Class 02: Outline

Answer questions

Hour 1:

- Review: Electric Fields
- Charge
- Dipoles

Hour 2:

- Continuous Charge Distributions

Last Time: Fields
Gravitational & Electric

Gravitational & Electric Fields

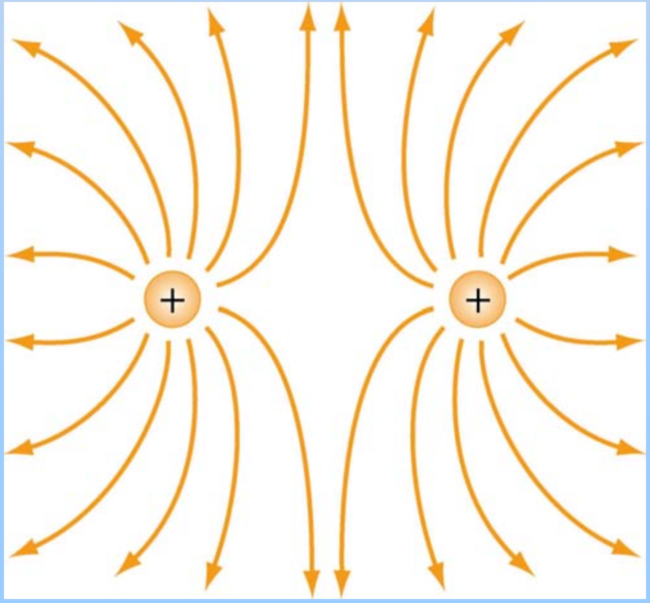
	Mass M	Charge q (±)
CREATE:	$\vec{\mathbf{g}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\frac{q}{r^2}\hat{\mathbf{r}}$
FEEL:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

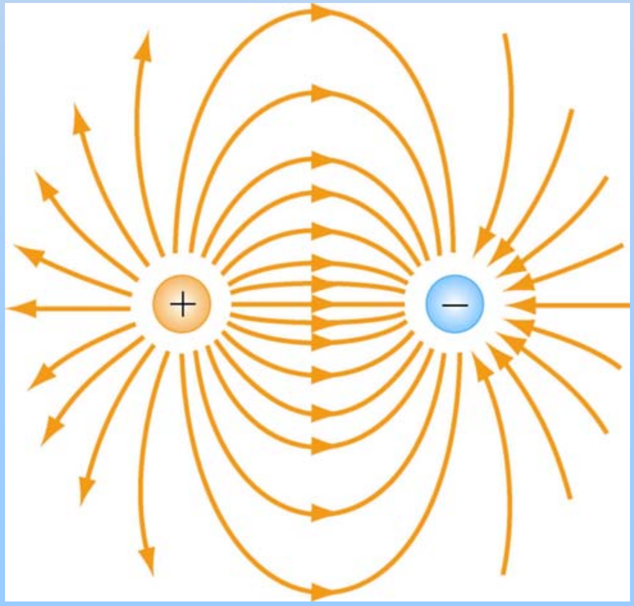
This is easiest way to picture field

PRS Questions: Electric Field

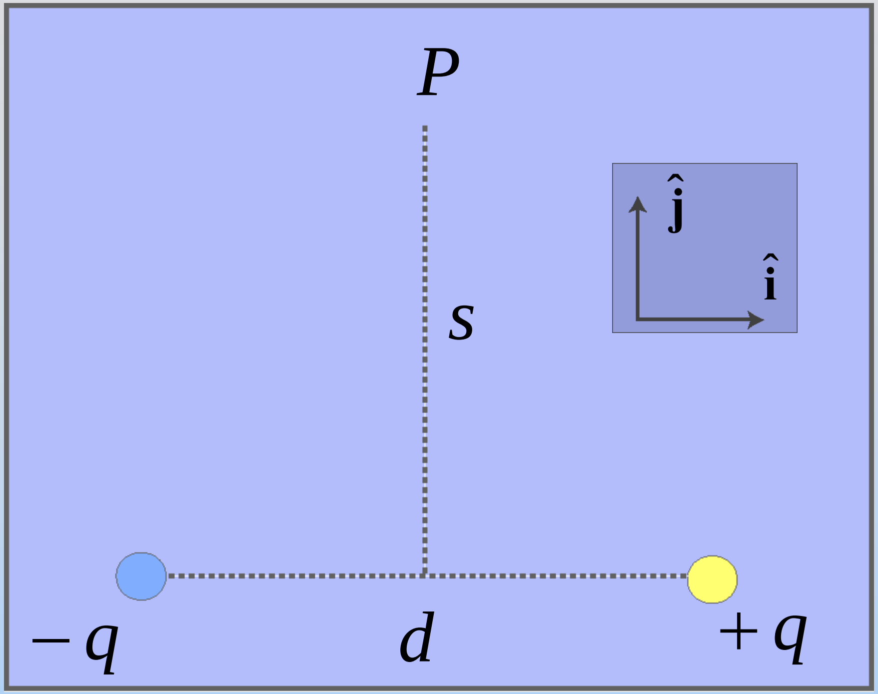
Electric Field Lines

1. Direction of field line at any point is tangent to field at that point
2. Field lines point away from positive charges and terminate on negative charges
3. Field lines never cross each other





In-Class Problem



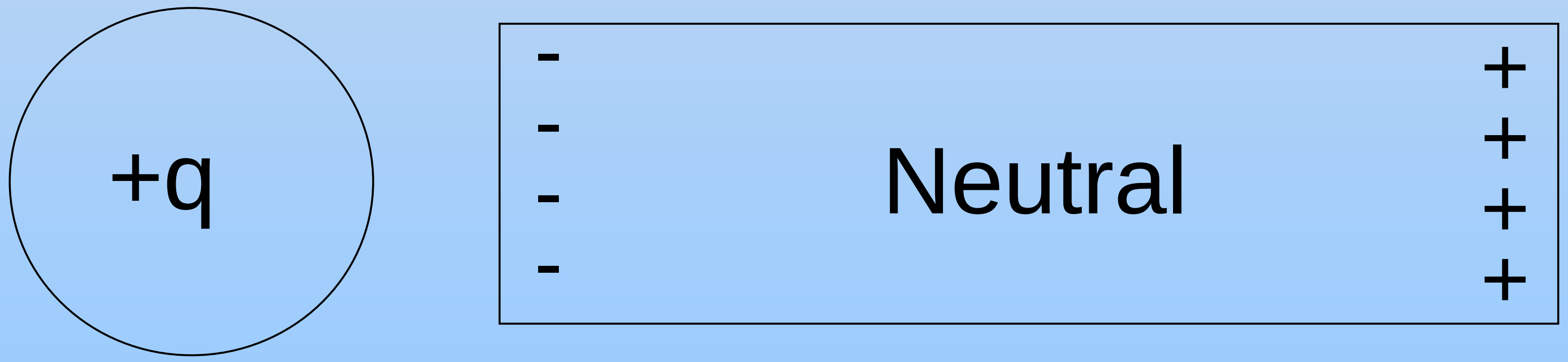
Consider two point charges of equal magnitude but opposite signs, separated by a distance *d*. Point *P* lies along the perpendicular bisector of the line joining the charges, a distance *s* above that line. What is the E field at *P*?

Two PRS Questions: E Field of Finite Number of Point Charges

Charging

How Do You Charge Objects?

- Friction
- Transfer (touching)
- Induction



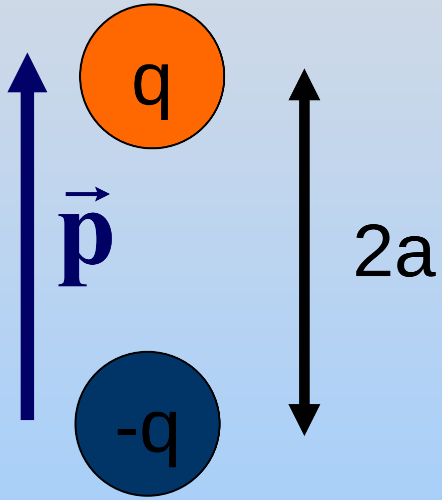
Demonstrations: Instruments for Charging

Electric Dipoles

A Special Charge Distribution

Electric Dipole

Two equal but opposite charges ***+q*** and ***–q***, separated by a distance **2a**



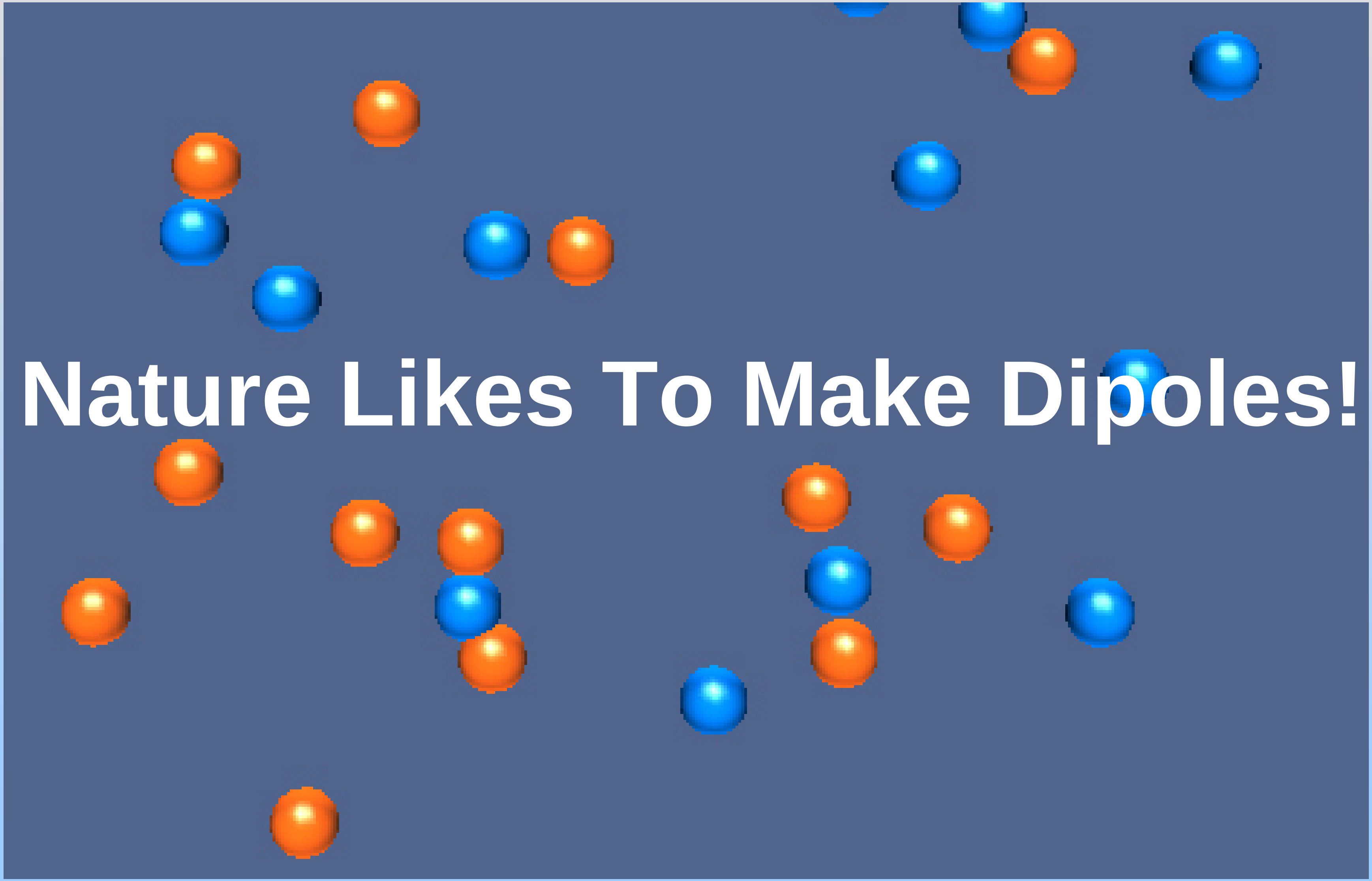
Dipole Moment

$$\vec{\mathbf{p}} \equiv \text{charge} \times \text{displacement}$$

$$= q \times 2a\hat{\mathbf{j}} = 2qa\hat{\mathbf{j}}$$

$\vec{\mathbf{p}}$ points from negative to positive charge

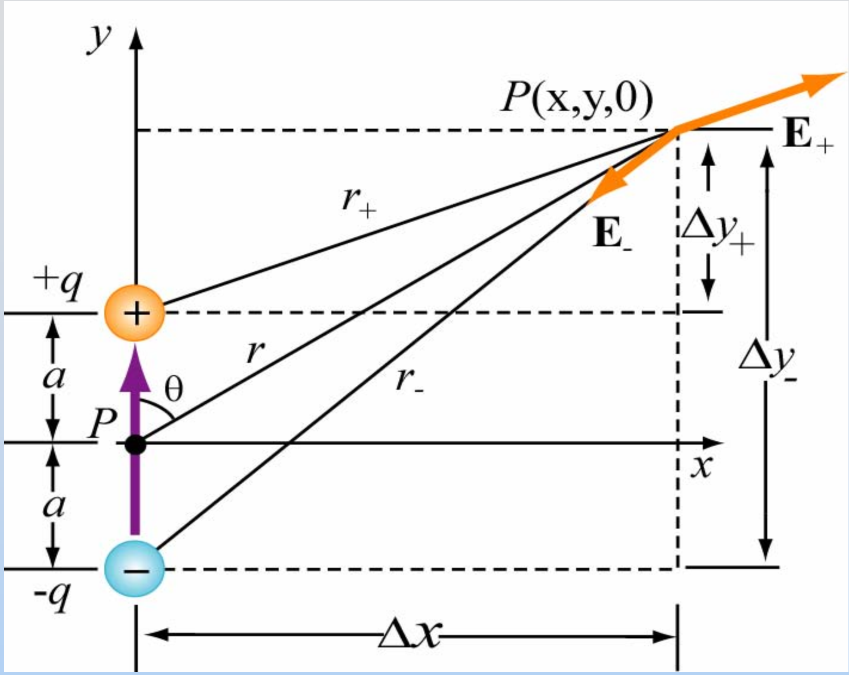
Why Dipoles?



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/20-Molecules2d/20-mole2d320.html

Dipoles *make* Fields

Electric Field Created by Dipole



Thou shalt use components!

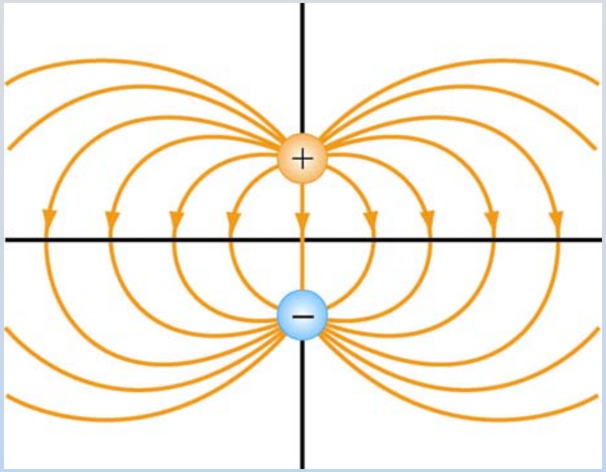
$$\frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{\Delta x}{r^3}\hat{\mathbf{i}} + \frac{\Delta y}{r^3}\hat{\mathbf{j}}$$

$$E_x = k_e q\left(\frac{\Delta x}{r_+^{\ 3}} - \frac{\Delta x}{r_-^{\ 3}}\right) = k_e q\left(\frac{x}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{x}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

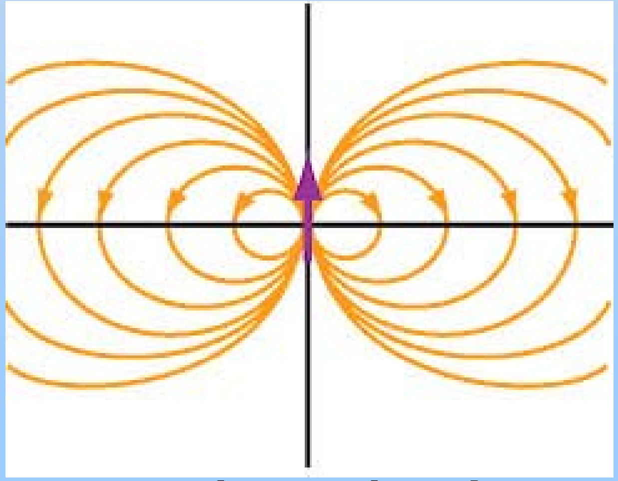
$$E_y = k_e q\left(\frac{\Delta y_+}{r_+^{\ 3}} - \frac{\Delta y_-}{r_-^{\ 3}}\right) = k_e q\left(\frac{y-a}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{y+a}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

PRS Question: Dipole Fall-Off

Point Dipole Approximation



Finite Dipole



Point Dipole

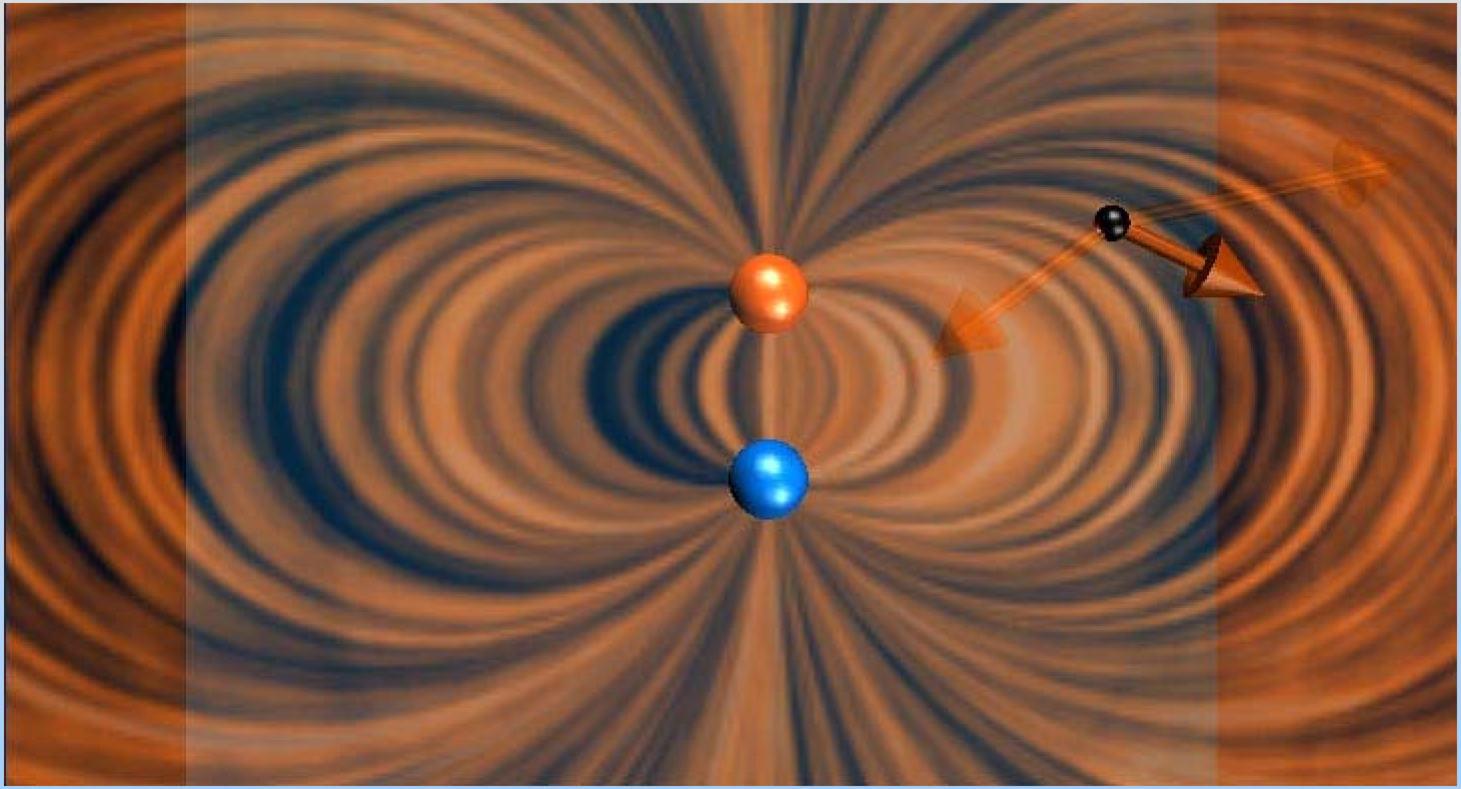
Take the limit $r >> a$

You can show…

$$E_x \rightarrow \frac{3p}{4\pi\varepsilon_0 r^3} \sin\theta\cos\theta$$

$$E_y \rightarrow \frac{p}{4\pi\varepsilon_0 r^3} \left(3\cos^2\theta - 1\right)$$

Shockwave for Dipole



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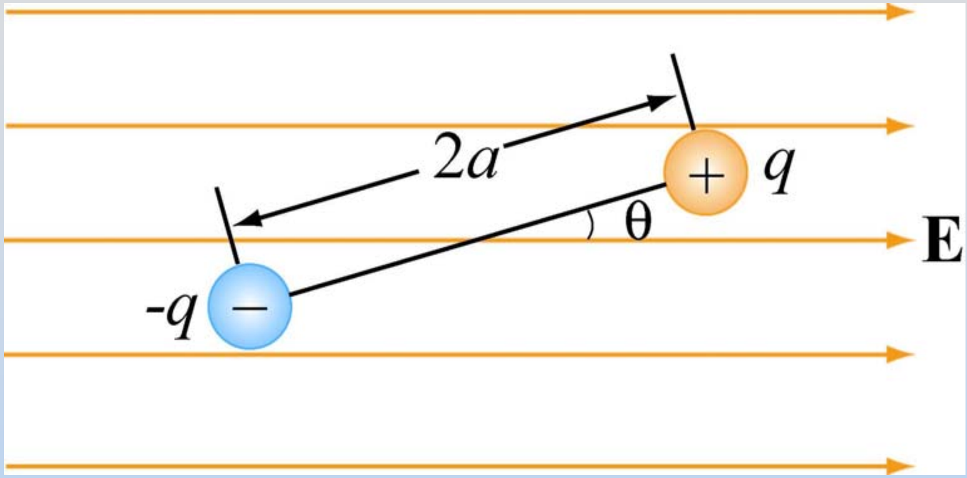
Dipoles *feel* Fields

Demonstration: Dipole in Field

Dipole in Uniform Field



$$\vec{\mathbf{E}} = E\hat{\mathbf{i}}$$

$$\vec{\mathbf{p}} = 2qa(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$$

Total Net Force: $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_{+} + \vec{\mathbf{F}}_{-} = q\vec{\mathbf{E}} + (-q)\vec{\mathbf{E}} = 0$

Torque on Dipole: $\boxed{\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}}$

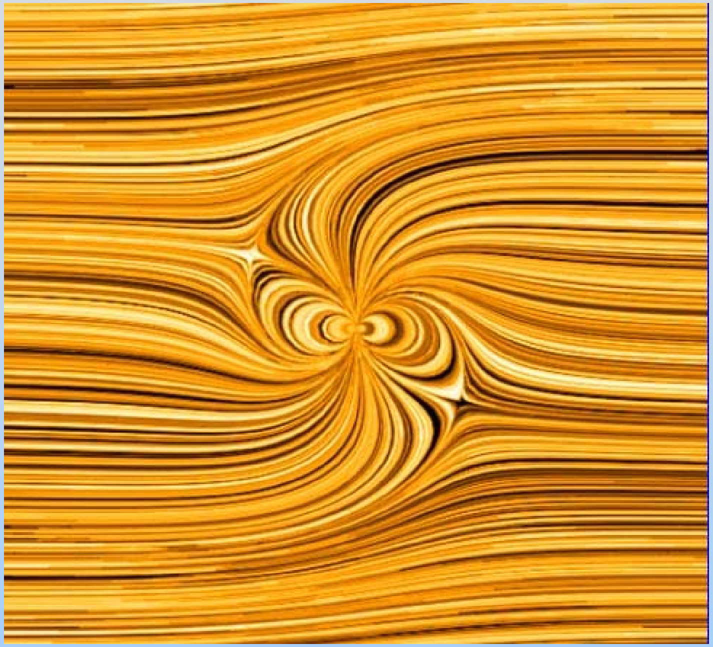
$$\tau = rF_{+}\sin(\theta) = \left(2a\right)\left(qE\right)\sin(\theta) = pE\sin(\theta)$$

$\vec{\mathbf{p}}$ tends to align with the electric field

Torque on Dipole

Total Field (dipole + background) shows torque:

http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/43-torqueondipolee/43-torqueondipolee320.html

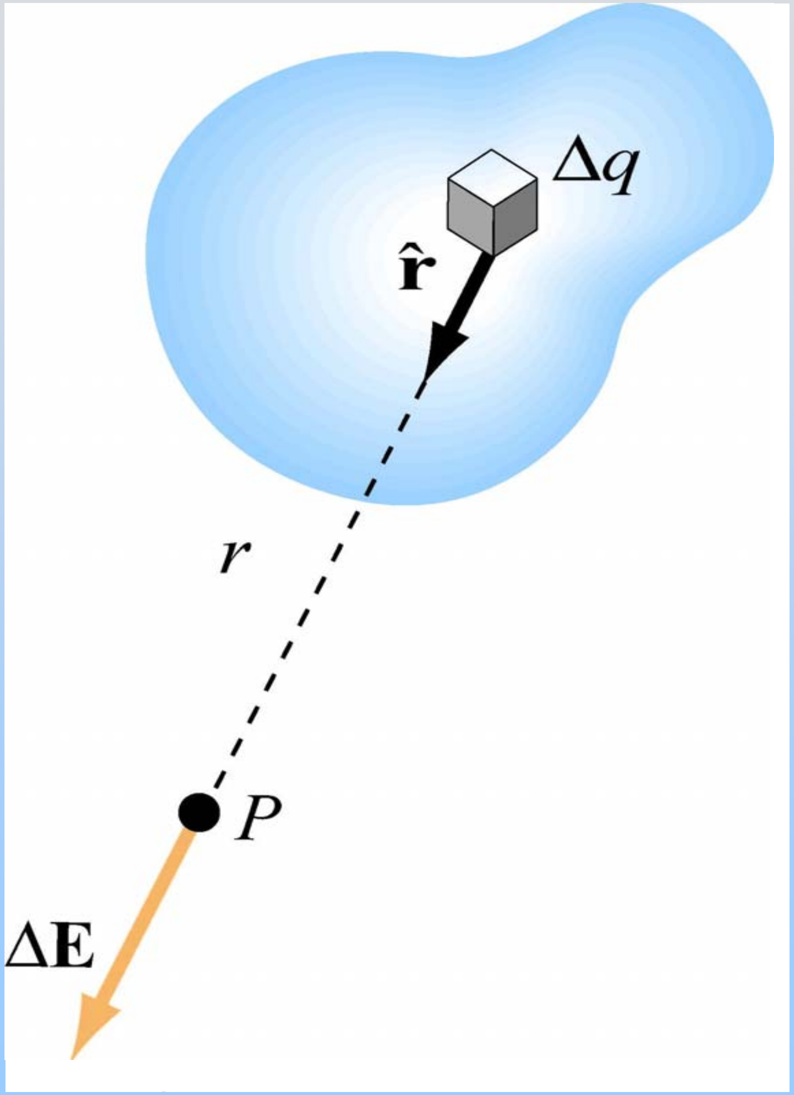


- Field lines transmit tension
- Connection between dipole field and constant field “pulls” dipole into alignment

PRS Question:
Dipole in Non-Uniform Field

Continuous Charge Distributions

Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \rightarrow \int_V dq$$

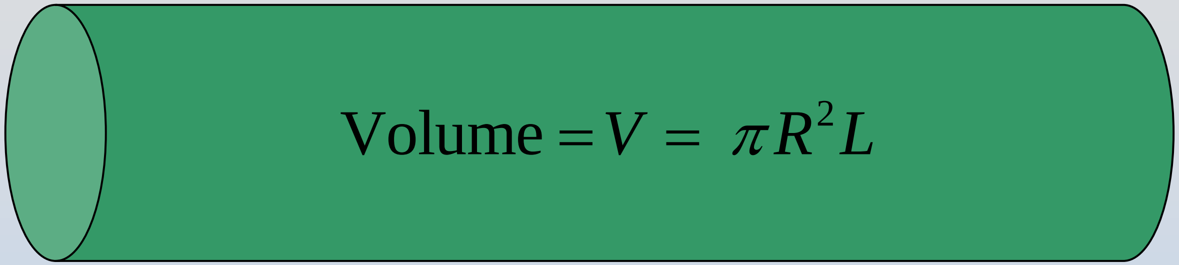
E field at P due to Δq

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}} \rightarrow d\vec{\mathbf{E}} = k_e \frac{dq}{r^2} \hat{\mathbf{r}}$$

Superposition:

$$\vec{\mathbf{E}} = \sum \Delta \vec{\mathbf{E}} \rightarrow \int d\vec{\mathbf{E}}$$

Continuous Sources: Charge Density



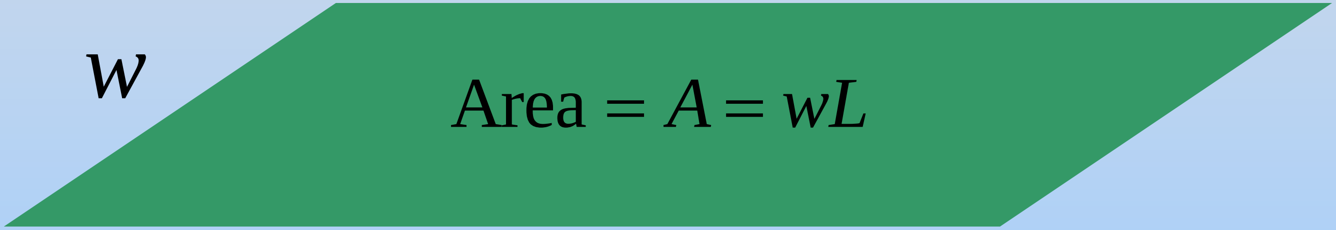
R

$\text{Volume} = V = \pi R^2 L$

L

$$dQ = \rho\, dV$$

$$\rho = \frac{Q}{V}$$



w

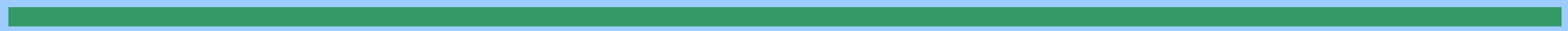
$\text{Area} = A = wL$

L

$$dQ = \sigma\, dA$$

$$\sigma = \frac{Q}{A}$$

$\text{Length} = L$



L

$$dQ = \lambda\, dL$$

$$\lambda = \frac{Q}{L}$$

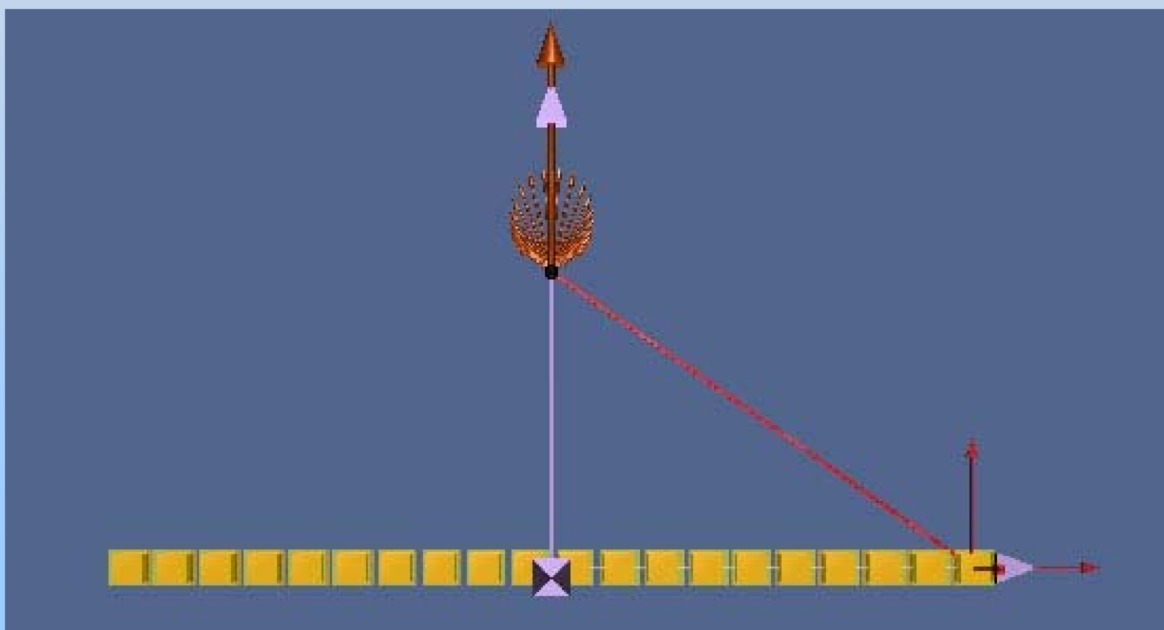
Examples of Continuous Sources: Line of charge

$\text{Length} = L$

L

$$dQ = \lambda \, dL$$

$$\lambda = \frac{Q}{L}$$



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/07-LineIntegration/07-LineInt320.html

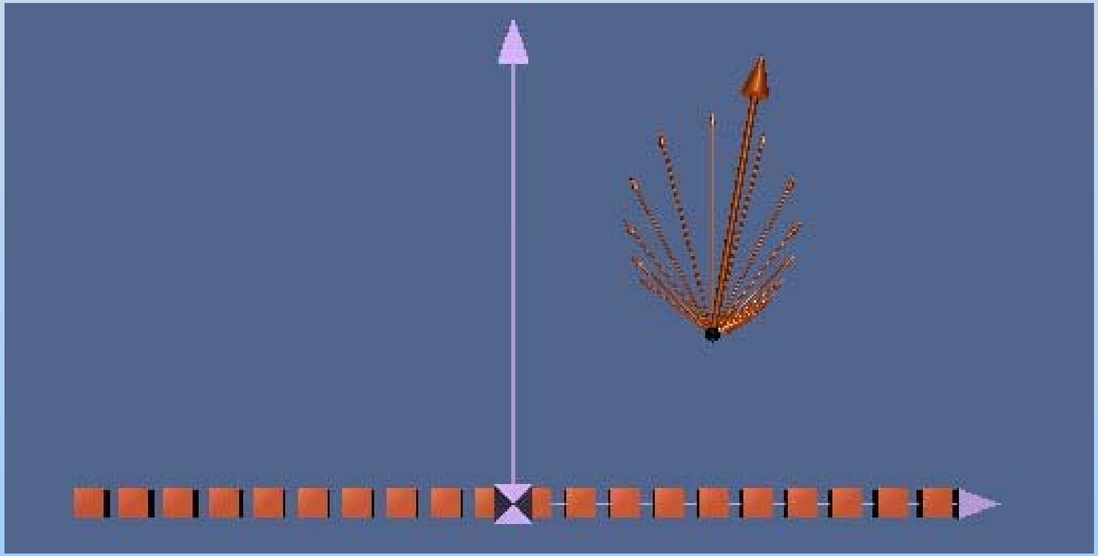
Examples of Continuous Sources: Line of charge

$$\text{Length} = L$$

$$L$$

$$dQ = \lambda\, dL$$

$$\lambda = \frac{Q}{L}$$



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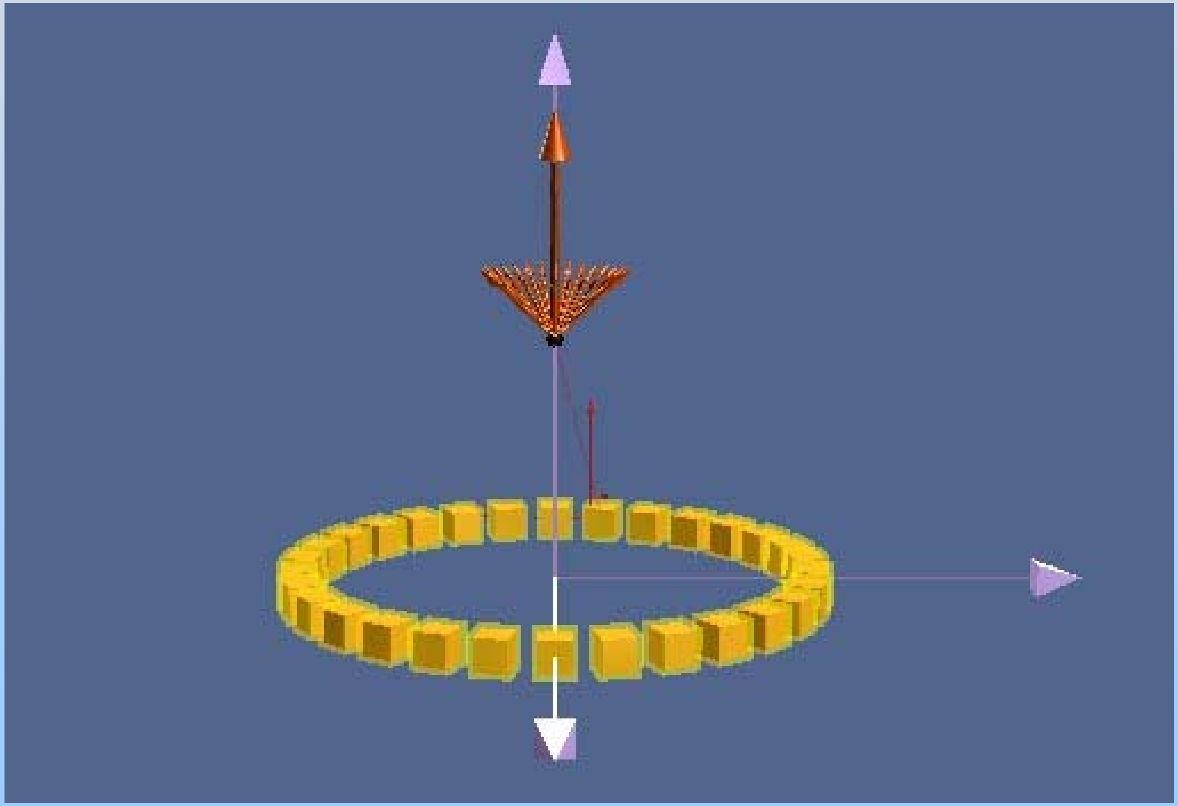
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{2\pi R}$$



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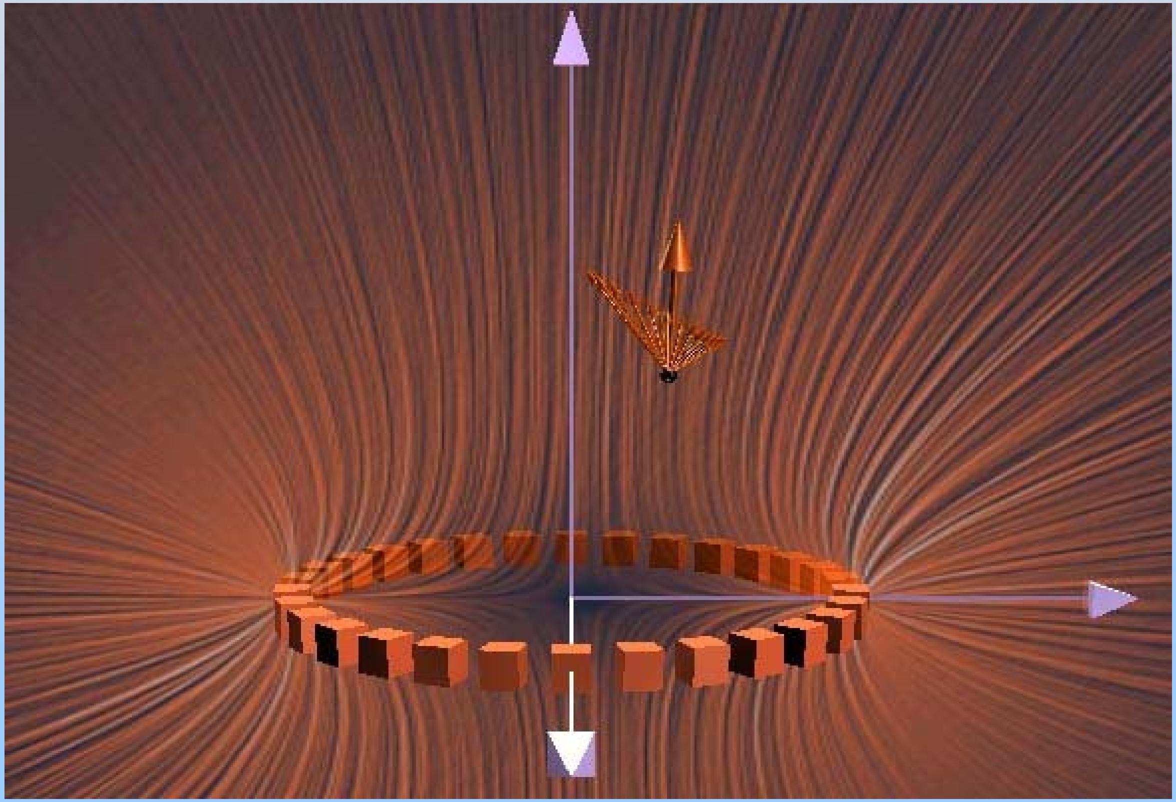
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{2\pi R}$$



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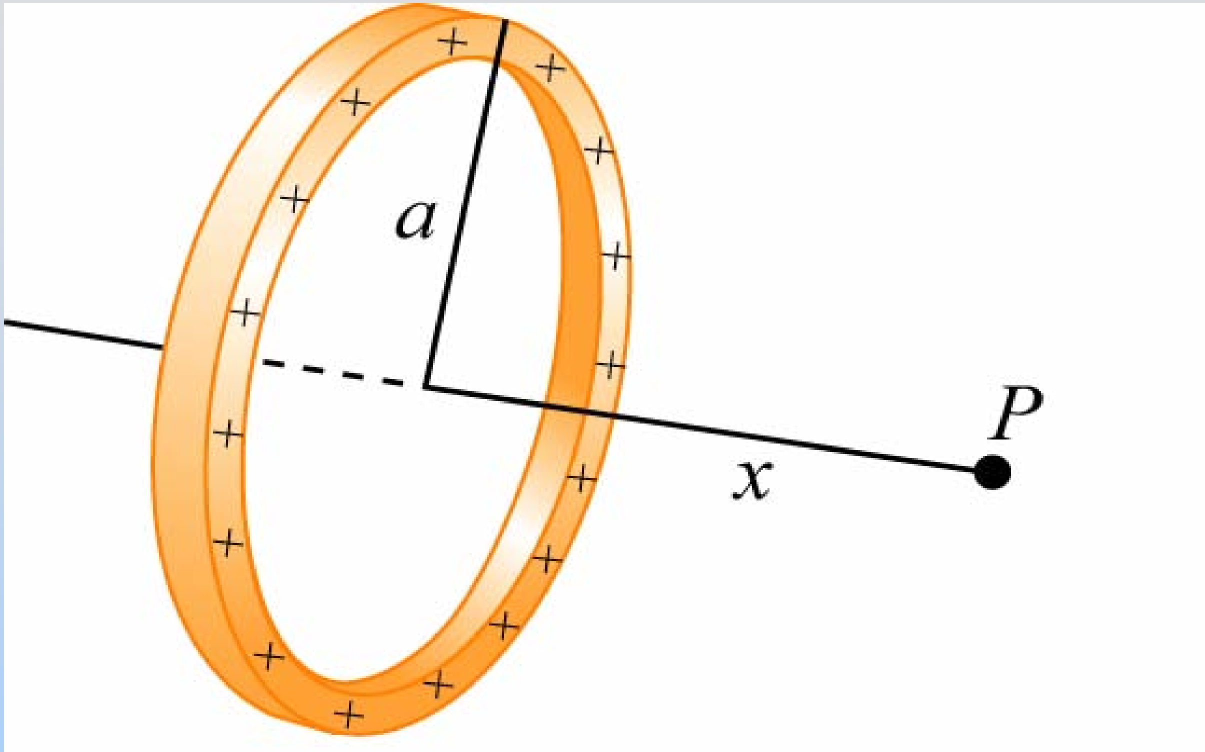
Example: Ring of Charge



P on axis of ring of charge, *x* from center
Radius a, charge density λ.

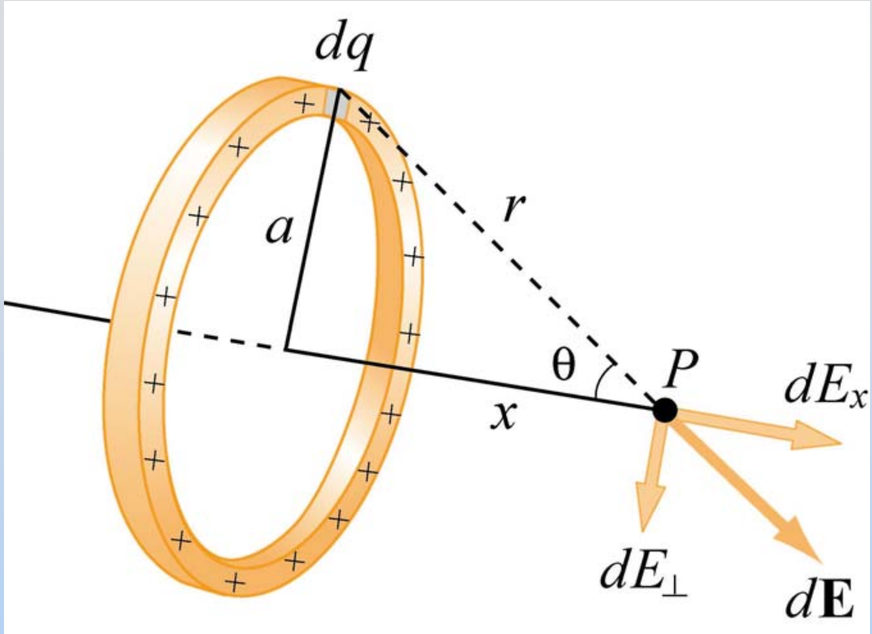
Find **E** at P

Ring of Charge

1) Think about it

$E_{\perp} = 0$ Symmetry!

http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/09-RingIntegration/09-ringInt320.html



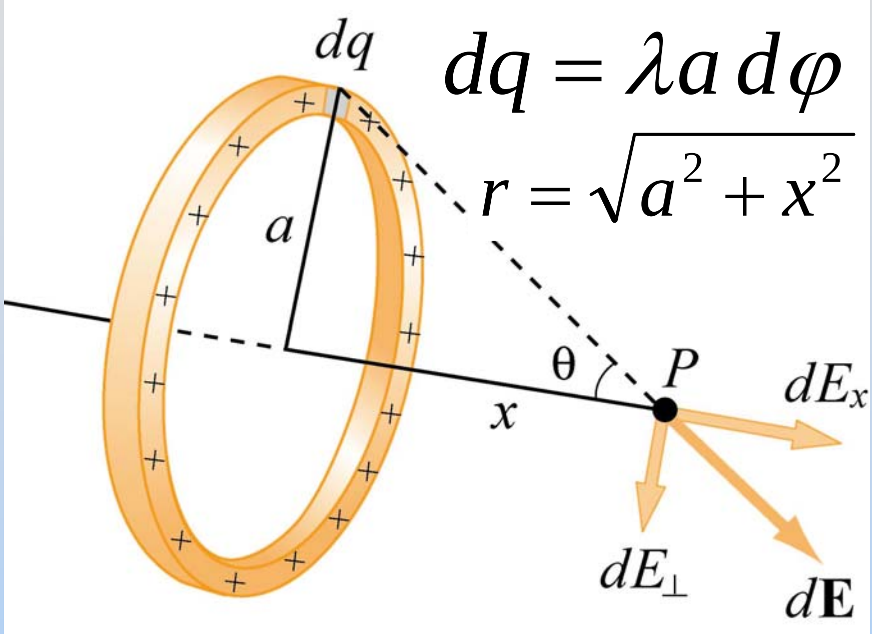
2) Define Variables

$$dq = \lambda\, dl = \lambda\left(a\, d\varphi\right)$$

$$r = \sqrt{a^2 + x^2}$$

Ring of Charge

3) Write Equation



$$d\vec{\mathbf{E}} = k_e dq \frac{\hat{r}}{r^2} = k_e\, dq \frac{\vec{r}}{r^3}$$

a) My way

$$dE_x = k_e dq \frac{x}{r^3}$$

b) Another way

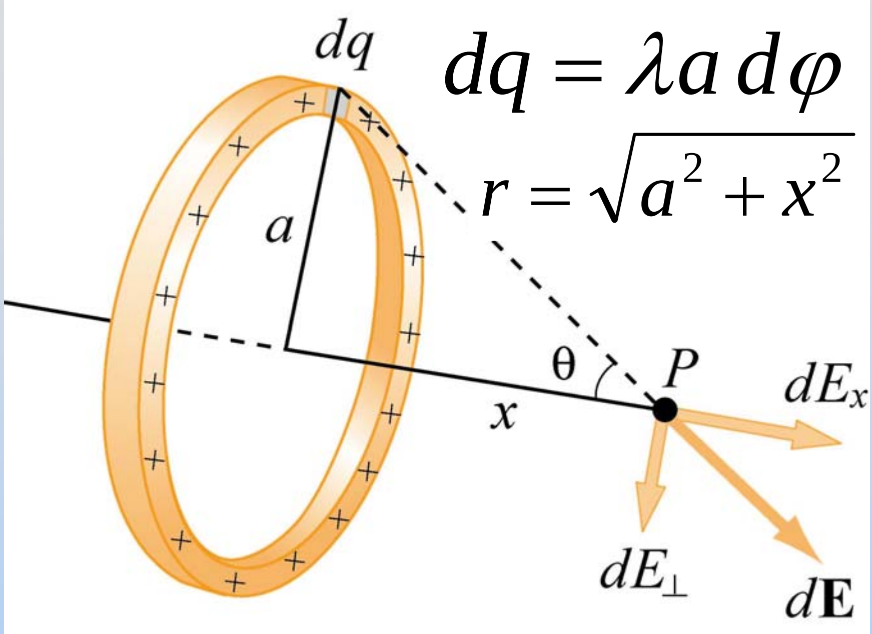
$$dE_x = \left| d\vec{\mathbf{E}} \right| \cos(\theta) = k_e dq \frac{1}{r^2} \cdot \frac{x}{r} = k_e dq \frac{x}{r^3}$$

Ring of Charge

4) Integrate

$$E_x = \int dE_x = \int k_e \, dq \frac{x}{r^3}$$

$$= k_e \frac{x}{r^3} \int dq$$



$$dq = \lambda a \, d\varphi$$

$$r = \sqrt{a^2 + x^2}$$

Very special case: everything except *dq* is constant

$$\int dq = \int_0^{2\pi} \lambda a \, d\varphi = \lambda a \int_0^{2\pi} d\varphi = \lambda a 2\pi$$

$$= Q$$

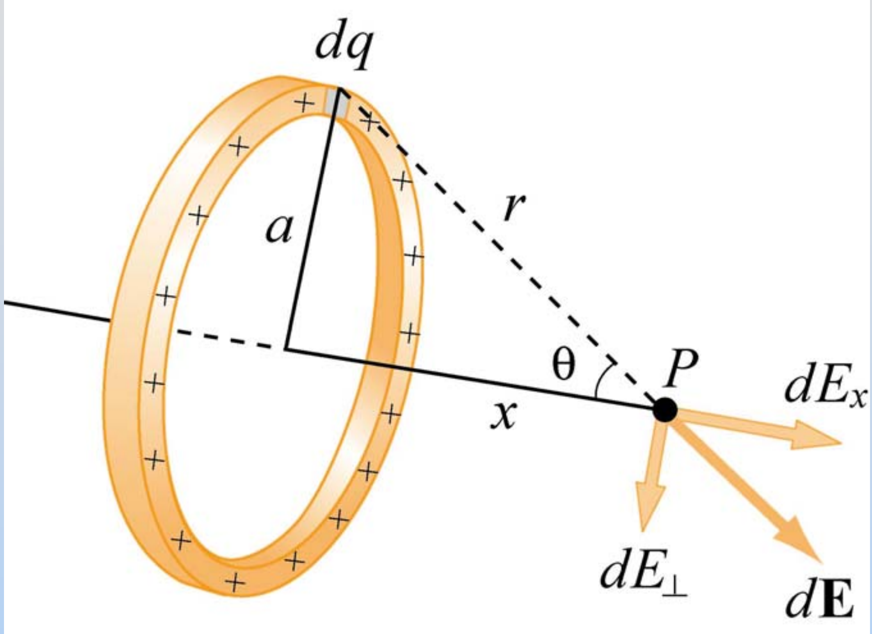
Ring of Charge

5) Clean Up

$$E_x = k_e Q \frac{x}{r^3}$$

$$E_x = k_e Q \frac{x}{\left(a^2 + x^2\right)^{3/2}}$$

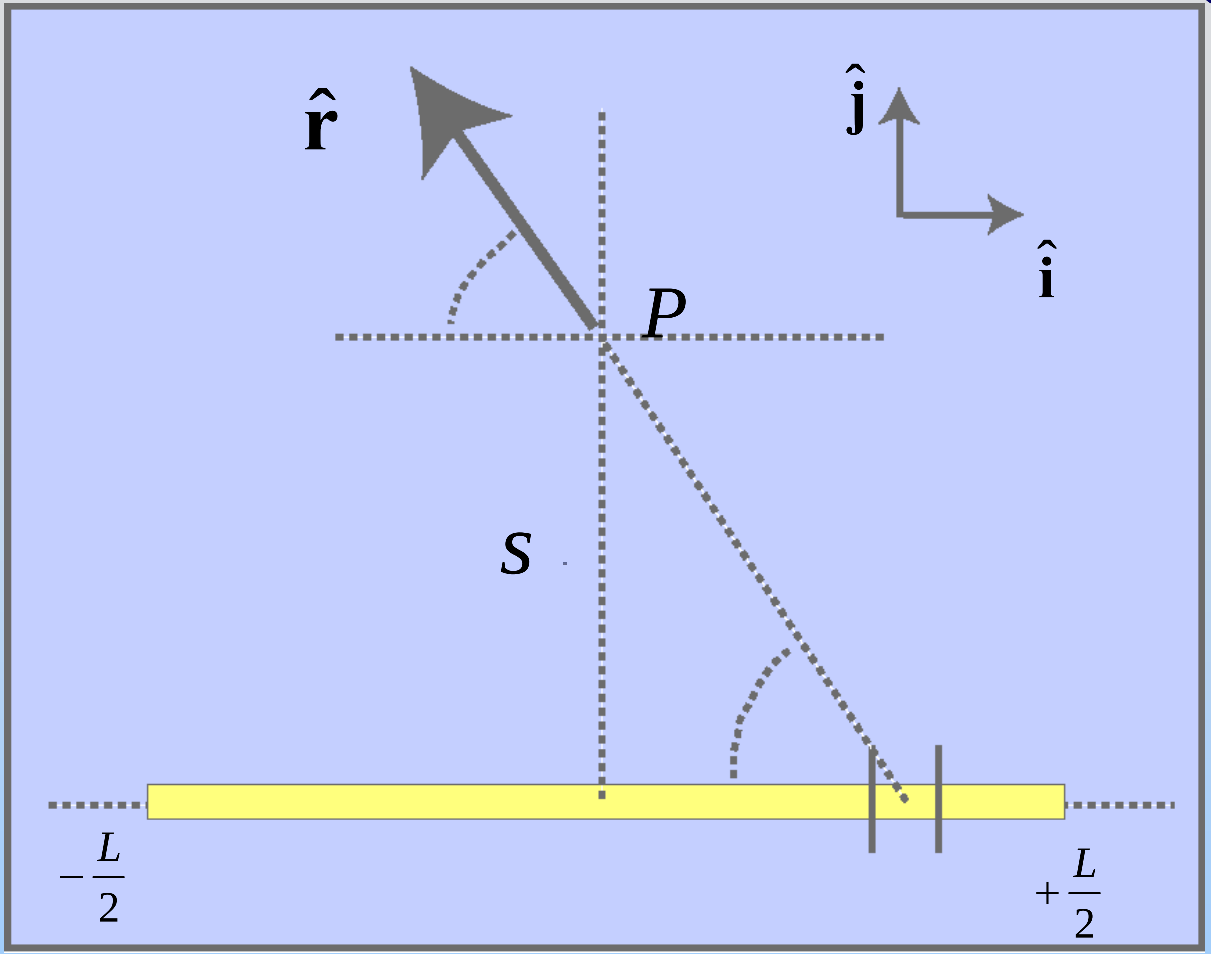
$$\vec{\mathbf{E}} = k_e Q \frac{x}{\left(a^2 + x^2\right)^{3/2}} \hat{\mathbf{i}}$$



6) Check Limit $a \to 0$

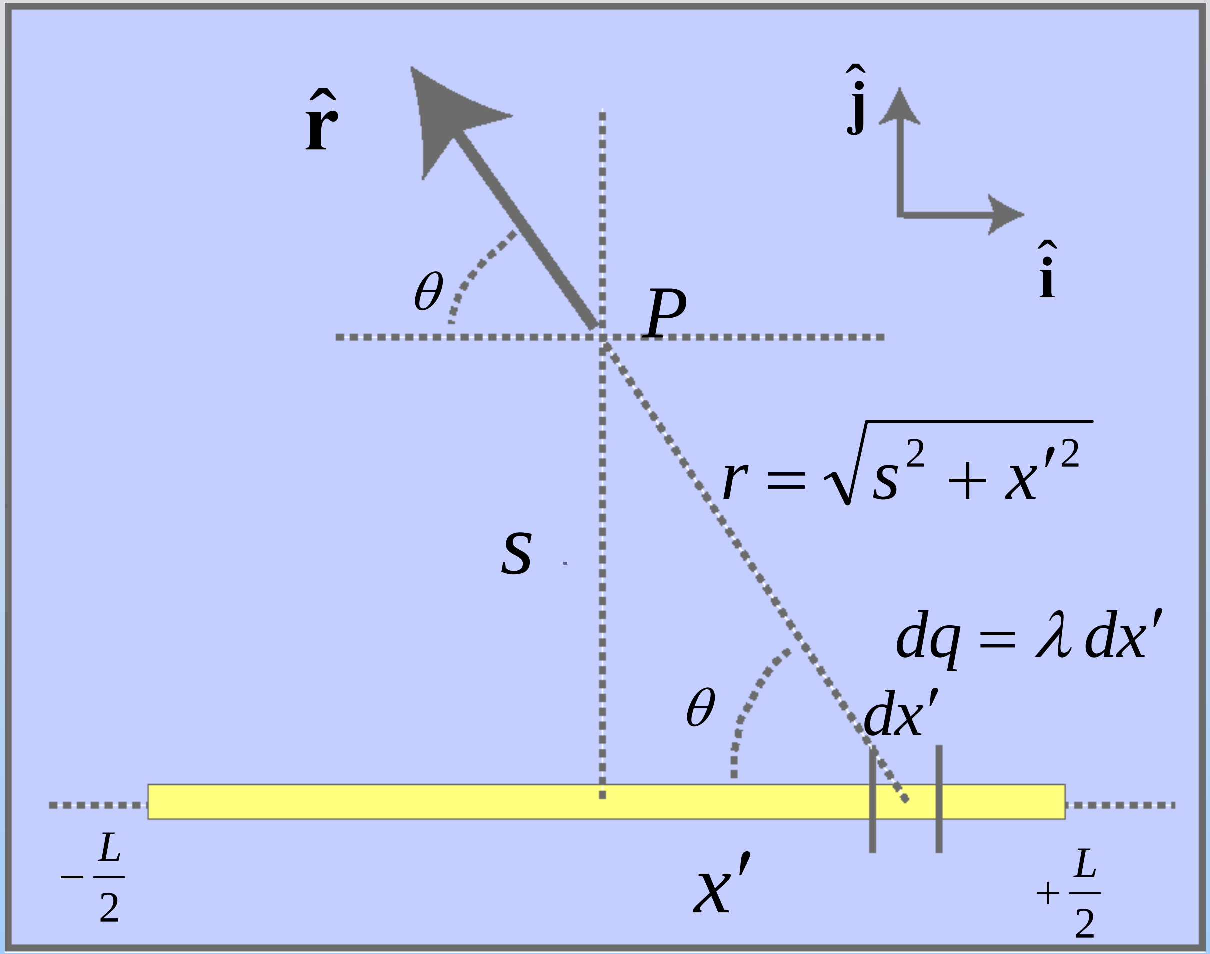
$$E_x \to k_e Q \frac{x}{\left(x^2\right)^{3/2}} = \frac{k_e Q}{x^2}$$

In-Class: Line of Charge



Point *P* lies on perpendicular bisector of uniformly charged line of length *L*, a distance *s* away. The charge on the line is *Q*. What is ***E*** at *P*?

Hint: http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/electrostatics/07-LineIntegration/07-LineInt320.html



Typically give the integration variable (*x'*) a "primed" variable name.

E Field from Line of Charge

$$\vec{\mathbf{E}} = k_e \frac{Q}{s(s^2 + L^2/4)^{1/2}} \hat{\mathbf{j}}$$

Limits:

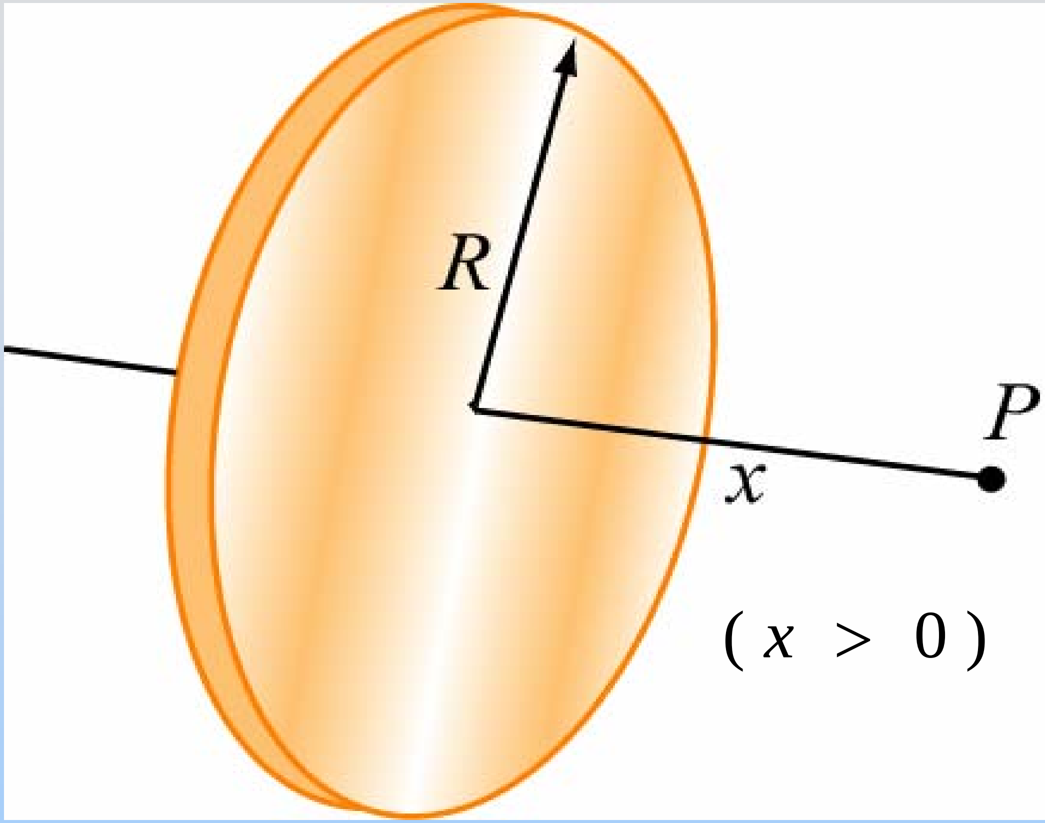
$$\lim_{s>>L} \vec{\mathbf{E}} \rightarrow k_e \frac{Q}{s^2} \hat{\mathbf{j}}$$

Point charge

$$\lim_{s<<L} \vec{\mathbf{E}} \rightarrow 2k_e \frac{Q}{Ls} \hat{\mathbf{j}} = 2k_e \frac{\lambda}{s} \hat{\mathbf{j}}$$

Infinite charged line

In-Class: Uniformly Charged Disk



P on axis of disk of charge, *x* from center

Radius R, charge density σ.

Find **E** at P

Disk: Two Important Limits

$$\vec{\mathbf{E}}_{disk} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right]\hat{\mathbf{i}}$$

<u>Limits:</u>

$$\lim_{x>>R} \vec{\mathbf{E}}_{disk} \overset{***}{\rightarrow} \frac{1}{4\pi\varepsilon_o}\frac{Q}{x^2}\hat{\mathbf{i}}$$

Point charge

$$\lim_{x<<R} \vec{\mathbf{E}}_{disk} \rightarrow \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{i}}$$

Infinite charged plane

E for Plane is Constant????

1) Dipole: E falls off like $1/r^3$
2) Point charge: E falls off like $1/r^2$
3) Line of charge: E falls off like $1/r$
4) Plane of charge: E constant