NICHOLAS DIBELLA: OK. So last week, I described the quest that theoretical physicists are on today in coming up with a theory of everything. We don't know what the theory of everything is yet. Nobody knows. It would somehow have to reconcile the conflict between quantum theory and general relativity. It would have to be some kind of theory of quantum gravity.

But many people are certainly trying. And many of the optimistic people think that we're almost there. I personally have no idea because I'm not an expert in any candidate theories of quantum gravity. But there's a sense that we've definitely been making progress over the past hundred years.

And so when you think of all the progress we've made, it seems maybe it's conceivable that sometime in the future, one day, we'll finally have the theory of quantum gravity. We'll finally have the theory of everything. Whether that's in 50 years, or 100 years, or 500 years, I have no idea. Nobody knows. But that day might eventually come.

So it seems like we can understand the universe. The universe is an extremely complicated thing, but someday we might be able to understand it. One, in fact, famous quote of Einstein is that the most incomprehensible thing about the universe is that it is comprehensible.

But is the universe really comprehensible? Can we really understand everything? We certainly understand some things about the universe. Actually, we think we know a lot about the universe, just not everything. But could we really understand everything about the universe?

What do you guys think? Do you think we can actually understand everything? Have the theory of everything? No?

AUDIENCE: No.

NICHOLAS DIBELLA: No. Why not? Why are you all so pessimistic?

AUDIENCE: I'll be the optimist today.

AUDIENCE: Yes.

NICHOLAS DIBELLA: Yes, we can. OK. Why are you so optimistic?

AUDIENCE: Because I'm an optimistic person.

NICHOLAS DIBELLA: OK. That's a valid position to take. Why are you so pessimistic, though? I mean, we've been able to describe things that we can't even see. Things that are a billion or a million billion times smaller than a centimeter. I mean, we can see things that are so small. We have theories about them that really seem to work. We can see things that are incredibly small.

We've been making headway in these questions, like [INAUDIBLE] the beginning of time. Is time travel possible? So it's conceivable that someday we'll understand everything. It's conceivable.

I mean, I'm not going to-- I wouldn't put forth an opinion. But yeah, it's natural. I mean, it's natural to think that maybe we won't be able to.

I mean, the universe is a really big thing, right? We're small things. So maybe something really small can't find out everything about something that's really big.

AUDIENCE: Wait. What was the question?

NICHOLAS DIBELLA: Do you think it's impossible in principle to know everything about the universe? I mean, maybe not information about every particle, every bit of matter. But the laws at least. Is it possible in principle to know all the laws with 100% accuracy?

AUDIENCE: I thought we [INAUDIBLE] only scratched at the surface of [INAUDIBLE].

NICHOLAS DIBELLA: OK. Yeah, it's true that we've kind of scratched the surface. But many people think that we're actually getting close. Many people worked on quantum gravity, like string theory and things like that. But also, many people don't think we're close at all. Many people don't think we've scratched the surface. We're going in all the wrong directions. OK. Yeah.

Pessimism is certainly a valid position to take. Optimism might be. I don't know. OK.

So today, I'd like to-- well, for the first part of today's class, I'd like to look at the question of whether it's actually possible in principle to have a theory of everything. Could we really know everything?

So first, we need to backtrack. We need to backtrack. Take a look at these theories. So one of my aims for this class has been to describe these physical theories-- quantum mechanics, relativity, string theory, and so forth-- as conceptually as I could, just using ideas.

And I think to a large extent, it's possible to convey the sense of ideas that you have in these theories, like extra dimensions or time going shorter for one person than for another. But deep down, these theories are mathematical in nature. I mean, deep down when you look at the theories, they're just a bunch of equations. Strictly speaking, physics is just a bunch of equations.

But what physicists do is they look at these equations and they try to make meaning out of them. They try to interpret them in some kind of a way. And they say, a-ha. These mathematical ideas actually correspond to something in reality. But deep down, they're mathematical in nature. And the real way to understand them is mathematically.

So if anything I've said over the past six weeks has sounded vague-- I'm sure there are a lot of things I've said that have sounded vague, or anything that sounded imprecise or unclear, you'll just have to take my word that if you studied the theories mathematically, then eventually things would make a lot more sense.

I tried to explain them as clearly as I could without math. But eventually, you need math. Actually, more speculative to theories, like theories of quantum gravity, even the math is not clear. We don't really know what the right math to use is. So there's still many ideas and our current candidate theories of everything that are still very vague and imprecise.

But definitely everything about quantum mechanics and relativity, those are all-- pretty much everything about those, those are all very clear and precise. OK. Everyone fine with that? OK.

Now, I'd like to describe one of the most surprising results of modern mathematics. But first, I just want to ask the question, well, what is mathematics? Most of us when we're first exposed to math, we just see it as a bunch of tools or a bag of tricks. Just some interesting things about numbers, shapes. I mean, a lot of these things seem to find their place in the real world.

I mean, we see shapes around us, for example. I mean, here is a rectangle. There is a circle. The sun looks like a circle. We can count objects-- 1, 2, 3, 4. We can count things. So when we're first exposed to math, we didn't really see it as much more than something that's not really worth studying. Well, many of us at least. But there do exist people that study math for a

living. They're called mathematicians.

And to them, math is much more exciting than just a bag of tricks or just a set of tools. Although, many people do get excited about certain bags of tricks.

But to mathematicians, to a pure mathematician, someone who studies purely math, who doesn't care about what it has to do with the real world, the ideas in math, the objects and notions that mathematicians study, they really take on a life of their own.

There are a lot of things about math that seem to find their place in the real world. But mathematicians often tend to pride themselves on things that actually don't have anything whatsoever to do with reality.

Now, I said before that there are a lot of things that-- a lot of mathematical concepts that find a place in reality. Like we can count. There are integers or whole numbers. We can count 1, 2, 3. There are shapes. But there are many things, many abstract mathematical ideas, that don't find themselves in reality.

For example, imaginary numbers. What's that?

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: Yeah. So imaginary numbers are numbers that are square roots of negative numbers. They're these things that are square roots of negative numbers.

So when we first learn about multiplication, we first learn about numbers that aren't imaginary. We call them real numbers. So numbers like 3, negative 4, 6.9. And we can take a number and we can square it. And to square a number, it just means to multiply it by itself.

So for example, 3 times 3 is 9. We say that 9 is the square root of 3.

Negative 1 times negative 1 is positive 1. So 1 is the square of negative 1. And so whenever you multiply a positive number, a number greater than 0 by itself, you get another positive number. And whenever you multiply a negative number by another negative number, you always get a positive number. So the square of a number is always something that's positive. Or else it's 0. 0 times 0 is 0. We call those real numbers. So these are the real numbers.

This sort of number.

AUDIENCE:	[INAUDIBLE]
NICHOLAS DIBELLA:	What's that?
AUDIENCE:	[INAUDIBLE]
NICHOLAS DIBELLA:	Oh. Reals. OK. Those are the real numbers. But you can ask the question, does there exist any kind of number which when you square it gives a negative number? Like does negative 1 have-- we call it the square root. So we say 3 is the square root of 9. Negative 1 is a square root of positive 1. But does negative 1 have a square root? Is there any number, which when you multiply by itself gives negative 1? And there doesn't seem to be any real number that gives negative 1 as its square. So you can invent a new kind of number.
	You can say, well, let's invent a new kind of number. Let's call it an imaginary number. And let's give it the symbol i. Then me say i times i equals negative 1. Problem solved. You can get square roots of negative numbers by making imaginary numbers.
	Similarly, 2i times 2i equals negative 4. So these are the imaginary numbers.
	Now, real numbers seem to appear everywhere in the world. I can count people-- 1, 2, 3. I can measure the length of some stick. Maybe this is 2.3 feet long. So real numbers appear everywhere.
	Imaginary numbers? Not so much. Does anyone see imaginary numbers when they look around the world? Maybe if you see imaginary people, then they might remind you of imaginary numbers. So imaginary numbers don't seem to find any place in the real world. But mathematicians studying things like-- yeah?
AUDIENCE:	The only place I've seen imaginary numbers would be my math team. We were all numbered imaginary numbers. I was number square root of negative 5.
NICHOLAS DIBELLA:	OK. Yeah, so imaginary numbers might show up in recreational activities involving math people in everyday life. Yeah, certainly. But you definitely don't count in imaginary numbers. We don't measure things in imaginary numbers. That's definitely true.
	But imaginary numbers are things that mathematicians study. And they don't seem to have anything to do with the real world. It's just an abstract idea. The imaginary number system. It's

this abstract idea that mathematicians study.

The very simple abstract system. I mean, this is all there is to it. But there are many other abstract mathematical systems that mathematicians study that don't seem to have anything to do with the real world.

Now, imaginary numbers actually turn out to be very useful in quantum mechanics. But you definitely don't measure anything to be imaginary. In the theory, imaginary numbers actually show up in the quantum states, in the wave function. It's actually an imaginary thing. OK.

So to a mathematician, there exists a mathematical kind of existence. There is some abstract mathematical truth that kind of exists out there independently of us. But I'll have a lot more to say about the nature of mathematical existence later. OK.

So all mathematical systems have some basic axioms or some basic postulates that make the systems. So remember a couple of weeks ago when I talked about special relativity? Special relativity had what I called two postulates, There are these two fundamental principles. And from these two fundamental principles, everything about the theory you can derive.

You can derive the fact that moving clocks run slow. The fact that lengths gets shorter for moving objects. All these facts can be derived from two fundamental principles. We call them postulates.

The situation is similar in math. All mathematical systems also are built from fundamental postulates or axioms. So I'll use both for it.

So axioms. That's a cool word, I think. So fundamental premises of math.

How many of you ever studied any geometry at all? OK. A good number of you. So how many of you have heard of Euclid's axioms or Euclid's postulates? Well, it turns out that all of geometry that you've probably studied can be traced back to some-- to a certain number of fundamental postulates. So all the geometry that is that you do on flat surfaces, like on this chalkboard.

You can draw a circle. Draw a line. A triangle. You can draw geometrical objects. All geometry that you can draw on a flat surface, we call that plane geometry. And all the facts that you can derive about these things can all be traced back down to some fundamental axioms.

And the axioms of plane geometry are called Euclid's axioms. So this is one type of mathematical system. I guess that's OK. So the first axiom is that any two points can be connected by a straight line. So I'll just write down these axioms in pictures instead of words if I can.

So you have two points. The first axiom says you can take these two points and form a line. It's very simple. A line.

The second axiom says that if you have a line segment-- not an infinitely long thing. But if you have a line segment like this, there are two endpoints. You can always take a line segment and turn it into a line.

The third axiom says that given a line segment-- move this down a little bit. Given a line segment, you can always take this line segment and turn it into a circle. It's supposed to be a circle.

The fourth axiom says that all right angles are the same. Here. So right angle is just 90 degrees. So this thing is the same as-- I can do a little better. This thing is the same as this thing. All right angles are the same.

And then, the fifth axiom is actually a more interesting axiom. Because if you change it, then you can actually look at geometry on curved surfaces. Like general relativity is based on geometries of curved surfaces.

But on plane services, like on a chalkboard, the fifth postulate says that parallel lines never intersect. I'll write this in words.

So this line and this line. Use your imagination to pretend that they go forever. They never hit each other. They never intersect. So those are the two postulates. Sorry, the five postulates of Euclidean geometry. This is the geometry that Euclid, a Greek mathematician from the BCs, wrote down. These were his axioms. OK.

Are there any questions about Euclid's axioms? OK.

From these axioms, you can then derive things. You can then prove things, like the Pythagorean theorem. Just take a right triangle. Triangle with a right angle. Call this a. Call the length of this b. Call this c. And then, you can prove this fact just from these axioms.

If you're smart enough, you can prove them. Makes sense, right? Yeah, something you can do.

If you're smart enough, you can use these axioms. Use logic and deduce these facts.

You can also prove that the area of a circle-- the way to find the area of a circle. You look at the radius of a circle, r. And then it turns out you can prove that the area was just pi, 3.14159, et cetera. It's pi times r squared. That's the area of the circle.

And if you're smart enough, you can-- there's no limit to what-- who knows what you can do? You'll prove all sorts of crazy things about circles, triangles, heptagons, lines, line segments, angles. You can prove all sorts of crazy things about it. OK. So that's geometry.

You can also write down some axioms for arithmetic. And I'm going to define arithmetic as the math of whole numbers. I'll just define it very loosely. 0, 1, 2, 3, et cetera. Those are all whole numbers.

And the math. What kinds of things can you do with them? Well, you can add them. You can multiply them. And you can have a lot of fun.

So that's arithmetic. That's how I define arithmetic.

There are a certain number of axioms. I'm not going to write them down. They're a little more complicated than these axioms. These axioms are very intuitive. We can just look around us and see that Euclid's axioms seem to be similar to the real world.

The axioms of arithmetic are a little more complicated, but the story's pretty much the same. There's a certain number of them. And then, from those axioms you can prove things.

Prove things about whole numbers. Like, an odd number plus an odd number is an even number. An even number plus an odd number is an odd. And a lot more complicated things, too. A lot more complicated things, but I'll just leave them like that. OK. So that's how I define arithmetic.

Now, are you ready to have your mind's blown? Blown away? Ready? OK.

No, you're not ready? Well, you have to be ready soon.

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: OK. I just had to. I couldn't control it. You were spread, and now you're destroyed. OK.

Now, it turns out when you look at arithmetic, you look at these things that you can prove from arithmetic. We call them theorems. We just call them facts. Theorems. Facts about whole numbers.

Turns out that there exists true things about these numbers that you can't prove. You can't prove from the axioms. So in a sense, truth is more fundamental than provability. And this is called Godel's incompleteness theorem. Very loosely put.

AUDIENCE: Don't you need provability to establish truth?

NICHOLAS DIBELLA: Well, yeah. So how did I prove this you mean? [INAUDIBLE]

AUDIENCE: Or to prove things that are true, don't you have to prove it?

NICHOLAS DIBELLA: OK. So there are two kinds of proofs going on. There are proofs inside of arithmetic. So you're working with the axioms of arithmetic and you prove things about the numbers. Well, then you can step outside or the arithmetic. Step outside the system. Then, you can prove things about arithmetic, not things about-- you can prove things about arithmetic, not the stuff that arithmetic describes, but arithmetic itself.

So this is called doing meta-mathematics. Kind of just take a step outside of math. And so you can prove things about the way mathematics is done itself. The way proof happens itself.

And this was a major discovery that happened in the early 20th century. And this result was proven by a German mathematician named Godel. I'm sure I'm mispronouncing it. That's the best approximation of the pronunciation that I can do. It's called Godel's incompleteness theorem.

Arithmetic is in the sense incomplete. It doesn't describe all the nature of mathematical truth. There are some mathematical truths. Some truths about whole numbers in arithmetic that you just can't reach. They're just unreachable truths. So there's a much more precise meaning to this all, but the gist of it is that there exists-- there are true mathematical statements. Well, there are true arithmetical statements that can't be proven.

And what I mean by "proven," what I mean by when you prove something, the way you prove a theorem, what I mean by prove is you start off with the axioms. You start off with Euclid's axioms, and then you just apply deductive reasoning. You just apply logic to those axioms, and then you see what you can get. That's what I mean by prove. There exists true arithmetical statements that can't be proven. That's the famous Godel's incompleteness theorem. Now, why it's true is a vast, complicated story that I can't describe now.

But just to kind of tantalize you about why it's true, an important idea that comes up in the proof is self-reference. So there exists statements that can kind of refer to themselves.

Consider the sentence, this sentence is false. The statement that's referring to itself. It's also a very paradoxical-sounding sentence. Because if it's true, then it says it's false. So if it's true, it's false. But if it's false, then it's saying it's false, so it's true. So it's the famous liar's paradox. I'll just tantalize you all with that for now. OK.

Godel's theorem is actually more general than just applying to arithmetic. It turns out not only is arithmetic incomplete, but any system is incomplete if it satisfies three conditions. So the first condition is that there are a finite number of axioms.

There are just five axioms of Euclid. So that's finite. There's also just a finite number of axioms of arithmetic. So that's good.

The second condition is that the system has to be large enough to include arithmetic. I defined arithmetic with all these whole numbers, and then with a plus-- addition and multiplication. A system that's not as large as arithmetic is just a system that only has plus and no multiplication, or a system that just has times and no addition. So that would be an example of a system that's not as big as arithmetic. It's simpler than arithmetic. It's not as complex.

And so for that kind of a system, Godel's theorem actually doesn't apply. So if you have arithmetic with just multiplication, then every mathematical truth about that arithmetic can be proven. All those mathematical truths are reachable.

Of course, you could also make a system larger than arithmetic. You can add extra stuff. That would be a system larger than arithmetic.

The final condition is that it has to be consistent. The system has to be consistent.

And by "consistent," what I mean is that there are no contradictions. For example, one way

that Euclid's axioms could contain a contradiction, one way they could be inconsistent, is, well, let's say axiom 4 I said was all right triangles are congruent. They're all the same.

Maybe there's a sixth axiom that says all right triangles are different. So that's an obvious contradiction. So if you had a system like that, then it won't be consistent. And so Godel's theorem wouldn't apply.

So any system that has the three properties contains truths about it. So arithmetical then is a generalization. A generalization depending on the type of system.

If the system satisfies these three conditions, then there are truths about it that can't be proven. OK.

Are there any questions about what Godel's theorem says? Not about why it's true, but what it says? Yes.

AUDIENCE: So if the theorem says that there are true-- OK. For any system of mathematics with postulates, by the definition of a postulate is that it is true, but cannot be proven?

NICHOLAS DIBELLA: Yes. Yes, good point. So these postulates, these axioms, they themselves are unprovable. So for any system, you actually have some statements that are just unprovable.

I mean, you accept your postulates, and then you work from there. That's a good point.

So what I should have said here is there are true arithmetical statements that aren't postulates. That aren't axioms. You can never prove the axiom. You just accept them for whatever reasons you want.

Maybe they model the physical world. Maybe you think they're kind of cute. But yeah, more precisely there are true arithmetical statements that aren't axioms. Good point.

Any other questions about Godel's theorem? OK.

As I said before, the theories that we use to describe nature, all the theories we've ever used, they've always been mathematical. I mean, math seems to be the language that the university uses. There's just no getting around using math to describe the universe. And so the system that described the theory of everything that describes the universe would have to have some kind of mathematical system.

And so you could ask the question, does that mathematical system satisfy these three conditions? If it does, then it would seem that there would be true statements about the universe that simply can't be explained.

We're used to explaining things that happen by ultimately going back down to a fundamental deep level. So for example, if I take a pen and I drop it, you can ask my why that happened and I can give you an answer. I can say, well, pens and pencils, they have this thing. They have this property. We call it mass.

And there are a lot of things that have mass. Pens have mass. The earth also has mass.

Now, there is a law of physics. There's a law of physics about the universe that says that anything that has mass attracts each other. Any two objects that have mass attract each other. This is the law of gravitation that Newton first described.

And when you ask me to explain why that happened, I can say, well, the pen has mass. The earth has mass. I'm no longer holding it, so they attract each other, and that's it. We can explain what happened. We can explain the observation.

If the theory of everything, if the mathematical system that describes the theory of everything satisfies the conditions of Godel's theorem, that there will be true things about the universe that we can't explain.

If that's the case, then that's really not good for people who want a theory of everything. If the theory is incomplete, then we couldn't explain everything. So it wouldn't be the theory of everything. And so if Godel's theorem applies to the mathematical system that described the theory of everything, then there is actually no theory of everything. There is stuff that we know about the universe, but then there's always going to be stuff that we don't know.

So now you can ask the question, well, does Godel's theorem apply? And the answer is maybe and maybe not. So we don't yet know what the mathematical system is that describes the universe.

People are certainly working on it. Maybe it's the mathematical system of string theory. Maybe it's the mathematical system of loop quantum gravity that I talked about last class.

There are certainly ways that these conditions could fail to be met. So the first condition that there are a finite number of axioms, that certainly seems like it would be the case for the

theory of everything.

I mean, every theory that we've ever written down has had a finite number of axioms. Remember, special relativity has two. Quantum mechanics has a few-ish. All these physical theories that we've ever written down, they've always had a finite number of axioms. So there's good reason to believe that the theory of everything also has a finite number of axioms. But it could very well not have a finite number of axioms.

And if it does, if it has an infinite number of axioms, then Godel's theorem wouldn't apply and it would be possible to know everything about the universe. It would be possible to have a theory of everything.

But of course, how you would actually store that theory of everything if it has an infinite number of items in it, that's another issue. OK.

The third condition could also fail to be met. Well, it could, in principle. But I personally have a hard time understanding what a system that's consistent would mean.

If a system is inconsistent, then it would say that contradictions are true. So something is true and it's not true simultaneously. I just have a hard time understanding that.

Does anybody have an intuition for understanding true contradictions? Yeah, I have a hard time. But actually, there exists philosophers and logicians. People that study logic that have actually come up with meaningful ways of understanding true contradictions.

I don't know that much about it, but I do know it has a name. So one way it could fail to be met is if there's this aspect about the mathematical system called dialetheism, or something like that. I don't actually know how it's pronounced because I haven't really talked about this in person with experts. But I found this word before, and I've read a little bit about it.

So I mean, there are some people that think you could have true contradictions and have contradictions manifest themselves in the real world in a meaningful way. I don't know. It's hard for me to understand what a true contradiction so.

So it seemed that the mathematical system should be consistent. I mean, just intuitively. It might not be meaningful to have an inconsistent system, that's all I'm saying.

Finally, the second axiom. The second axiom is the one that's most likely to have a chance of

not being met. So if the system is large enough to include arithmetic, and if the other two axioms are met-- I'm sorry, the other two conditions are met, then Godel's theorem would apply.

So it turns out that real numbers are actually simpler than whole numbers. Whole numbers seem pretty simple. Just 0, 1, 2, 3.

But if you tried doing arithmetic with real numbers, with numbers like negative 5, 6.8, pi, square root of 2. Rational numbers, irrational numbers, you put them all in the mix, and tried to do arithmetic with those numbers, it turns out that it's actually simpler than arithmetic. And the system ends up being complete, it turns out.

And actually for a similar reason, Euclidean geometry, Euclid's axiom, that's also a complete system. So it could be that the mathematical system just isn't very complex. It's not as complex as arithmetic. And so it could fail to meet that requirement.

There are a lot of other mathematical systems that are simpler than arithmetic. And if that system is the system that the theory of everything uses, then Godel's theorem wouldn't apply and there would be hope in getting a theory of everything.

But there are many other systems that are more complex than arithmetic, and that might be the system that you use for the theory of everything. And so then, Godel's theorem would apply if the other two conditions were also met. And so then the dream of the theory of everything would be down the hall and impossible. OK.

Anybody have any questions about that? Some people would be sad, I think, if it's impossible to have a theory of everything. Some people want to know everything about the universe. But Godel's theorem says that there's a chance that that might not be possible. It might not be possible in principle to have a theory of everything.

And so a lot of people, I'm sure would be sad, if we found that out to be the case. I mean, we don't know if it's the case. It might be the case. It might not. It depends on the mathematical structure of the universe how it all works out. Yeah.

AUDIENCE: I think people need a quest for something. If we find out everything, we find out everything about the universe [INAUDIBLE].

NICHOLAS Yeah, that's a good point.

DIBELLA:

AUDIENCE: Because I think we all need-- whatever quest. We always need to attain something. It doesn't have to be a physical thing, but we need to reach a goal. If there are no goals, people will quickly become kind of bored or--

NICHOLAS DIBELLA: Yeah, that's a good point. I mean, what's more important? Is it the conclusion of the journey or the journey itself that's more important?

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: Yeah. So that would be good news for physicists. Physicists would always have something to do. Similarly, with mathematicians. I mean, mathematicians knew this a long time ago. So mathematicians have this set of truths that they can always-- I mean, there's always more math to discover. There's always more math to discover. And the same might be the same for physicists. There might always be new physics to discover, new things to learn about the universe.

AUDIENCE: You wouldn't need physicists [INAUDIBLE].

NICHOLAS DIBELLA: Yeah. Physics would be out of a job.

AUDIENCE: Exactly.

NICHOLAS DIBELLA: But then again, physicists are very clever and very skillful. So they'd have no problem finding a new job. But they might not be happy. OK. Let's see. OK.

So [INAUDIBLE] mentioned a good point before, which is that even if the system is complete, even if Godel's theorem doesn't apply, then there would still be some things that were left explaining, some unprovable things according to the theory. The axioms. There would still be the axioms leftover.

You still might be wondering, well, why is this the mathematical system? Why these axioms? Why not others?

The theory itself has no explanation for it. The theory explains the physical universe, but it doesn't explain itself. So to explain it, to explain the theory, you need some other kind of reasoning. And I'll get to that. I'll get to that probably next class why those axioms, not others.

So now we'd like to take a brief digression and talk about types of existence. So up to now, I've been pretty casual in referring to the theory of everything. And last class, I said there's matter. There's energy. There's forces. That's it. That's everything. It's everything. Let's get a theory for that stuff.

But you can still ask the question, what is energy? What is matter? What are forces? And these are actually the types of questions, these deep questions-- "what is" questions. These kinds of questions, scientists tend to have a habit of shying away from them. Because they'll say, oh, it's not a scientific question.

We only care about things that are scientific. We only care about experiments and observations. The questions like, what are forces, what is energy-- those Kind of questions-- those are philosophical questions, so we don't care about them. That's what many scientists say

but I personally am kind of confused about this response. How can anybody do science when you don't really understand what science is about? I mean, if I have a theory about an electron, I'd like to know what the electron is. I'd be a lot more comfortable knowing what the electron is. So this leads to a big question.

Namely, what is reality? What is all this stuff? Where shall I write it? Because it's very big, I'm going to write it in big letters. What is reality?

So now, I'd like to discuss a few kinds of existence, types of existence. So one type of existence that people think about is mathematical existence. Well, let's start with mathematical existence. One type of existence is mathematical existence. This is types of existence.

So before, when I was describing math, how mathematicians think of it, a lot of mathematicians like to think of math as something that exists out there. There are these mathematical truths that we try to discover. There are these abstract things that are true. There's the abstract concept of a line, the abstract concept of a number that exists out there in a mathematical world. So there is a mathematical kind of existence.

The second kind of existence? A physical existence. Well, let's just abbreviate this by math. Let's abbreviate this by matter.

When we look around us, we see things. We see matter. We see energy. We see forces. And

generally, we call them physical things. We just give them a name. Not really explaining what they are, we just kind of give them a name. We call them physical things.

So the theory of everything aims to describe the physical universe. It tries to describe physical existence. So I'll abbreviate that by matter.

And finally, a third kind of existence that people think about is mental existence. Mind. This is the world inside of our heads.

We have a conscious experience. We have perceptions. We have thoughts. We have ideas. We have emotions. And it seems like this is like a completely separate kind of thing from physical existence or a mathematical existence, at least on the face of it. It seems to be a wholly separate kind of thing. So mental existence. So we have math, matter, and mind.

Now, there are a lot of ways that these different kinds of existence can be related to each other. I'll start off with mind and matter.

So probably most scientists who study the brain, most neuroscientists, most cognitive scientists, they would say that the mind is actually something that emerges from something that's physical. So there are these physical systems. We call them humans. There's this part, this region inside of humans-- this area has a physical thing called the brain, which consists of all these atoms and molecules and electrical signals, and so forth. And when everything's put together the right way, you get a mind. That's how a mind forms.

It's true that there's still a lot to be learned, the scientists say, about the nature of the conscious experience, how it really emerges from physical things. Nonetheless, the view among most neuroscientists and cognitive scientists is that the mind is a physical thing.

And so mind actually arises from matter. So matter gives rise to mind.

Now, there are some people that don't believe this. Probably, minority. They believe that mind and body are actually separate things. This is the mind-body problem.

AUDIENCE: [INAUDIBLE] talking about a soul?

NICHOLAS DIBELLA: Maybe it's some kind of a soul or some kind of a spirit. Or maybe it's just some kind of-- just a different thing. Just a different thing. Who knows what kind of thing it is? It just has a different kind of exist-- it's a different kind of substance, different kind of existence.

I won't try to go into the mind-body debate. It's sufficiently complex to make me afraid of trying to describe it all in full detail. But there's a debate. And most scientists would say that it's in this direction-- mind emerges from matter. But some people would say that these are actually separate. They call themselves dualist. There's a dual nature to mind and body.

Let's look at another possible connection. Let's see.

Mathematical existence and mental existence. So there's a question that philosophers of mathematics have asked for a long time, which is, is math discovered or invented? Is math something that exists out there as a lot of mathematicians believe? A lot of non-mathematicians as well. Is it something out there that is uncovered, the truth is uncovered, it's discovered, exists out there? Or is math something that's invented? Is it a product of the human mind? Is it simply something that creative people come up with? Something that emerges from human creativity?

I'm also not going to try to go into this in any detail, but there's a debate whether math is discovered or invented. If it's invented-- that is, if math is something that is a product of the mind-- then it would seem to be that mind gives rise to math. So you can draw an arrow like this.

On the other hand, it could be that math is something that's discovered. It's something that's completely different from mental existence. And so they could actually be separate from one another.

So we can draw an interesting diagram of these three kinds of existence. We could put mind here. A little more space. Mind here. Let's put math here. And matter here.

So if both of these types of connections are true for example, if matter gives rise to mind and mind gives rise to math, then we can draw it an arrow this way. Matter gives rise to mind, and then mind gives rise to math, so we can draw an arrow this way. And then, this would be the connection between the three of them.

Now, you can also draw other possible combinations or get rid of the arrows all together. You could say, for example, that matter and mind are different, but mind does give rise to matter. You could say something weird. Most people wouldn't say it, but mind gives rise to matter. The math is something different. You can do all sorts of combinations. All sorts of them.

But there's a big question. There's a big question confronting all these kinds of connections,

which is, why is math so darn effective at describing the universe? Why is it so effective at describing reality? It's puzzling in pretty much all types of the combinations.

If math is separate from matter, that's probably the most puzzling. Probably the most puzzling case. Because in that case, there is a physical universe, which we usually think of as being real, more real than all the other kinds of existences, if they're real at all. And there is this abstract thing called math that describes it. Why should the real universe be described by something that doesn't really exist? Why should it be described as some abstract thing?

So you have this puzzling question, why is math so effective at describing reality? Can anybody give an explanation? Well, don't worry if you can't because many people have thought about this over the years and they've all been pretty puzzled s it. They've all been really puzzled by the unreasonable effectiveness of mathematics as Eugene Wigner put it.

So you have a choice. You can either accept this puzzling fact or you can try to explain it. So I'd like to give one possible explanation for it now. Hopefully, I'll have enough time to explain it.

But first-- OK. So I'll give one explanation for it, which is essentially that the universe is mathematical. It's a mathematical thing. And therefore, you shouldn't be surprised that math describes it so well. And this is called the mathematical universe hypothesis. It's proposed by Max Tegmark, who's a cosmologist here at MIT.

So it starts out by assuming that mental existence isn't the only kind of existence. So it starts off with something called the external reality hypothesis.

I mean in principle, strictly speaking, the only thing we know for sure exists is our mind. I think, therefore I am. Famous words of Descartes. So it's quite conceivable that the mind is all that exists.

Ordinarily, we like to think that there's a whole lot of reality in addition to just our mind. I like to think there are people out there that also have minds. There are things that-- the things that exist in addition to me. So there is this external reality hypothesis. An external world exists. So it's based on this hypothesis. But you might not believe this hypothesis.

You might want to believe that your mind is the only thing that exists. In other words, might want to believe in something called solipsism. I'm all that exists, and that's it. I exist and nothing else.

So the external reality hypothesis rejects solipsism. How many of you are solipsists? How many of you believe in the solipsism? You in the green shirt? Or is your friend joking? OK. So most of us don't believe it.

So why not? Why don't you believe in solipsism? Why don't you believe that you're all that exists? Yeah.

AUDIENCE: I don't know. I think that's kind of an egocentric way to look at the universe. I'm the only thing that exists. And therefore, nothing else does. I mean, I understand it-- the idea. But the whole idea that you're the only thing that exists is nothing else. And there's no [INAUDIBLE] possibility of something else existing, be it another person or matter, whatever. I find that a little egocentric. You know, I'm the only thing.

NICHOLAS DIBELLA: So one objection is that solipsism is egocentric. Yeah. Just in case they can't hear it.

Yeah. I mean, it's kind of an arrogant position to take. But what's wrong with arrogance? What's wrong with egocentrism?

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: Another justification?

AUDIENCE: I don't think it's egocentric. It's depressing. Who wants to be the only guy out there?

NICHOLAS DIBELLA: So yeah, it's depressing. Why should reality care about what's depressing for you? Why is that a valid argument?

AUDIENCE: I reserve my right to respond-- never mind.

NICHOLAS DIBELLA: I mean, you have to think a bit about this to really give a good explanation for or against him. The first responses we get are usually emotional ones.

I mean, it's egocentric or that would be depressing. I'm not going to believe it. But let me ask for some others. Yeah.

AUDIENCE: It seems like really unreal. Because like, if I'm the only that exists, then everything [INAUDIBLE].

NICHOLAS DIBELLA: Everything else is what?

AUDIENCE: Everything else that we're seeing must be false.

NICHOLAS DIBELLA: So if solipsism is true, then our reality is kind of like an illusion? Why should reality care about whether it's an illusion for you?

AUDIENCE: Well, it's sad.

NICHOLAS DIBELLA: It's sad, yeah.

AUDIENCE: [INAUDIBLE] lied to and stuff.

NICHOLAS DIBELLA: Yeah. Well, maybe reality is a liar.

AUDIENCE: [INAUDIBLE] then don't you exist to?

NICHOLAS DIBELLA: Well, I mean, I can certainly say I exist. I don't know if you exist. I mean, I definitely know I exist, but I don't know if you exist. Yeah.

AUDIENCE: It's not possible to have two [INAUDIBLE].

NICHOLAS DIBELLA: It's not possible to have what?

AUDIENCE: It's not possible to have two people that believe in that because one of them shouldn't exist [INAUDIBLE].

NICHOLAS DIBELLA: So it's not possible to have two people believe in solipsism you're saying? Well, the way that solipsists would respond is by saying, well, I'm the only person that exists. I mean, there are all these other figments of my imagination essentially. They don't actually exist. They just exist in my mind. And these things, these objects in my mind, think that they exist. But really, I'm all that exist.

Define exist. Hard question. I can't define. I don't know how to define that.

I don't know how to define it. But that's how solipsists would respond to your defense

[INAUDIBLE]. Yeah.

AUDIENCE: There's no way to prove that anything but yourself exists. So isn't more solipsism more plausible than the external reality?

NICHOLAS DIBELLA: OK. So the only thing that we know for sure exists, the only thing that we can definitely prove, at least to ourselves, is that we exist. So you're saying, well, isn't it more plausible?

Well, it's definitely more conservative. I mean, it's actually-- I mean, it's quite a leap-- you say, well, I exist. Does anything else exist? It's quite a leap to say, well, other things also exist. So solipsism is actually a very conservative viewpoint to take. The external reality of all this is quite liberal in this respect. So you might want to stick just with conservative positions. I mean, you don't want to make any radical assumptions. Yeah.

AUDIENCE: Well, one way I look at is there's a lot of things in the world that I don't know how to do, or languages I don't know. And I don't think my mind would invent things that I don't know. Like, why would my mind come up with French if I don't speak French? Why would my mind come up with something going to the moon if I can't go to the moon.

NICHOLAS DIBELLA: Yeah. So you're saying, why would my mind come up with all this stuff that's seems to be out of my control, or something like that, right? That's also a good response.

Any other takes? Yeah.

AUDIENCE: Things don't go the way we want it to go.

NICHOLAS DIBELLA: Things don't go the way we want it to go. But then, it could be that there are really two types of-- there are really two parts about it. There's a conscious part and there's an unconscious part. The conscious part is what we can control, the unconscious part is what we can't control.

I mean, there's a thing that we call "we" or "I." It can control some things, but it can't control other things. So that would be a way of explaining it from a solipsist's perspective. Yeah.

AUDIENCE: It's for our mind to develop this amazingly complex reality where the human race quote unquote, "knows everything," or at least a whole darn lot. Our minds would need to be super-powerful, or something.

AUDIENCE: [INAUDIBLE]

AUDIENCE: I don't think that's true.

AUDIENCE: No. It actually-- that actually has been debunked. I'm just saying, the thing is we use 10% of our brains. But we use different parts of it at different times [INAUDIBLE].

NICHOLAS DIBELLA: I think we only use 10% of our hearts. I forgot what that was from.

AUDIENCE: *Wedding Crashers.*

NICHOLAS DIBELLA: OK.

AUDIENCE: Anyway.

NICHOLAS DIBELLA: Yeah, anyway. Yeah. So one response is that, well, the mind would have to be this really complex thing to account for what we observe, right? Physics gives a quite simple explanation of everything. Whereas, proposing some kind of a very complex mind that has a conscious part, an unconscious part, that's a very complicated explanation of everything.

And remember what I talked about last time. I talked about Occam's razor, which is that the simplest theory is more likely to be true. And so you might say, well, solipsism is actually a more complicated theory than non-solipsism-- a theory that has an external reality. And so by Occam's razor, you would say that solipsism is less likely to be true.

AUDIENCE: How is that more complex?

NICHOLAS DIBELLA: How is it more complex? OK. So according to solipsism-- well, as you pointed out before, there are parts of our experience that we can control and there are parts that we don't seem to be able to control. We have a conscious part and an unconscious part.

The unconscious part would seem to be what a non-solipsist would call the external world. It's a part of the world that we do scientific experiments on and that we try to learn about.

To a solipsist, he would call that an unconscious thing. He would call it the unconscious mind. So I'm going to actually explain my argument against solipsism, and then I'll slightly-- that will in the process explain-- answer your question. Yeah.

So I personally feel that solipsism is more complicated, more complex than non-solipsism. So

you start off by looking at the conscious versus the unconscious. The unconscious being the part that you can't control. And it's similar to what non-solipsists would call the external reality.

Well, if they're essentially the same. I mean, the unconscious mind, and then the external reality just then seem to be just different names for the same thing then. There's still the conscious mind in both cases, but the external reality just gets a different name.

So then, solipsism reduces to a case of non-solipsism with respect to the unconscious mind. But solipsism has a different view for the conscious mind.

In non-solipsism, the conscious mind is a physical thing that emerges from physical laws. When physical things are put together in some sort of way-- in the right way, you get a conscious mind. And so you have a simple interplay between the conscious and the unconscious in the non-solipsist theory.

In solipsist theory, you have this unconscious mind. You also have a conscious mind. And to have a comprehensive solipsist theory, you have to have a theory for how the conscious mind works, a theory for how the unconscious mind works, and also a theory for how they work together, how they interact with each other. Whereas, in the non-solipsist theory, you just have physics. Everything is physics, and that's all.

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: Yeah. So in that respect then, solipsism seems to be more complicated than non-solipsism. Non-solipsism, you just get physics. Gets the job done. Solipsism, you need all these extra rules.

Well, then you could say, well, maybe there's an overarching rule that describes how the conscious and the unconscious both work and how they both interact. Well, then you might as well call it physics.

And then if you have these overall arching rules, then you've essentially got physics again. So then, solipsism became non-solipsism. So that's my argument against solipsism.

Does that makes sense? It has more complex. Were you going to say something?

AUDIENCE: [INAUDIBLE] dreams at night sort of a [INAUDIBLE]?

NICHOLAS Are dreams at night sort of a solipsist thing?

DIBELLA:

AUDIENCE: Yeah. [INAUDIBLE].

NICHOLAS DIBELLA: What?

AUDIENCE: [INAUDIBLE]

NICHOLAS DIBELLA: So I guess when you're dreaming, you kind of become an effective solipsist, sort of. Effective solipsist. And actually, sometimes it's fun to be recreational solipsist. Like someone talks to you. They ask you questions. You just-- I'm a solipsist. I'm a solipsist. I'm a solipsist. Sometimes, it's fun. You know, if you're stressed or if you have nothing else better to do. Just be a recreational solipsist. OK.

Any questions about solipsism? I mean, I gave my justification for why I think-- I've read a number of justifications and I've kind of combined them all together to get this one, what I think is reasonable. But maybe you're smarter than me and you can think of a better argument for solipsism than for against it.

If you can, then I'd be very interested in hearing it. Because I think solipsism is a very cool idea. It'd be cool if I'm the only thing that exists. But yeah, maybe it would also be depressing. And what was the other? Egocentric. It's interesting in any case, though. OK.

So the mathematical universe hypothesis starts off by rejecting solipsism and accepting the external reality hypothesis. So according to the way that this argument works, you start off by saying there's an external reality. So we humans can't perceive this external reality. But if extraterrestrial intelligent aliens exist, then they'd also be able to observe this external reality.

If it's possible to create artificial intelligence, then that artificial intelligence would be able to observe this external reality. So there's this external reality that's accessible to all intelligent beings.

Now, we, humans, have a very particular way of describing that external reality. We've defined concepts like mass and energy, protons. And so we have this set of language that we use in describing the world. Aliens could very well have a different language. I mean, who knows how alien psychology works? They could have a very different set of concepts that they use.

If humans and aliens and all-- everyone else finally comes across the theory of everything, they should all be similar. The theories that they form, they should all be similar.

And in particular, the theory should be free of the language that we try to use to describe the theories. It shouldn't matter whether a human is describing it or whether an alien describing, and so forth. So when we try to describe the external reality, we see things and we see things that are related to each other in sorts of ways. Things that have relations between them.

The theory of everything would have to be free from human language or alien language, free from what we call baggage. And so therefore, it would have to be abstract. It would have to be an abstract description of things that are related to each other in some sort of way.

Now, it turns out that the notion of a set of things, a set of abstract things that are related to each other in a particular way, has a name for it. It's called the mathematical structure. Abstract objects related in some way.

For example, a square is a very primitive type of mathematical structure. There exists these things called points, or things called line segments. And they're related to each other in a sort of way. So you have a very primitive mathematical structure of a square.

If an external reality exists, then-- oh, I'll be done. Give me two more minutes.

If an external reality exists, then it should be free from baggage. And so it should be some kind of abstract description of nature with objects related in some sort of way. So the external realities should be a mathematical structure.

And so the mathematical universe hypothesis is that the universe is actually a mathematical structure. We live in a mathematical structure. We're mathematical stuff. And so the mathematical universe hypothesis has a very simple explanation for why math is so effective at describing the world. It's a very simple answer.

The world is mathematical, so it's no surprise that it should describe it very well. We ran out of time today. But next class, I'll continue. I'll continue from here and discuss another level of parallel universes. Discuss something even crazier, which is that our universe is actually a computer simulation. So that's next class.

OK, see you next week.