .	58
<b>5</b>	<b>Eulerian to Lagrangian transformation by approximate methods.</b>	<b>60</b>

5.1	Tracking parcels around a steady vortex given limited Eulerian data . . . . .	60
5.1.1	The zeroth order approximation, or PVD . . . . .	60
5.1.2	A first order approximation, and the velocity gradient tensor . . . . .	61
5.2	Tracking parcels in gravity waves . . . . .	63
5.2.1	The zeroth order approximation, closed loops . . . . .	64
5.2.2	The first order approximation yields the wave momentum and Stokes drift . . . . .	64
<b>6</b>	<b>Aspects of advection, the Eulerian representation of fluid flow.</b>	<b>67</b>
6.1	The modes of a two-dimensional thermal advection equation . . . . .	68
6.2	The method of characteristics implements parcel tracking as a solution method . . . . .	70
6.3	A systematic look at deformation due to advection; the Cauchy-Stokes Theorem . . . . .	74
6.3.1	The rotation rate tensor . . . . .	77
6.3.2	The deformation rate tensor . . . . .	79
6.3.3	The Cauchy-Stokes Theorem collects it all together . . . . .	81
<b>7</b>	<b>Lagrangian observation and diagnosis of an oceanic flow.</b>	<b>82</b>
<b>8</b>	<b>Concluding remarks; where next?</b>	<b>86</b>
<b>9</b>	<b>Appendix: A Review of Composite Functions</b>	<b>87</b>
9.1	Definition . . . . .	88
9.2	Rules for differentiation and change of variables in integrals . . . . .	89

MIT OpenCourseWare  
<http://ocw.mit.edu>

Resource: Online Publication.Fluid Dynamics  
James Price

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.