

**Chapter 21**  
**Modern Portfolio Theory**  
**&**  
**Chapter 22**  
**Equilibrium Asset Pricing**

## 21.2 Basic Mean-Variance Portfolio Theory

### **"MODERN PORTFOLIO THEORY"**

**(aka "Mean-Variance Portfolio Theory", or "Markowitz Portfolio Theory" – Either way: "MPT" for short)**

➤ **DEVELOPED IN 1950s (by MARKOWITZ, SHARPE, LINTNER)**

(Won Nobel Prize in Economics in 1990.)

➤ **WIDELY USED AMONG PROFESSIONAL INVESTORS**

➤ **FUNDAMENTAL DISCIPLINE OF PORTFOLIO-LEVEL INVESTMENT STRATEGIC DECISION MAKING.**

**I. REVIEW OF STATISTICS ABOUT PERIODIC TOTAL RETURNS:**

(Note: these are all "time-series" statistics: measured across time, not across assets within a single point in time.)

**"1st Moment" Across Time (measures "central tendency"):**

"MEAN", used to measure:

- ☞ Expected Performance ("ex ante", usually arithmetic mean: used in portf ana.)
- ☞ Achieved Performance ("ex post", usually geometric mean)

**"2nd Moments" Across Time (measure characteristics of the deviation around the central tendency). They include...**

1) "STANDARD DEVIATION" (aka "volatility"), which measures:

- ☞ Square root of variance of returns across time.
- ☞ "Total Risk" (of exposure to asset if investor not diversified)

2) "COVARIANCE", which measures "Co-Movement", aka:

- ☞ "Systematic Risk" (component of total risk which cannot be "diversified away")
- ☞ Covariance with investor's portfolio measures asset contribution to portfolio total risk.

3) "CROSS-CORRELATION" (just "correlation" for short). Based on contemporaneous covariance between two assets or asset classes. Measures how two assets "move together":

- ☞ important for Portfolio Analysis.

4) "AUTOCORRELATION" (or "serial correlation": Correlation with itself across time), which reflects the nature of the "Informational Efficiency" in the Asset Market; e.g.:

- ☞ Zero → "Efficient" Market (prices quickly reflect full information; returns lack predictability) → Like securities markets (approximately).
- ☞ Positive → "Sluggish" (inertia, inefficient) Market (prices only gradually incorporate new info.) → Like private real estate markets.
- ☞ Negative → "Noisy" Mkt (excessive s.r. volatility, price "overreactions") → Like securities markets (to some extent).

# "Picture" of 1st and 2nd Moments . . .

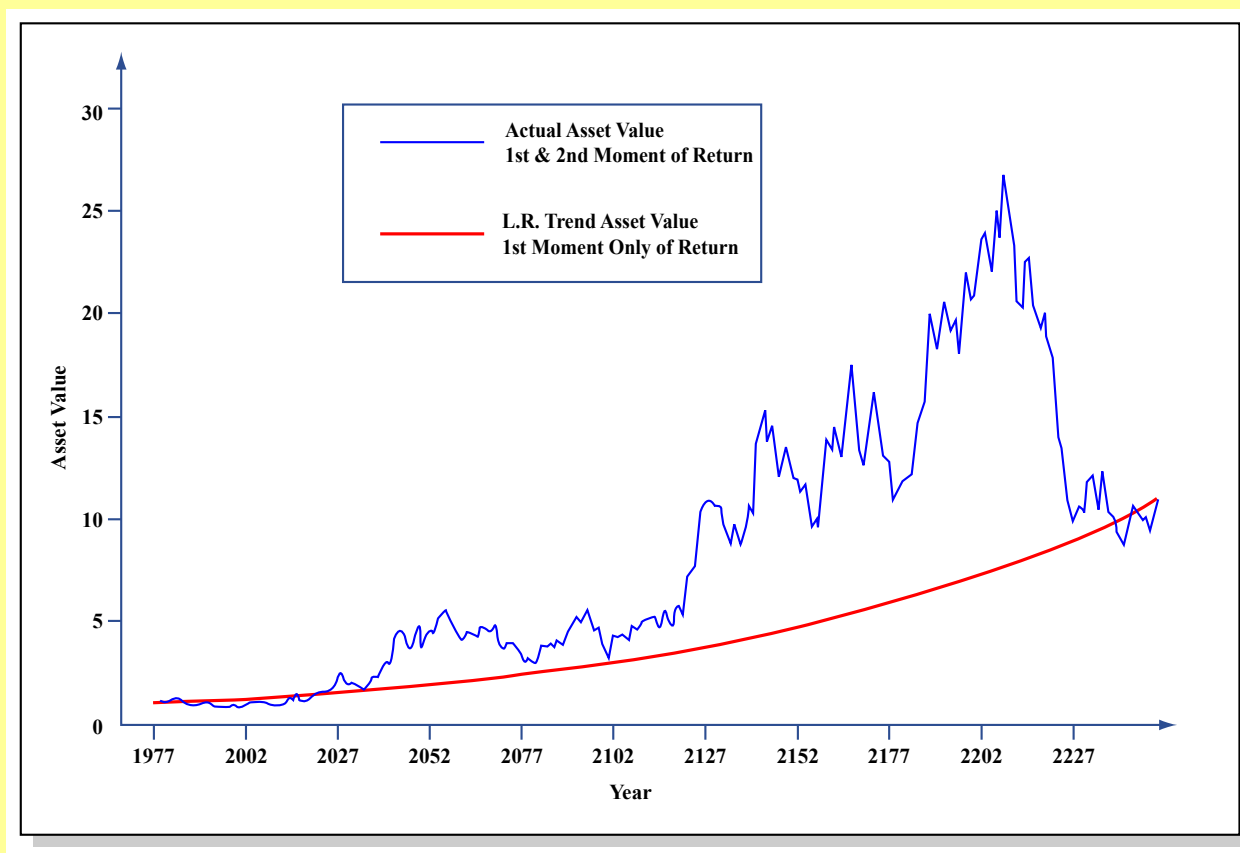


Figure by MIT OCW.

First Moment is "Trend". Second Moment is "Deviation" around trend.

***Food for Thought Question:***

**IF THE TWO LINES ABOVE WERE TWO DIFFERENT ASSETS, WHICH WOULD YOU PREFER TO INVEST IN, OTHER THINGS BEING EQUAL? . . .**

**Historical statistics, annual periodic total returns:  
Stocks, Bonds, Real Estate, 1970-2003...**

	<i>S&amp;P500</i>	<i>LTG Bonds</i>	<i>Private Real Estate</i>
<i>Mean (arith)</i>	<b>12.7%</b>	<b>9.7%</b>	<b>9.9%</b>
<i>Std.Deviation</i>	<b>17.5%</b>	<b>11.8%</b>	<b>9.0%</b>
<i>Correlations:</i>			
<i>S&amp;P500</i>	<b>100%</b>	<b>27.2%</b>	<b>16.6%</b>
<i>LTG Bonds</i>		<b>100%</b>	<b>-21.0%</b>
<i>Priv. Real Estate</i>			<b>100%</b>

← *1<sup>st</sup> Moments*

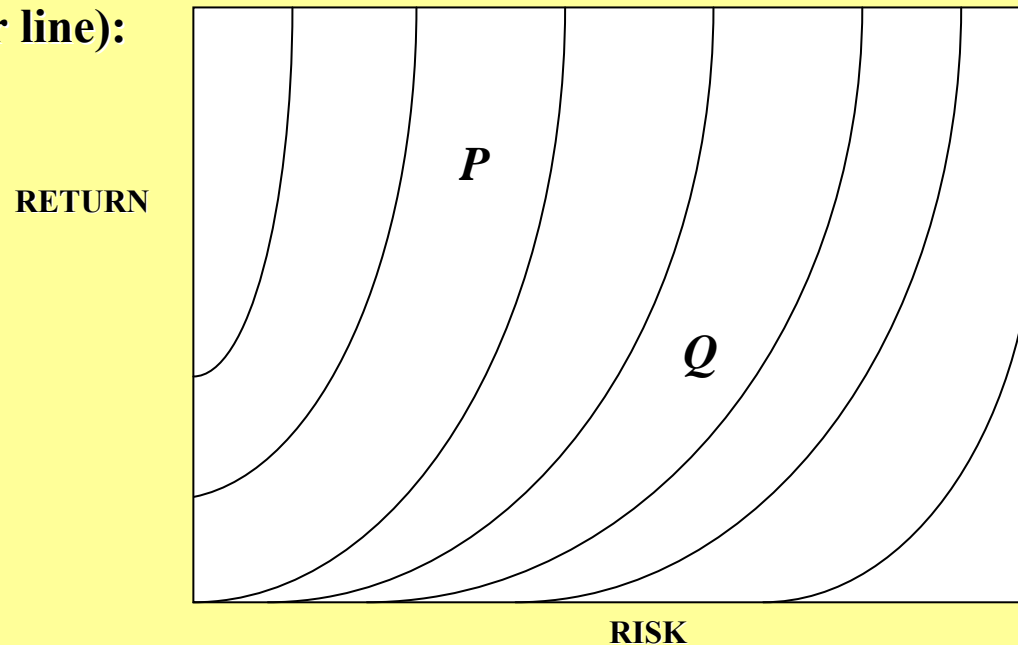
*2<sup>nd</sup> Moments*

***PORTFOLIO THEORY IS A WAY TO CONSIDER BOTH THE 1ST & 2ND MOMENTS (& INTEGRATE THE TWO) IN INVESTMENT ANALYSIS.***

### ***21.2.1. Investor Preferences & Dominant Portfolios***

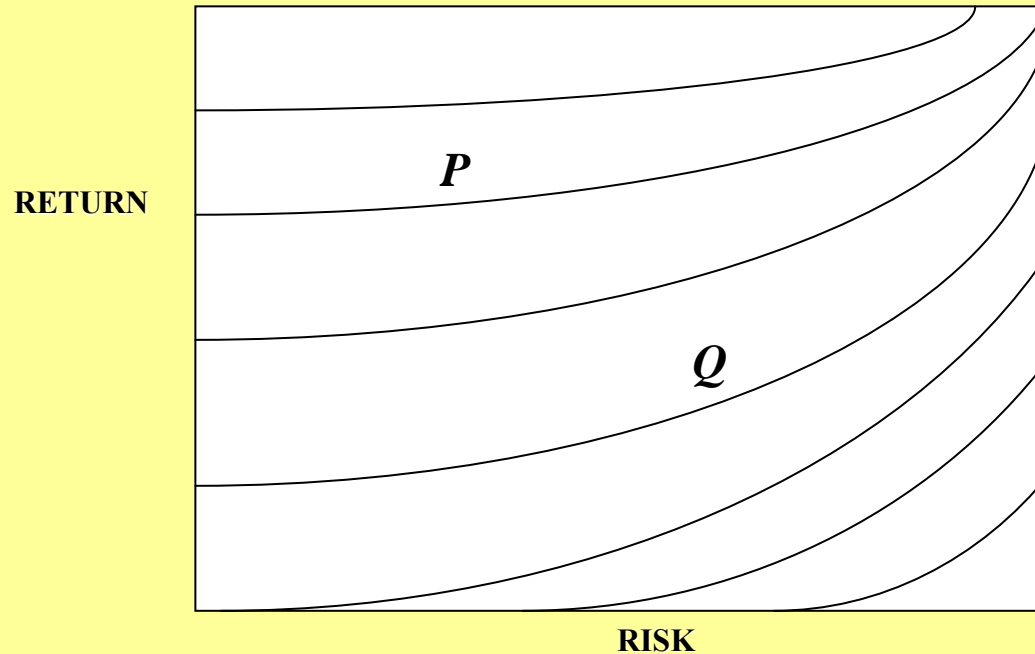
**SUPPOSE WE DRAW A 2-DIMENSIONAL SPACE WITH RISK (2ND-MOMENT) ON HORIZONTAL AXIS AND EXPECTED RETURN (1ST MOMENT) ON VERTICAL AXIS.**

**A RISK-AVERSE INVESTOR MIGHT HAVE A UTILITY (PREFERENCE) SURFACE INDICATED BY CONTOUR LINES LIKE THESE (investor is indifferent along a given contour line):**



**THE CONTOUR LINES ARE STEEPLY RISING AS THE RISK-AVERSE INVESTOR WANTS MUCH MORE RETURN TO COMPENSATE FOR A LITTLE MORE RISK.**

**A MORE AGGRESSIVE INVESTOR MIGHT HAVE A UTILITY (PREFERENCE) SURFACE INDICATED BY CONTOUR LINES LIKE THESE.**



**THE SHALLOW CONTOUR LINES INDICATE THE INVESTOR DOES NOT NEED MUCH ADDITIONAL RETURN TO COMPENSATE FOR MORE RISK.**

***BUT BOTH INVESTORS WOULD AGREE THEY PREFER POINTS TO THE "NORTH" AND "WEST" IN THE RISK/RETURN SPACE. THEY BOTH PREFER POINT "P" TO POINT "Q".***

**FOR ANY TWO PORTFOLIOS "P" AND "Q" SUCH THAT:**

**EXPECTED RETURN "P"  $\geq$  EXPECTED RETURN "Q"**

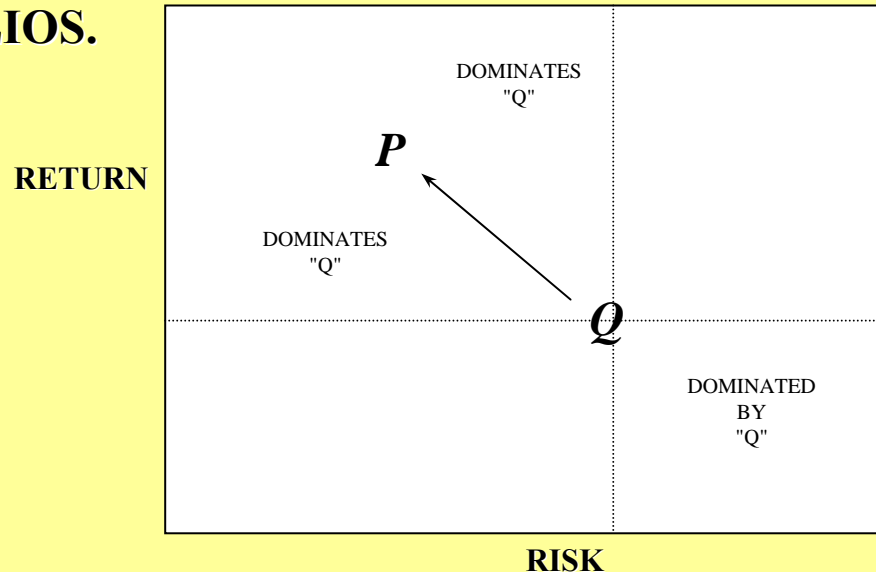
**AND (SIMULTANEOUSLY): RISK "P"  $\leq$  RISK "Q"**

**IT IS SAID THAT: "Q" IS *DOMINATED* BY "P".**

**THIS IS INDEPENDENT OF RISK PREFERENCES.**

**→ BOTH CONSERVATIVE AND AGGRESSIVE INVESTORS WOULD AGREE ABOUT THIS.**

**IN ESSENCE, PORTFOLIO THEORY IS ABOUT HOW TO AVOID INVESTING IN DOMINATED PORTFOLIOS.**



**PORTFOLIO THEORY TRIES TO MOVE INVESTORS FROM POINTS LIKE "Q" TO POINTS LIKE "P".**



## Section 21.2.2...

### III. **PORTFOLIO THEORY AND DIVERSIFICATION...**

"PORTFOLIOS" ARE "COMBINATIONS OF ASSETS".

PORTFOLIO THEORY FOR (or *from*) YOUR GRANDMOTHER:

***“DON'T PUT ALL YOUR EGGS IN ONE BASKET!”***

WHAT MORE THAN THIS CAN WE SAY? . . .

*(e.g., How many “eggs” should we put in which “baskets”.)*

***In other words,***

**GIVEN YOUR OVERALL INVESTABLE WEALTH, PORTFOLIO THEORY TELLS YOU HOW MUCH YOU SHOULD INVEST IN DIFFERENT TYPES OF ASSETS. FOR EXAMPLE:**

**WHAT % SHOULD YOU PUT IN REAL ESTATE?**

**WHAT % SHOULD YOU PUT IN STOCKS?**

***TO BEGIN TO RIGOROUSLY ANSWER THIS QUESTION, CONSIDER...***

## **AT THE HEART OF PORTFOLIO THEORY ARE TWO BASIC MATHEMATICAL FACTS:**

**1) PORTFOLIO RETURN IS A LINEAR FUNCTION OF THE ASSET WEIGHTS:**

$$r_P = \sum_{n=1}^N w_n r_n$$

**IN PARTICULAR, THE PORTFOLIO EXPECTED RETURN IS A WEIGHTED AVERAGE OF THE EXPECTED RETURNS TO THE INDIVIDUAL ASSETS. E.G., WITH TWO ASSETS ("i" & "j"):**

$$r_p = \omega r_i + (1-\omega)r_j$$

**WHERE  $\omega_i$  IS THE SHARE OF PORTFOLIO TOTAL VALUE INVESTED IN ASSET i.**

e.g., If Asset A has  $E[r_A]=5\%$  and Asset B has  $E[r_B]=10\%$ , then a 50/50 Portfolio (50% A + 50% B) will have  $E[r_p]=7.5\%$ .

## **THE 2<sup>ND</sup> FACT:**

**2) PORTFOLIO VOLATILITY IS A NON-LINEAR FUNCTION OF THE ASSET WEIGHTS:**

$$VAR_P = \sum_{I=1}^N \sum_{J=1}^N w_i w_j COV_{ij}$$

**SUCH THAT THE PORTFOLIO VOLATILITY IS LESS THAN A WEIGHTED AVERAGE OF THE VOLATILITIES OF THE INDIVIDUAL ASSETS. E.G., WITH TWO ASSETS:**

$$\begin{aligned} s_p &= \sqrt{[\omega^2(s_i)^2 + (1-\omega)^2(s_j)^2 + 2\omega(1-\omega)s_i s_j C_{ij}]} \\ &\leq \omega s_i + (1-\omega)s_j \end{aligned}$$

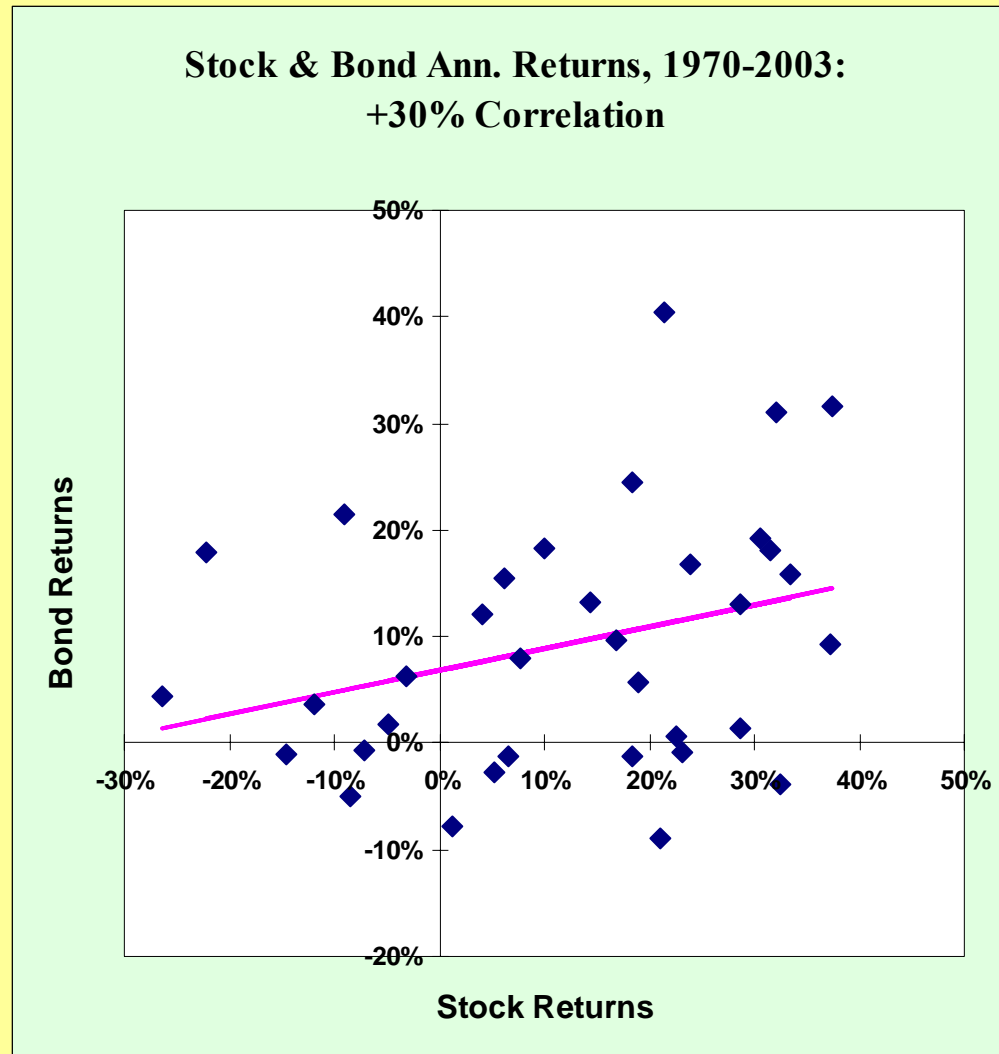
**WHERE  $s_i$  IS THE RISK (MEASURED BY STD.DEV.) OF ASSET  $i$ .**

e.g., If Asset A has  $\text{StdDev}[r_A]=5\%$  and Asset B has  $\text{StdDev}[r_B]=10\%$ , then a 50/50 Portfolio (50% A + 50% B) will have  $\text{StdDev}[r_p] < 7.5\%$  (conceivably even  $< 5\%$ ).

***→ This is the beauty of Diversification. It is at the core of Portfolio Theory. It is perhaps the only place in economics where you get a “free lunch”: In this case, less risk without necessarily reducing your expected return!***

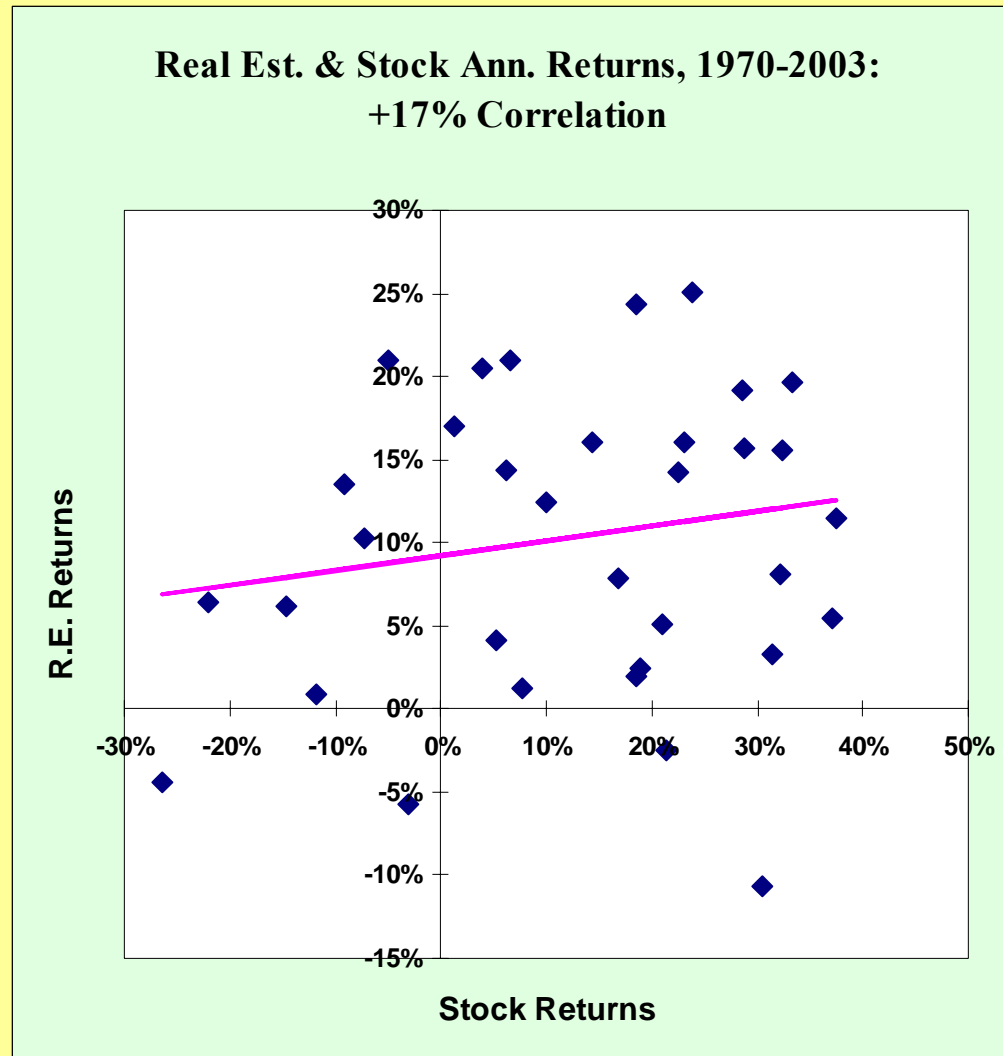
The diversification effect is greater the less correlated are the assets...

**Stocks & bonds (+30% correlation): Each dot is one year's returns.**



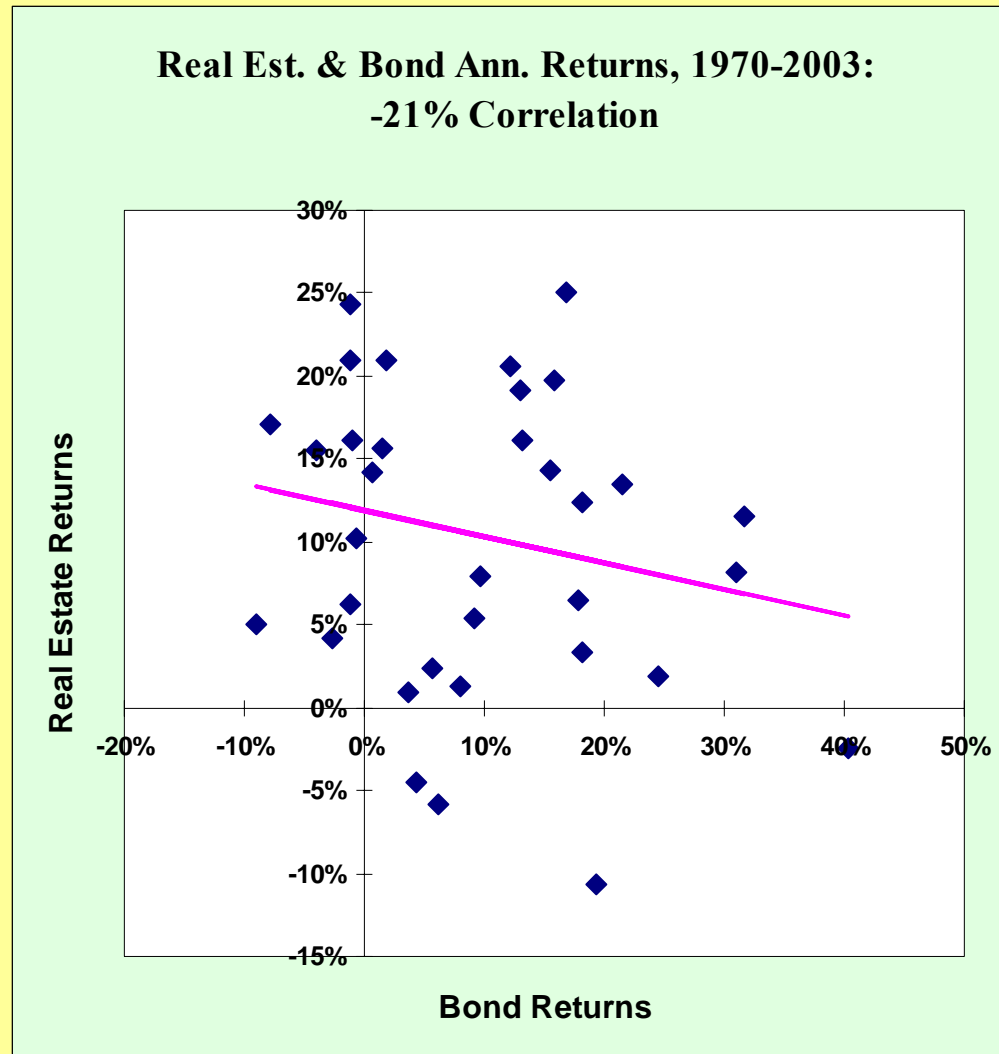
The diversification effect is greater the less correlated are the assets...

**Stocks & real estate (+17% correlation): Each dot is one year's returns.**

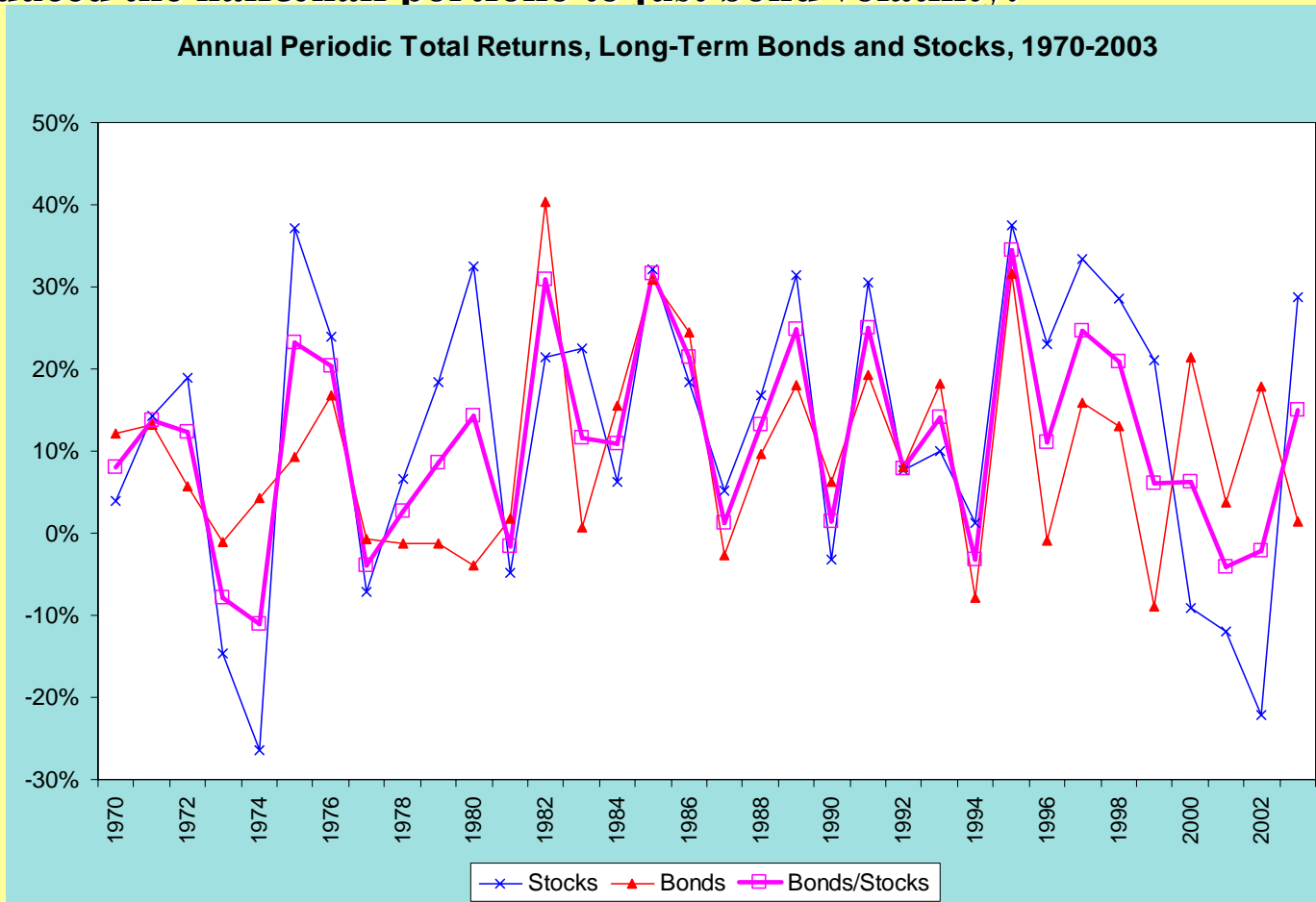


The diversification effect is greater the less correlated are the assets...

**Bonds & real estate (-21% correlation): Each dot is one year's returns.**

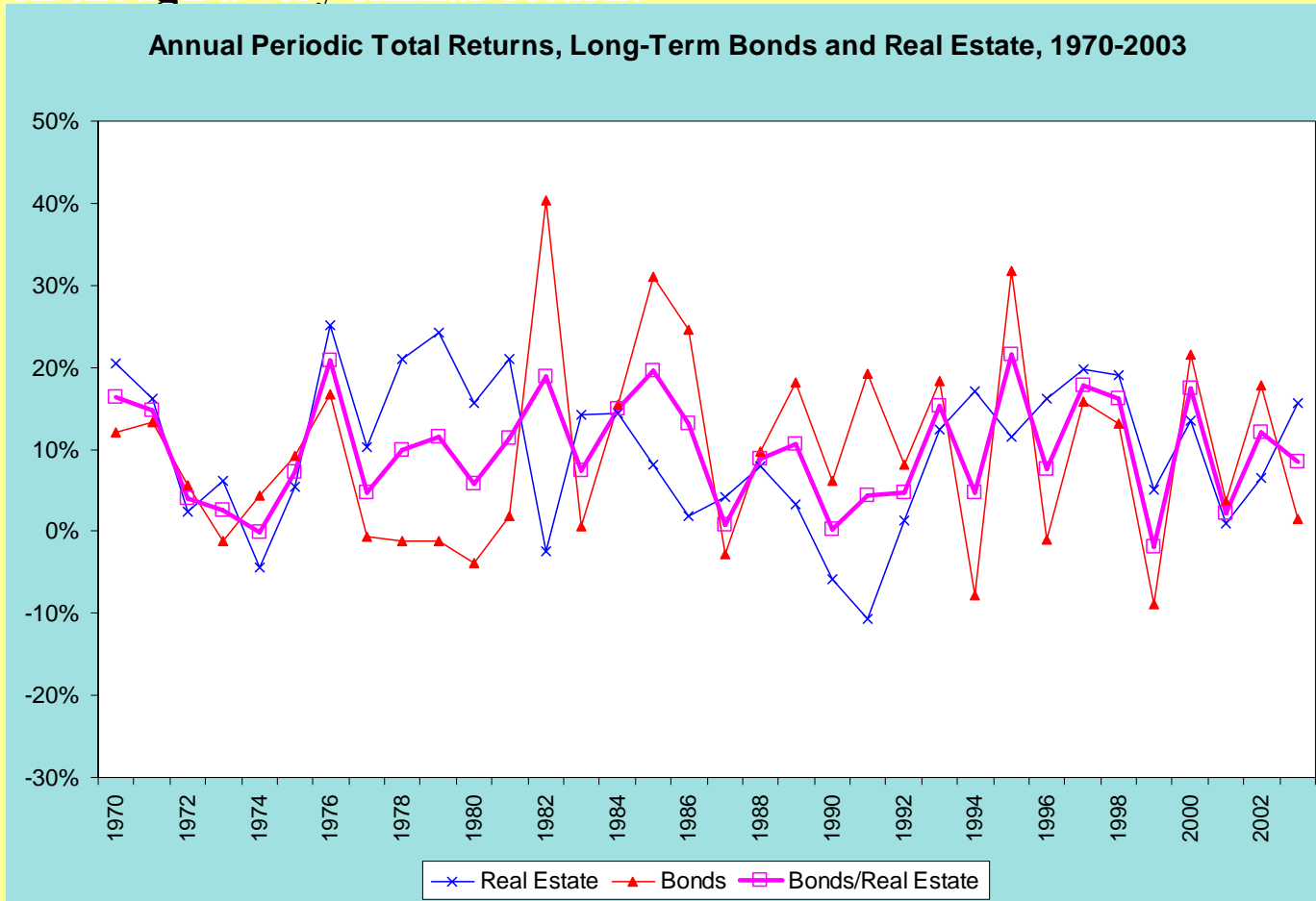


**For example, a portfolio of 50% bonds & 50% stocks would not have provided much volatility reduction during 1981-98, though over the longer 1970-2003 period it would have reduced the half&half portfolio to just bond volatility:**



Returns:	Bonds	Stocks	Half&Half
Mean	9.7%	12.7%	11.2%
Std.Dev.	11.8%	17.5%	11.8%

**Here the portfolio of 50% bonds & 50% real estate would have provided a more consistent diversification during 1970-2003, with less volatility than either asset class alone even though a very similar return:**



Returns:	<b>Bonds</b>	<b>R.Estate</b>	<b>Half&amp;Half</b>
<b>Mean</b>	<b>9.7%</b>	<b>9.9%</b>	<b>9.8%</b>
<b>Std.Dev.</b>	<b>11.8%</b>	<b>9.0%</b>	<b>6.6%</b>



**This “Diversification Effect” is greater, the lower is the correlation among the assets in the portfolio.**

**NUMERICAL EXAMPLE . . .**

SUPPOSE REAL ESTATE HAS:  
 EXPECTED RETURN = 8%  
 RISK (STD.DEV) = 10%

SUPPOSE STOCKS HAVE:  
 EXPECTED RETURN = 12%  
 RISK (STD.DEV) = 15%

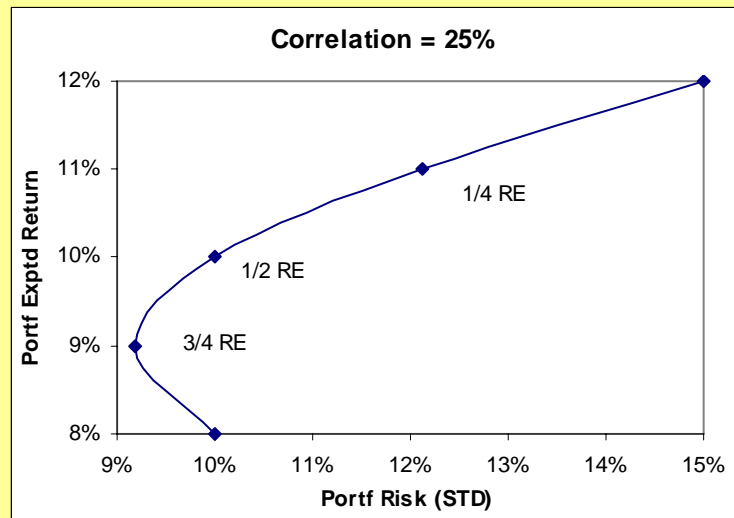
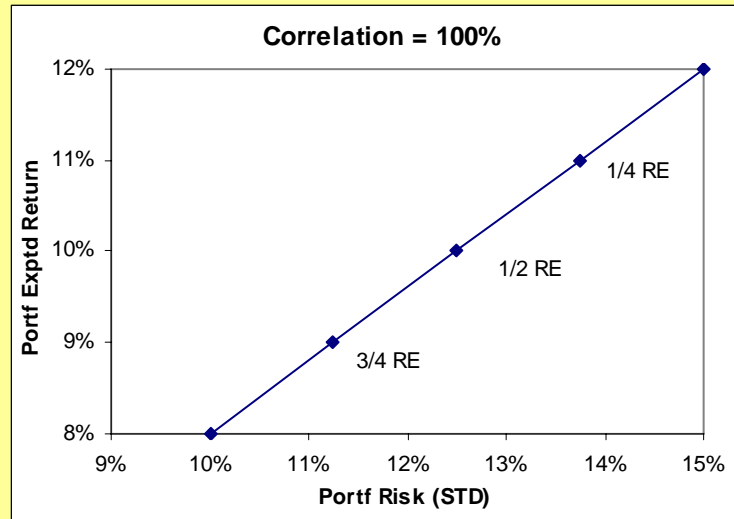
THEN A PORTFOLIO WITH  $\omega$  SHARE IN REAL ESTATE &  $(1-\omega)$  SHARE IN STOCKS WILL RESULT IN THESE RISK/RETURN COMBINATIONS, DEPENDING ON THE CORRELATION BETWEEN THE REAL ESTATE AND STOCK RETURNS:

$\omega$	<b>C = 100%</b>		<b>C = 25%</b>		<b>C = 0%</b>		<b>C = -50%</b>	
	$r_P$	$s_P$	$r_P$	$s_P$	$r_P$	$s_P$	$r_P$	$s_P$
<b>0%</b>	<b>12.0%</b>	<b>15.0%</b>	<b>12.0%</b>	<b>15.0%</b>	<b>12.0%</b>	<b>15.0%</b>	<b>12.0%</b>	<b>15.0%</b>
<b>25%</b>	<b>11.0%</b>	<b>13.8%</b>	<b>11.0%</b>	<b>12.1%</b>	<b>11.0%</b>	<b>11.5%</b>	<b>11.0%</b>	<b>10.2%</b>
<b>50%</b>	<b>10.0%</b>	<b>12.5%</b>	<b>10.0%</b>	<b>10.0%</b>	<b>10.0%</b>	<b>9.0%</b>	<b>10.0%</b>	<b>6.6%</b>
<b>75%</b>	<b>9.0%</b>	<b>11.3%</b>	<b>9.0%</b>	<b>9.2%</b>	<b>9.0%</b>	<b>8.4%</b>	<b>9.0%</b>	<b>6.5%</b>
<b>100%</b>	<b>8.0%</b>	<b>10.0%</b>	<b>8.0%</b>	<b>10.0%</b>	<b>8.0%</b>	<b>10.0%</b>	<b>8.0%</b>	<b>10.0%</b>

**where: C = Correlation Coefficient between Stocks & Real Estate.**

(This table was simply computed using the formulas noted previously.)

This “Diversification Effect” is greater, the lower is the correlation among the assets in the portfolio.



**IN ESSENCE,**

**PORTFOLIO THEORY ASSUMES:**

**YOUR OBJECTIVE FOR YOUR OVERALL WEALTH  
PORTFOLIO IS:**

**→ MAXIMIZE EXPECTED FUTURE RETURN**

**→ MINIMIZE RISK IN THE FUTURE RETURN**

**GIVEN THIS BASIC ASSUMPTION, AND THE EFFECT OF  
DIVERSIFICATION, WE ARRIVE AT THE FIRST MAJOR  
RESULT OF PORTFOLIO THEORY...**

To the investor, the risk that matters in an investment is that investment's contribution to the risk in the investor's overall portfolio, not the risk in the investment by itself. This means that covariance (correlation and variance) may be as important as (or more important than) variance (or volatility) in the investment alone.

*(e.g., if the investor's portfolio is primarily in stocks & bonds, and real estate has a low correlation with stocks & bonds, then the volatility in real estate may not matter much to the investor, because it will not contribute much to the volatility in the investor's portfolio. Indeed, it may allow a reduction in the portfolio's risk.)*

**THIS IS A MAJOR SIGNPOST ON THE WAY TO FIGURING OUT "HOW MANY EGGS" WE SHOULD PUT IN WHICH "BASKETS".**

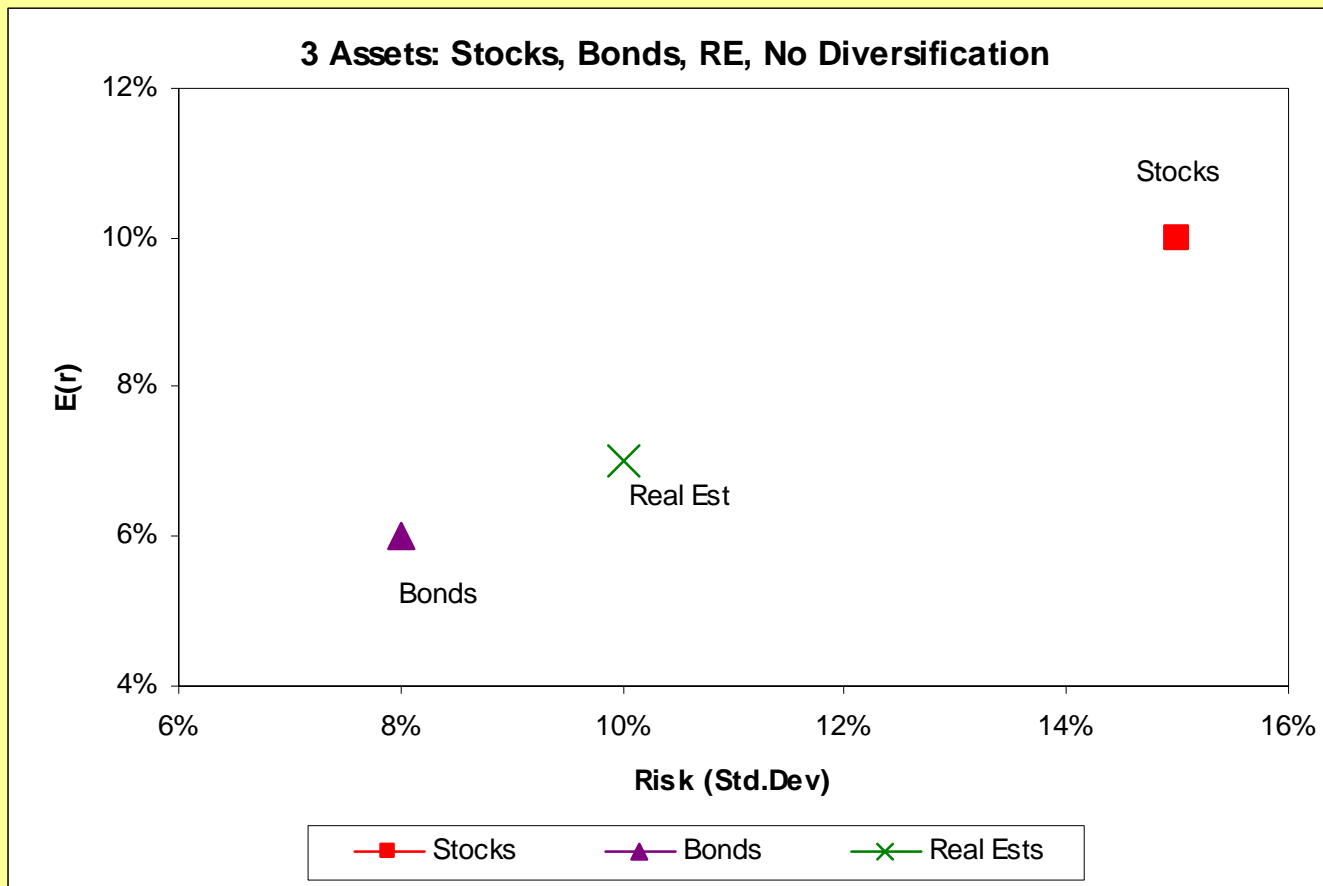
## 21.2.4 STEP 1: FINDING THE EFFICIENT FRONTIER

SUPPOSE WE HAVE THE FOLLOWING RISK & RETURN EXPECTATIONS...

	Stocks	Bonds	RE
Mean	10.00%	6.00%	7.00%
STD	15.00%	8.00%	10.00%
Corr			
Stocks	100.00%	30.00%	25.00%
Bonds		100.00%	15.00%
RE			100.00%

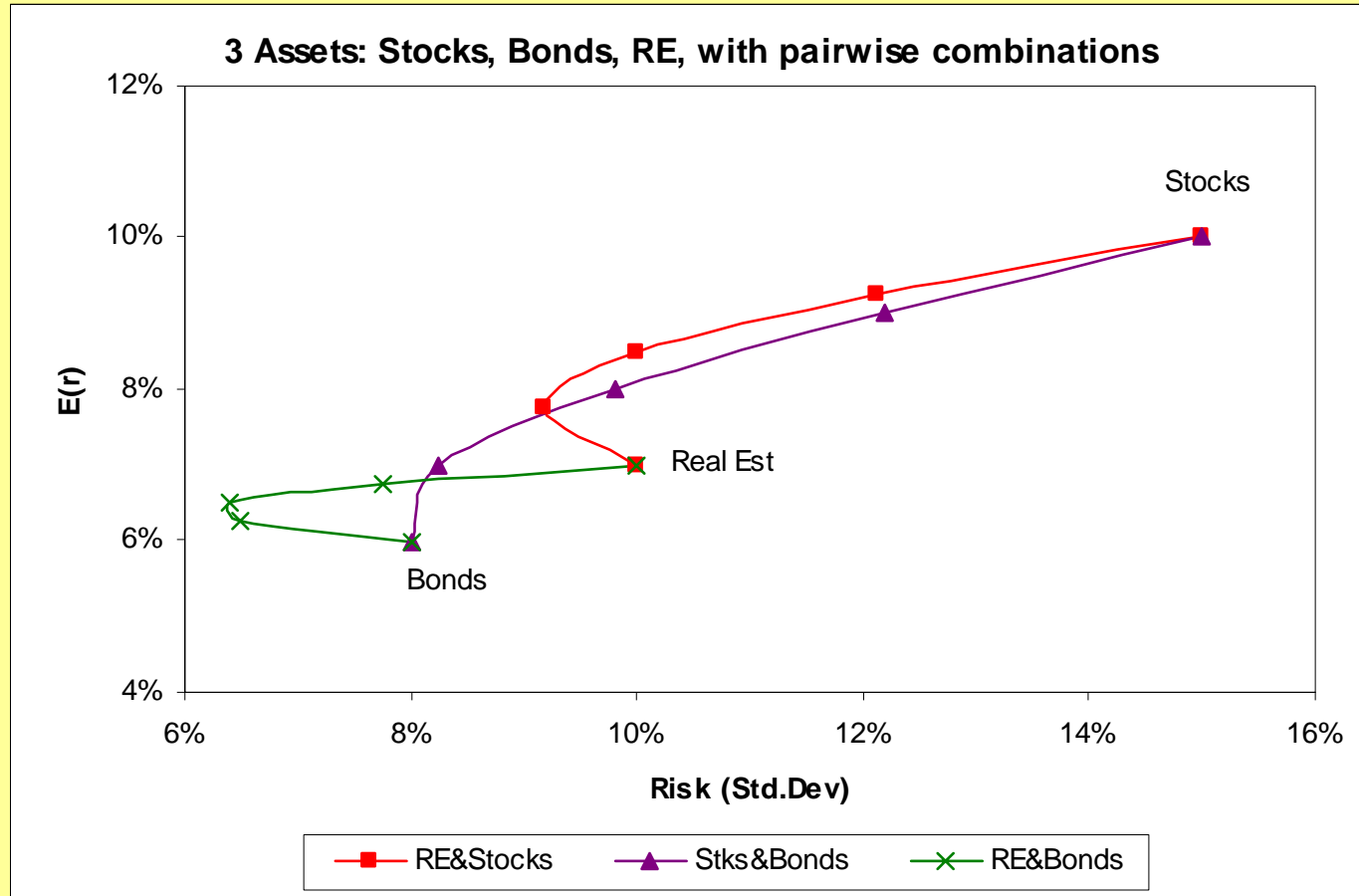
INVESTING IN ANY ONE OF THE THREE ASSET CLASSES WITHOUT DIVERSIFICATION ALLOWS THE INVESTOR TO ACHIEVE ONLY ONE OF THREE POSSIBLE RISK/RETURN POINTS...

**INVESTING IN ANY ONE OF THE THREE ASSET CLASSES WITHOUT DIVERSIFICATION ALLOWS THE INVESTOR TO ACHIEVE ONLY ONE OF THE THREE POSSIBLE RISK/RETURN POINTS DEPICTED IN THE GRAPH BELOW...**

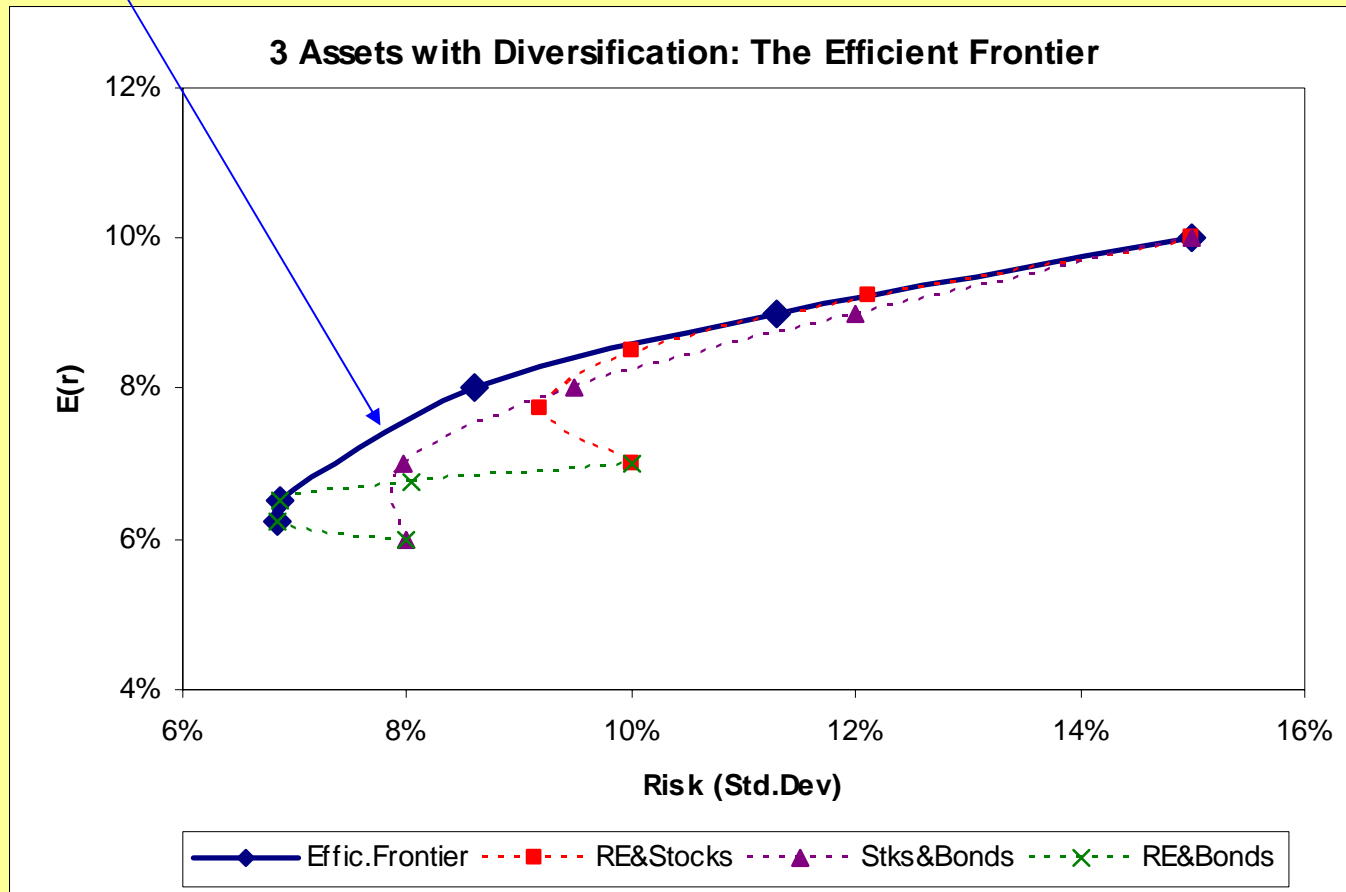


**IN A RISK/RETURN CHART LIKE THIS, ONE WANTS TO BE ABLE TO GET AS MANY RISK/RETURN COMBINATIONS AS POSSIBLE, *AS FAR TO THE "NORTH" AND "WEST" AS POSSIBLE.***

# ALLOWING PAIRWISE COMBINATIONS (AS WITH OUR PREVIOUS STOCKS & REAL ESTATE EXAMPLE), INCREASES THE RISK/RETURN POSSIBILITIES TO THESE...



FINALLY, IF WE ALLOW UNLIMITED DIVERSIFICATION AMONG ALL THREE ASSET CLASSES, WE ENABLE AN INFINITE NUMBER OF COMBINATIONS, THE “BEST” (I.E., MOST “NORTH” AND “WEST”) OF WHICH ARE SHOWN BY THE OUTSIDE (ENVELOPING) CURVE.



THIS IS THE “EFFICIENT FRONTIER” IN THIS CASE (OF THREE ASSET CLASSES).



**IN PORTFOLIO THEORY THE “*EFFICIENT FRONTIER*” CONSISTS OF ALL ASSET COMBINATIONS (PORTFOLIOS) WHICH MAXIMIZE RETURN AND MINIMIZE RISK.**

***THE EFFICIENT FRONTIER IS AS FAR “NORTH” AND “WEST” AS YOU CAN POSSIBLY GET IN THE RISK/RETURN GRAPH.***

**A PORTFOLIO IS SAID TO BE “*EFFICIENT*” (i.e., represents one point on the efficient frontier) IF IT HAS THE MINIMUM POSSIBLE VOLATILITY FOR A GIVEN EXPECTED RETURN, AND/OR THE MAXIMUM EXPECTED RETURN FOR A GIVEN LEVEL OF VOLATILITY.**

*(Terminology note: This is a different definition of "efficiency" than the concept of informational efficiency applied to asset markets and asset prices.)*

## SUMMARY UP TO HERE:

### DIVERSIFICATION AMONG RISKY ASSETS ALLOWS:

- GREATER EXPECTED RETURN TO BE OBTAINED  
FOR ANY GIVEN RISK EXPOSURE, &/OR;
- LESS RISK TO BE INCURRED  
FOR ANY GIVEN EXPECTED RETURN TARGET.

*(This is called getting on the "efficient frontier".)*

### PORTFOLIO THEORY ALLOWS US TO:

- QUANTIFY THIS EFFECT OF DIVERSIFICATION
- IDENTIFY THE "OPTIMAL" (BEST) MIXTURE OF RISKY ASSETS

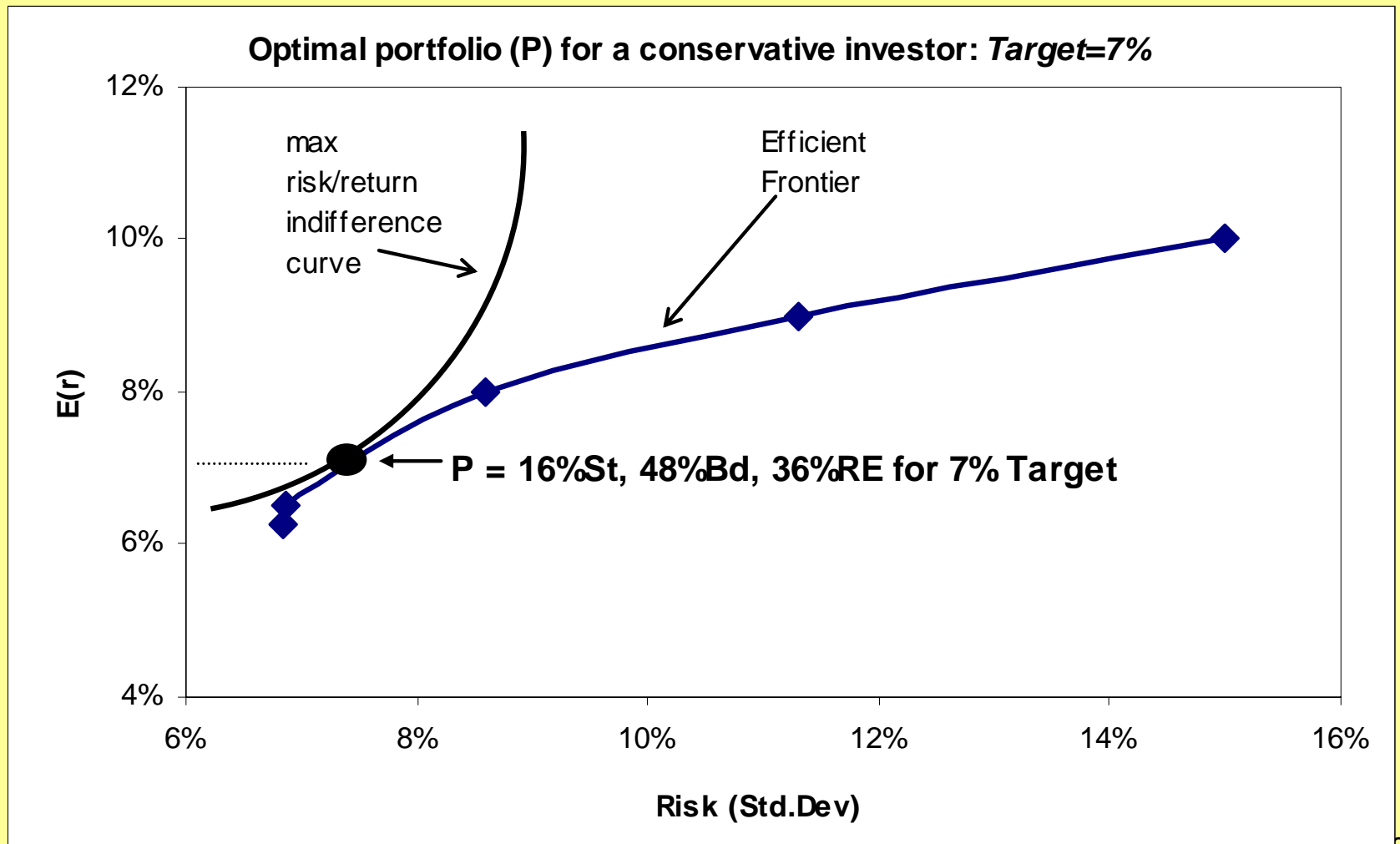
# **MATHEMATICALLY, THIS IS A "CONSTRAINED OPTIMIZATION" PROBLEM**

**==> Algebraic solution using calculus**

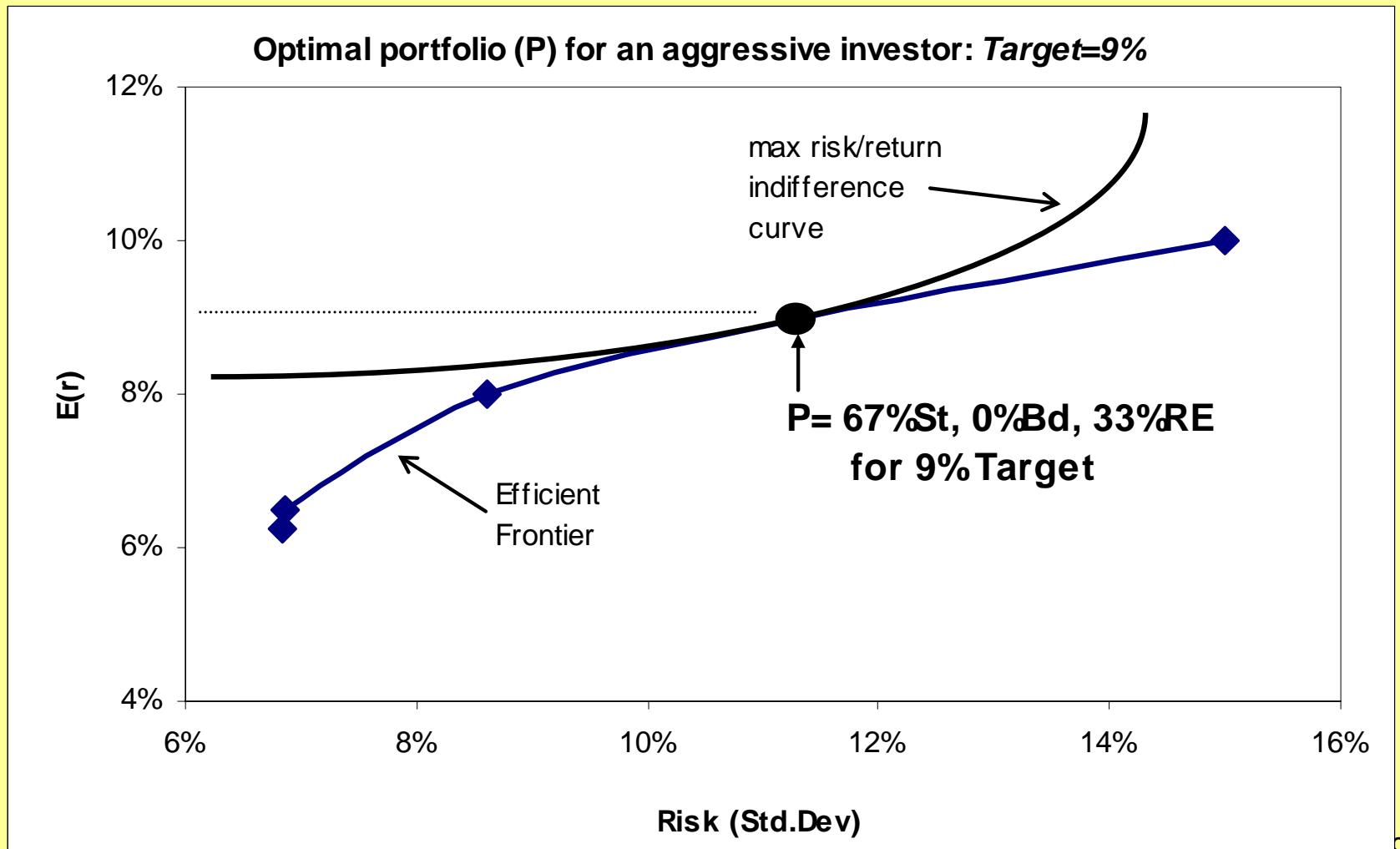
**==> Numerical solution using computer and "quadratic programming". Spreadsheets such as Excel include "Solvers" that can find optimal portfolios this way.**

## 21.2.5. STEP 2: PICK A RETURN TARGET FOR YOUR OVERALL WEALTH THAT REFLECTS YOUR RISK PREFERENCES...

*E.G., ARE YOU HERE (7%)?...*

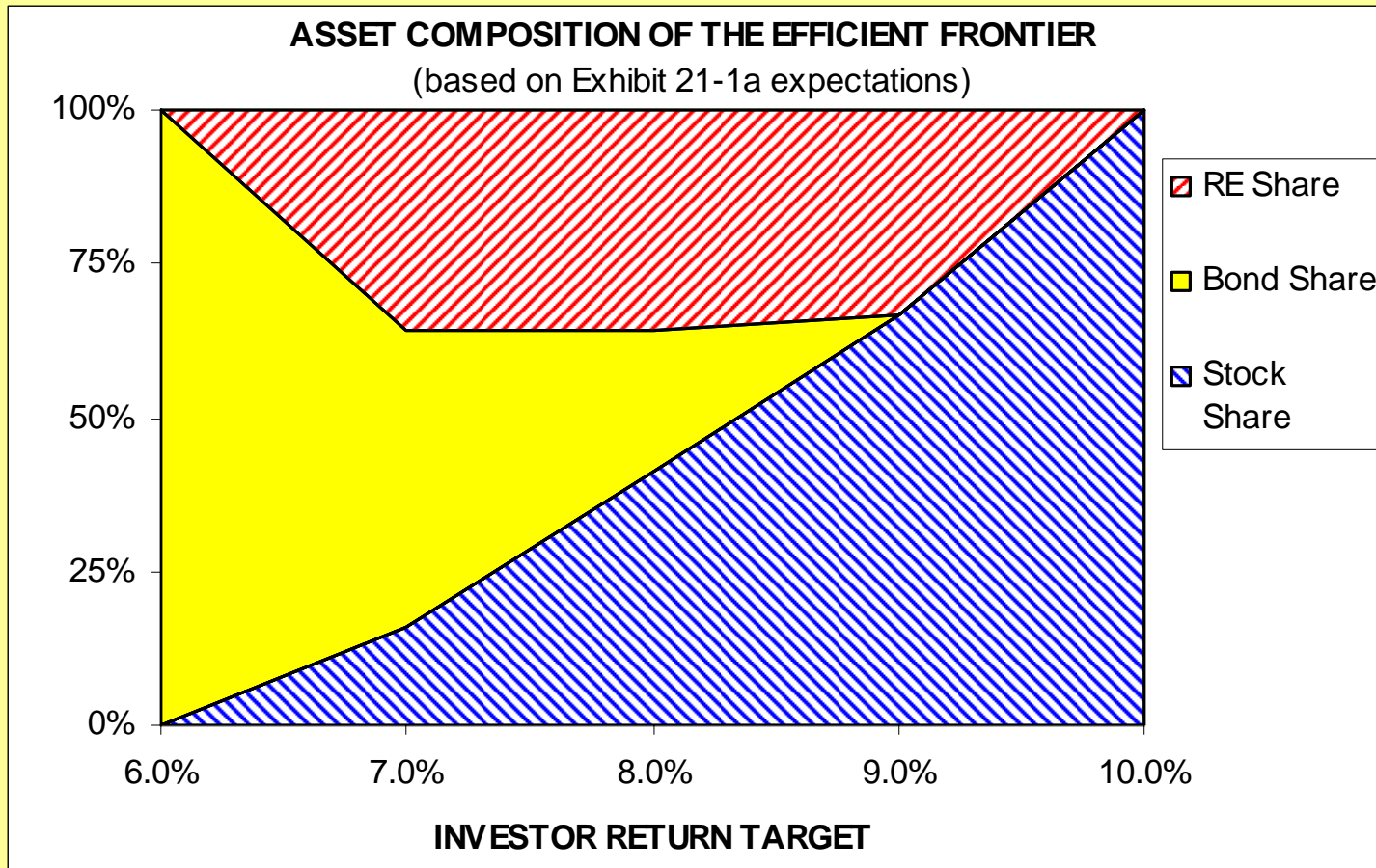


## ***OR ARE YOU HERE (9%)?...***



## 21.2.6

### Major Implications of Portfolio Theory for Real Estate Investment



**Core real estate assets typically make up a large share of efficient (non-dominated) portfolios for conservative to moderate return targets.**

## **GENERAL QUALITATIVE RESULTS OF PORTFOLIO THEORY**

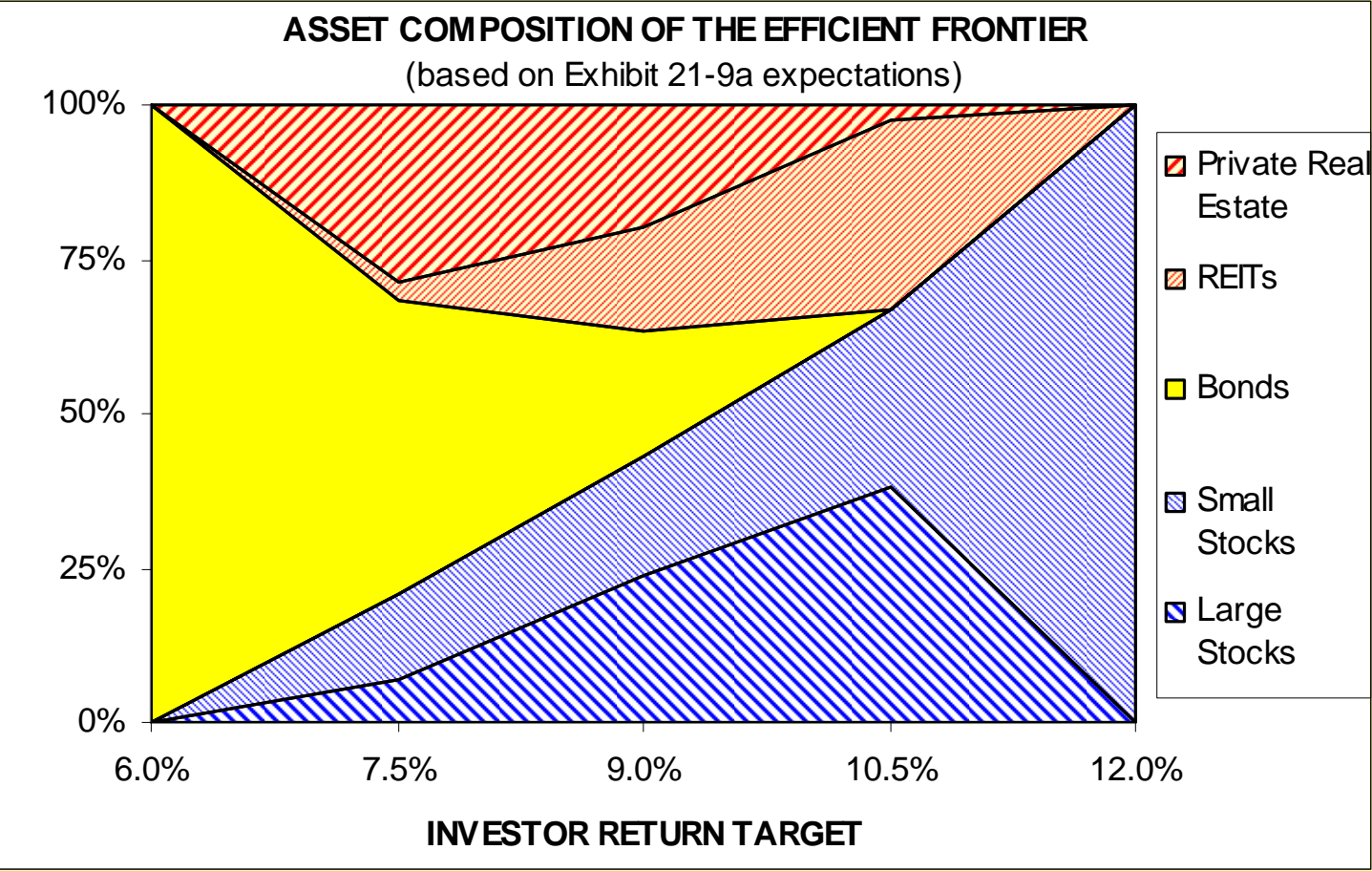
- 1) THE OPTIMAL REAL ESTATE SHARE DEPENDS ON HOW CONSERVATIVE OR AGGRESSIVE IS THE INVESTOR;
- 2) FOR MOST OF THE RANGE OF RETURN TARGETS, REAL ESTATE IS A SIGNIFICANT SHARE. (COMPARE THESE SHARES TO THE AVERAGE U.S. PENSION FUND REAL ESTATE ALLOCATION WHICH IS LESS THAN 5%. THIS IS WHY PORTFOLIO THEORY HAS BEEN USED TO TRY TO GET INCREASED PF ALLOCATION TO REAL ESTATE.)
- 3) THE ROBUSTNESS OF REAL ESTATE'S INVESTMENT APPEAL IS DUE TO ITS LOW CORRELATION WITH BOTH STOCKS & BONDS, THAT IS, WITH ALL OF THE REST OF THE PORTFOLIO. (NOTE IN PARTICULAR THAT OUR INPUT ASSUMPTIONS IN THE ABOVE EXAMPLE NUMBERS DID NOT INCLUDE A PARTICULARLY HIGH RETURN OR PARTICULARLY LOW VOLATILITY FOR THE REAL ESTATE ASSET CLASS. THUS, THE LARGE REAL ESTATE SHARE IN THE OPTIMAL PORTFOLIO MUST NOT BE DUE TO SUCH ASSUMPTIONS.)

## 21.2.7 SUPPOSE WE EXPAND THE PORTFOLIO CHOICE SET BY ADDING ADDITIONAL SUB-CLASSES OF ASSETS...

For example, suppose we add the following expectations for an additional sub-class of stocks (small stocks) and an additional sub-class of real estate (REITs)...

Exhibit 21-9a: Possible Risk & Return Expectations for 5 Asset Classes					
	Large Stocks	Small Stocks	Bonds	REITs	Private Real Estate
Expected Return ( $E[r]$ )	10.00%	12.00%	6.00%	10.00%	7.00%
Volatility	15.00%	20.00%	8.00%	15.00%	10.00%
Correlation with:					
Large Stocks	100.00%	60.00%	30.00%	45.00%	25.00%
Small Stocks		100.00%	0.00%	70.00%	25.00%
Bonds			100.00%	20.00%	15.00%
REITs				100.00%	40.00%
Private Real Estate					100.00%





## Section 21.3

### VII. INTRODUCING A "RISKLESS ASSET"...

IN A COMBINATION OF A **RISKLESS** AND A RISKY ASSET, BOTH RISK AND RETURN ARE WEIGHTED AVERAGES OF RISK AND RETURN OF THE TWO ASSETS:

*Recall:*

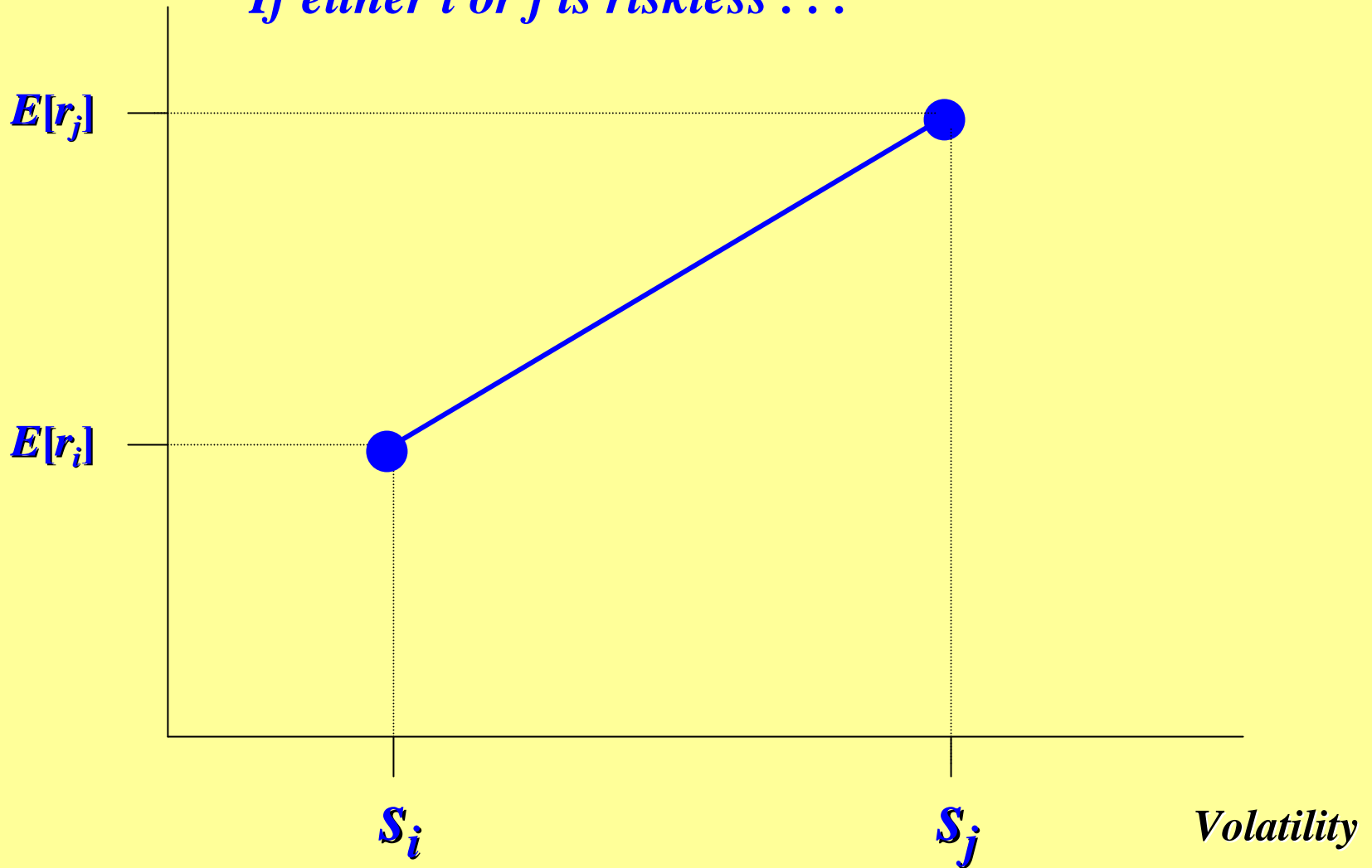
$$s_p = \sqrt{[\omega^2(s_i)^2 + (1-\omega)^2(s_j)^2 + 2\omega(1-\omega)s_i s_j C_{ij}]}$$

If  $s_j=0$ , this reduces to:

$$s_p = \sqrt{[\omega^2(s_i)^2] = \omega s_i}$$

SO THE RISK/RETURN COMBINATIONS OF A MIXTURE OF INVESTMENT IN A RISKLESS ASSET AND A RISKY ASSET LIE ON A **STRAIGHT LINE**, PASSING THROUGH THE TWO POINTS REPRESENTING THE RISK/RETURN COMBINATIONS OF THE RISKLESS ASSET AND THE RISKY ASSET.

*If either  $i$  or  $j$  is riskless . . .*



→ IN PORTFOLIO ANALYSIS, THE "RISKLESS ASSET" REPRESENTS BORROWING OR LENDING BY THE INVESTOR...

BORROWING IS LIKE "SELLING SHORT" OR HOLDING A NEGATIVE WEIGHT IN THE RISKLESS ASSET. *BORROWING IS "RISKLESS" BECAUSE YOU MUST PAY THE MONEY BACK "NO MATTER WHAT".*

LENDING IS LIKE BUYING A BOND OR HOLDING A POSITIVE WEIGHT IN THE RISKLESS ASSET. *LENDING IS "RISKLESS" BECAUSE YOU CAN INVEST IN GOVT BONDS AND HOLD TO MATURITY.*

SUPPOSE YOU COMBINE RISKLESS BORROWING OR LENDING WITH YOUR INVESTMENT IN THE RISKY PORTFOLIO OF STOCKS & REAL ESTATE.

YOUR OVERALL EXPECTED RETURN WILL BE:

$$r_W = v r_P + (1-v)r_f$$

AND YOUR OVERALL RISK WILL BE:

$$s_W = v s_P + (1-v)0 = v s_P$$

Where:  $v$  = Weight in risky portfolio  
 $r_W, s_W$  = Return, Std.Dev., in overall wealth  
 $r_P, s_P$  = Return, Std.Dev., in risky portfolio  
 $r_f$  = Riskfree Interest Rate

$v$  NEED NOT BE CONSTRAINED TO BE LESS THAN UNITY.  
 $v$  CAN BE GREATER THAN 1 ("**leverage**" , "**borrowing**"), OR  
 $v$  CAN BE LESS THAN 1 BUT POSITIVE ("**lending**", investing in bonds, in addition to investing in the risky portfolio).

**THUS, USING BORROWING OR LENDING, IT IS POSSIBLE TO OBTAIN ANY RETURN TARGET OR ANY RISK TARGET. THE RISK/RETURN COMBINATIONS WILL LIE ON THE STRAIGHT LINE PASSING THROUGH POINTS  $r_f$  AND  $r_P$ .**

## NUMERICAL EXAMPLE

SUPPOSE:

RISKFREE INTEREST RATE = 5%

STOCK EXPECTED RETURN = 15%

STOCK STD.DEV. = 15%

---

IF RETURN TARGET = 20%,

BORROW \$0.5

INVEST \$1.5 IN STOCKS ( $v = 1.5$ ).

EXPECTED RETURN WOULD BE:

$$(1.5)15\% + (-0.5)5\% = 20\%$$

RISK WOULD BE

$$(1.5)15\% + (-0.5)0\% = 22.5\%$$

---

IF RETURN TARGET = 10%,

LEND (INVEST IN BONDS) \$0.5

INVEST \$0.5 IN STOCKS ( $v = 0.5$ ).

EXPECTED RETURN WOULD BE:

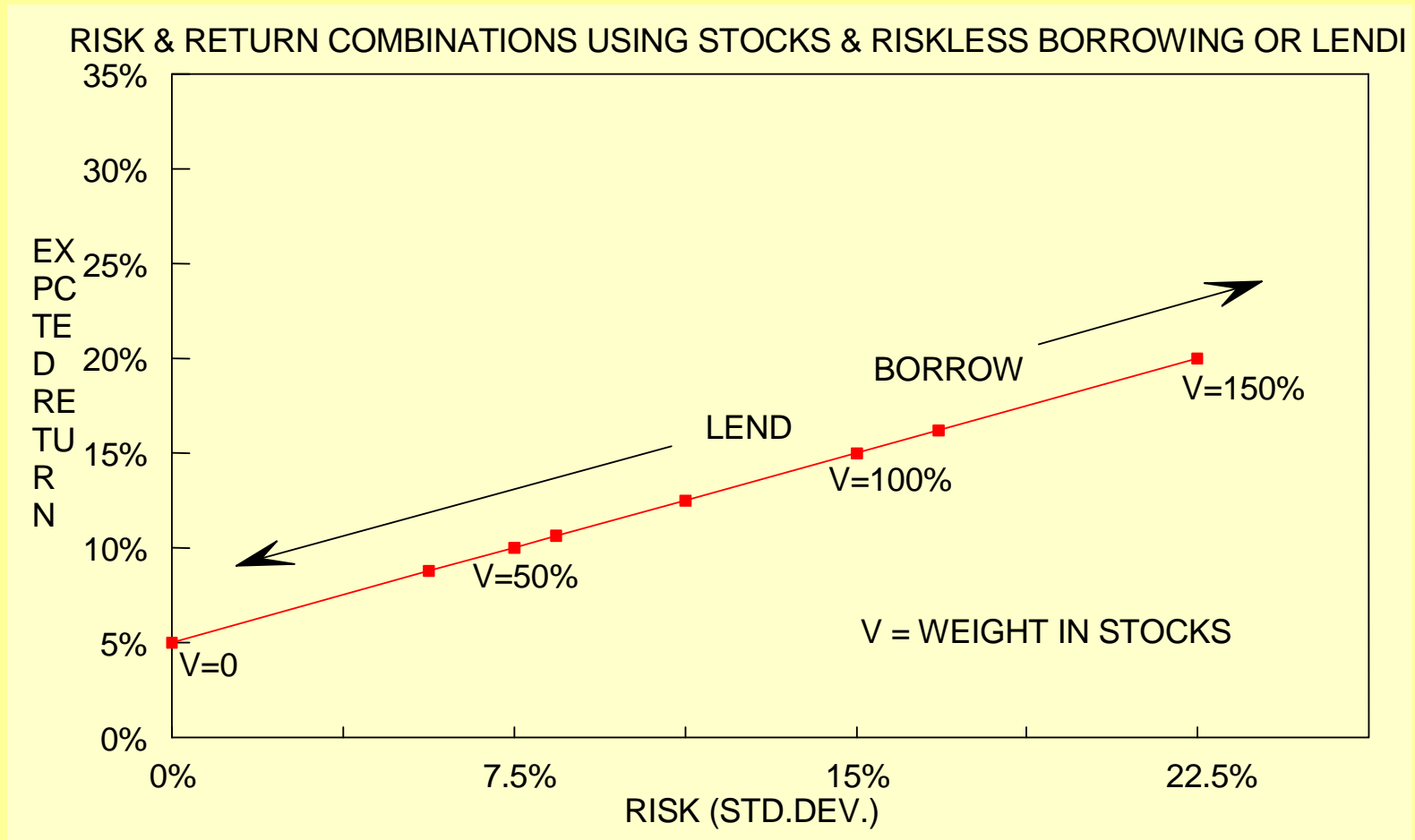
$$(0.5)15\% + (0.5)5\% = 10\%$$

RISK WOULD BE

$$(0.5)15\% + (0.5)0\% = 7.5\%$$

---

NOTICE THESE POSSIBILITIES LIE ON A STRAIGHT LINE IN RISK/RETURN SPACE . . .



BUT NO MATTER WHAT YOUR RETURN TARGET, YOU CAN DO BETTER BY PUTTING YOUR RISKY MONEY IN A DIVERSIFIED PORTFOLIO OF REAL ESTATE & STOCKS . . .

SUPPOSE:

REAL ESTATE EXPECTED RETURN = 10%

REAL ESTATE STD.DEV. = 10%

CORRELATION BETWEEN STOCKS & REAL ESTATE = 25%

THEN 50% R.E. / STOCKS MIXTURE WOULD PROVIDE:

EXPECTED RETURN = 12.5%;      STD.DEV. = 10.0%

---

IF RETURN TARGET = 20%,

BORROW \$1.0

INVEST \$2.0 IN RISKY MIXED-ASSET PORTFOLIO ( $v = 2$ ).

EXPECTED RETURN WOULD BE:

$$(2.0)12.5\% + (-1.0)5\% = 20\%$$

RISK WOULD BE:

$$(2.0)10.0\% + (-1.0)0\% = 20\% < 22.5\%$$

---

IF RETURN TARGET = 10%,

LEND (INVEST IN BONDS) \$0.33

INVEST \$0.67 IN RISKY MIXED-ASSET PORTFOLIO ( $v = 0.67$ ).

EXPECTED RETURN WOULD BE:

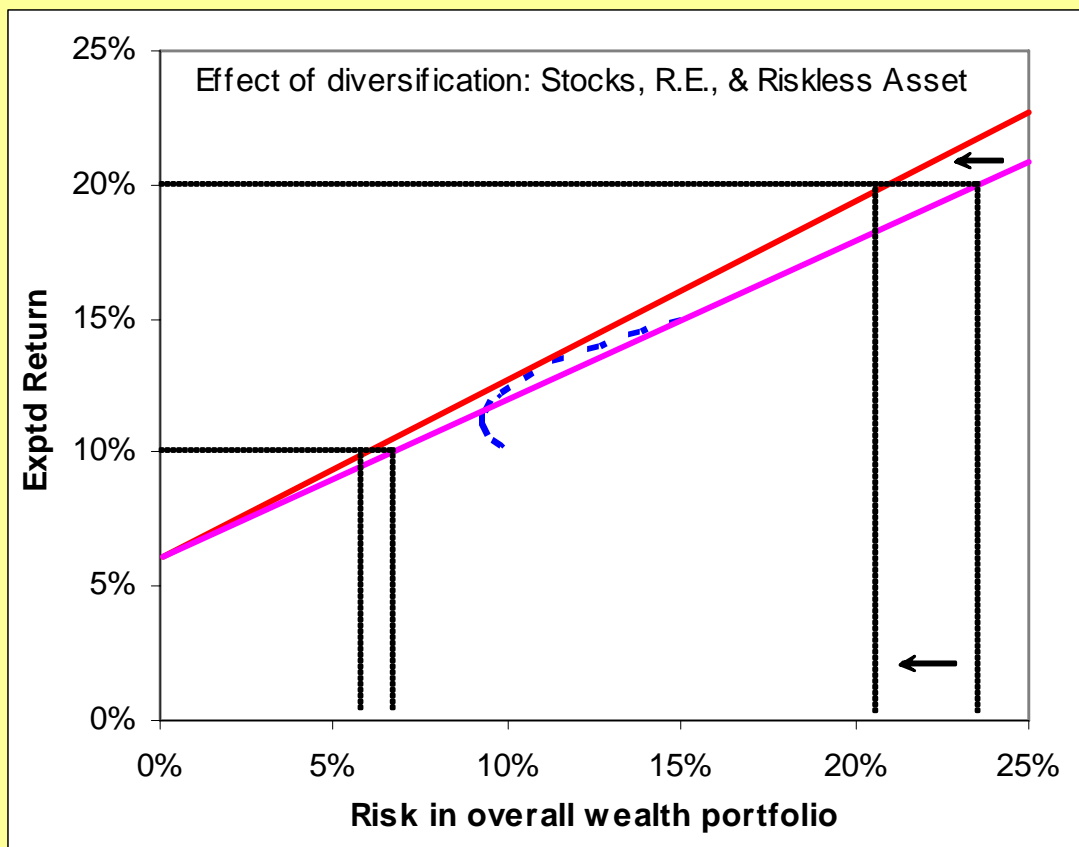
$$(0.67)12.5\% + (0.33)5\% = 10\%$$

RISK WOULD BE:

$$(0.67)10.0\% + (0.33)0\% = 6.7\% < 7.5\%$$



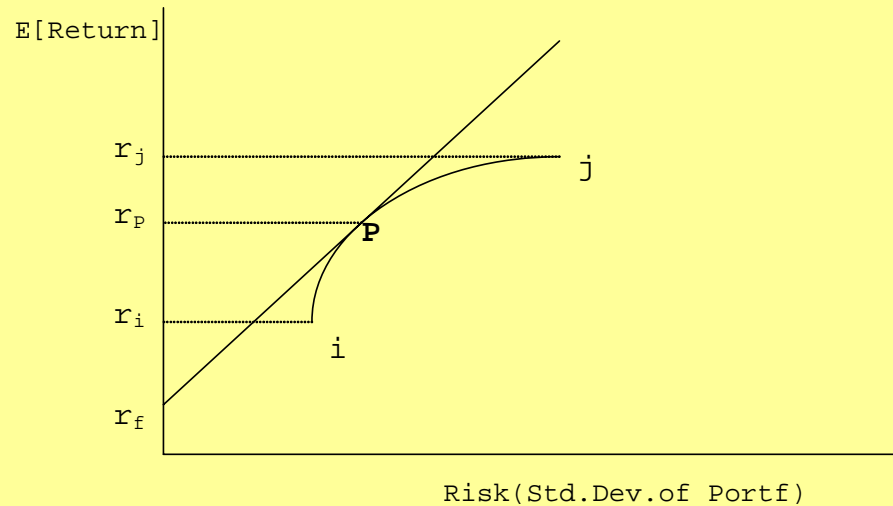
THE GRAPH BELOW SHOWS THE EFFECT DIVERSIFICATION IN THE RISKY PORTFOLIO HAS ON THE RISK/RETURN POSSIBILITY FRONTIER.



**THE FRONTIER IS STILL A STRAIGHT LINE ANCHORED ON THE RISKFREE RATE, BUT THE LINE NOW HAS A GREATER “SLOPE”, PROVIDING MORE RETURN FOR THE SAME AMOUNT OF RISK, ALLOWING LESS RISK FOR THE SAME EXPECTED RETURN.**

# THE "OPTIMAL" RISKY ASSET PORTFOLIO WITH A RISKLESS ASSET

(aka "TWO-FUND THEOREM")



CURVED LINE IS FRONTIER OBTAINABLE INVESTING ONLY IN RISKY ASSETS

STRAIGHT LINE PASSING THRU  $r_f$  AND PARABOLA IS OBTAINABLE BY MIXING RISKLESS ASSET (LONG OR SHORT) WITH RISKY ASSETS.

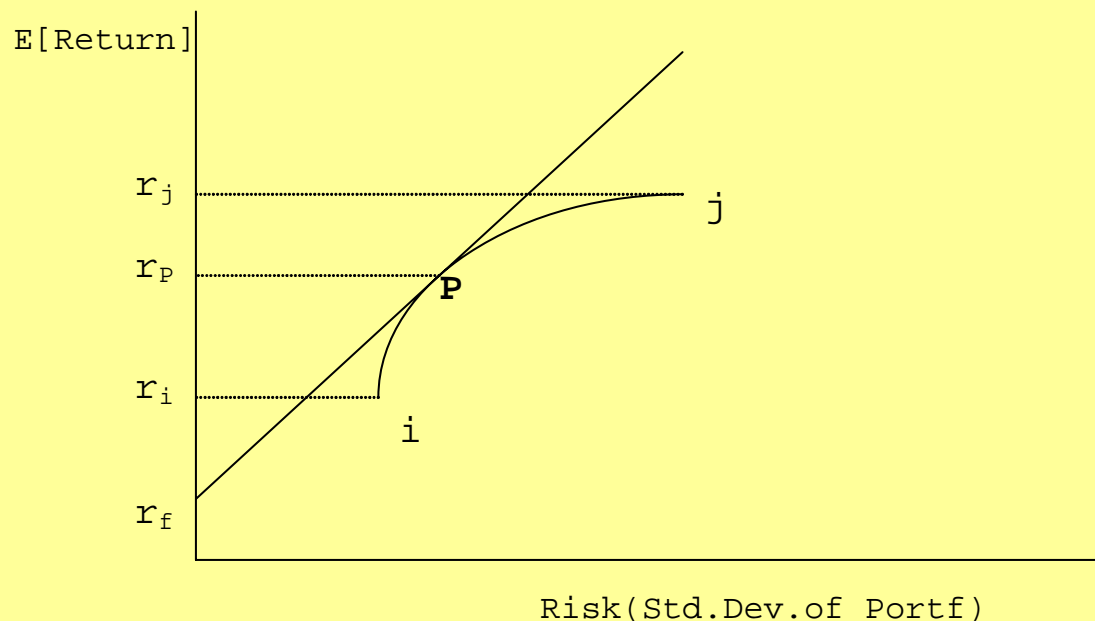
YOU WANT "HIGHEST" STRAIGHT LINE POSSIBLE (NO MATTER WHO YOU ARE!).

OPTIMAL STRAIGHT LINE IS THUS THE ONE PASSING THRU POINT "P".

IT IS THE STRAIGHT LINE ANCHORED IN  $r_f$  WITH THE MAXIMUM POSSIBLE SLOPE.

THUS, THE STRAIGHT LINE PASSING THROUGH "P" IS THE EFFICIENT FRONTIER. THE FRONTIER TOUCHES (AND INCLUDES) THE CURVED LINE AT ONLY ONE POINT: THE POINT "P".

THUS, THE **"2-FUND THEOREM"** TELLS US THAT THERE IS A SINGLE PARTICULAR COMBINATION OF RISKY ASSETS (THE PORTFOLIO "P") WHICH IS "OPTIMAL" NO MATTER WHAT THE INVESTOR'S RISK PREFERENCES OR TARGET RETURN.



THUS,  
ALL EFFIC. PORTFS ARE COMBINATIONS OF JUST 2 FUNDS:  
RISKLESS FUND (long or short position) + RISKY FUND "P" (long position).  
HENCE THE NAME: "2-FUND THEOREM".

## 21.3.2

*HOW DO WE KNOW WHICH COMBINATION OF RISKY ASSETS IS THE OPTIMAL ALL-RISKY PORTFOLIO “P”?*

IT IS THE ONE THAT MAXIMIZES THE SLOPE OF THE STRAIGHT LINE FROM THE RISKFREE RETURN THROUGH “P”. THE SLOPE OF THIS LINE IS GIVEN BY THE RATIO:

$$\text{Portfolio Sharpe Ratio} = (r_p - r_f) / s_p$$

MAXIMIZING THE **SHARPE RATIO** FINDS THE OPTIMAL RISKY ASSET COMBINATION. THE SHARPE RATIO IS ALSO A GOOD INTUITIVE MEASURE OF “RISK-ADJUSTED RETURN” FOR THE INVESTOR’S WEALTH, AS IT GIVES THE RISK PREMIUM PER UNIT OF RISK (MEASURED BY ST.DEV).

THUS, IF WE ASSUME THE EXISTENCE OF A RISKLESS ASSET, WE CAN USE THE 2-FUND THEOREM TO FIND THE OPTIMAL RISKY ASSET MIXTURE AS THAT PORTFOLIO WHICH HAS THE HIGHEST "SHARPE RATIO".

## **BACK TO PREVIOUS 2-ASSET NUMERICAL EXAMPLE...**

USING OUR PREVIOUS EXAMPLE NUMBERS, THE OPTIMAL COMBINATION OF REAL ESTATE & STOCKS CAN BE FOUND BY EXAMINING THE SHARPE RATIO FOR EACH COMBINATION . . .

$\omega =$ RE share	$r_p$	$r_p - r_f$	$s_p$	Sharpe Ratio
0	15.0%	10.0%	15.0%	66.7%
0.1	14.5%	9.5%	13.8%	68.9%
0.2	14.0%	9.0%	12.6%	71.2%
0.3	13.5%	8.5%	11.6%	73.2%
0.4	13.0%	8.0%	10.7%	74.6%
0.5	12.5%	7.5%	10.0%	75.0%
0.6	12.0%	7.0%	9.5%	73.8%
0.7	11.5%	6.5%	9.2%	70.5%
0.8	11.0%	6.0%	9.2%	65.1%
0.9	10.5%	5.5%	9.5%	58.0%
1.0	10.0%	5.0%	10.0%	50.0%

OF THE 11 MIXTURES CONSIDERED ABOVE, THE 50% REAL ESTATE WOULD BE BEST BECAUSE IT HAS THE HIGHEST SHARPE MEASURE.

***BUT SUPPOSE YOU ARE NOT SATISFIED WITH THE 12.5%  $E_r$  THAT WILL GIVE YOU FOR YOUR OVERALL WEALTH? ...***

***OR YOU DON'T WANT TO SUBJECT YOUR OVERALL WEALTH TO 10% VOLATILITY?...***

THEN YOU CAN INVEST PROPORTIONATELY 50% IN REAL ESTATE AND 50% IN STOCKS, ...

AND THEN ACHIEVE A GREATER RETURN THAN 12.5% BY BORROWING (LEVERAGE,  $v > 1$ ),

OR YOU CAN INCUR LESS THAN 10.0% RISK BY LENDING (INVESTING IN GOVT BONDS,  $v < 1$ )...

***(BUT YOU CAN'T DO BOTH. THE "FREE LUNCH" OF PORTFOLIO THEORY ONLY GETS YOU SO FAR, THAT IS, TO THE EFFICIENT FRONTIER, BUT ON THAT FRONTIER THERE WILL BE A RISK/RETURN TRADEOFF. THAT TRADEOFF WILL BE DETERMINED BY THE MARKET...)***

## Example of Difference Between Markowitz and Sharpe Optimal Portfolios

Exhibit 21-11: Comparison of Optimal 7%-Return-Target Portfolio Allocations, Variance-Minimization vs Sharpe Ratio-Maximization...

	Return & Risk Expectations*:			Portfolio Allocations:	
	Return	Volatility	Sharpe Ratio	Var-Min	Sharpe-Max
Cash (T-bills)	3.00%	NA**	NA	NA	10%
Bonds	6.00%	8.00%	0.38	48%	33%
Real Estate	7.00%	10.00%	0.40	36%	32%
Stocks	10.00%	15.00%	0.47	16%	25%
Var-Min Portfolio	7.00%	6.89%	0.58	100%	NA
Shape-Max Portf.	7.00%	NA**	0.59	NA	100%

\*Also includes correlations:

Stock-Bond +30%, Stock-Real Estate +25%, Bond-Real Estate +15%.

\*\*From the Sharpe-maximization perspective, T-bills are viewed as having zero volatility, but as this is not exactly true in reality, it would be misleading to calculate and show a Sharpe-maximizing portfolio volatility juxtaposed with that of the variance-minimized portfolio.

## 2-FUND THEOREM SUMMARY:

- 1) THE 2-FUND THEOREM ALLOWS AN ALTERNATIVE, INTUITIVELY APPEALING DEFINITION OF THE OPTIMAL RISKY PORTFOLIO: THE ONE WITH THE **MAXIMUM SHARPE RATIO**.
- 2) THIS CAN HELP **AVOID "SILLY"** OPTIMAL PORTFOLIOS THAT PUT TOO LITTLE WEIGHT IN HIGH-RETURN ASSETS JUST BECAUSE THE INVESTOR HAS A CONSERVATIVE TARGET RETURN. (*OR TOO LITTLE WEIGHT IN LOW-RETURN ASSETS JUST BECAUSE THE INVESTOR HAS AN AGGRESSIVE TARGET.*)
- 3) IT ALSO PROVIDES A GOOD **FRAMEWORK** FOR ACCOMMODATING THE POSSIBLE USE OF **LEVERAGE**, OR OF RISKLESS INVESTING (BY **HOLDING BONDS TO MATURITY**), BY THE INVESTOR.



## Chapter 21 Summary: MPT & Real Estate . . .

- **The classical theory suggests a fairly robust, substantial role for the real estate asset class in the optimal portfolio (typically 25%-40% without any additional assumptions), either w or w/out riskless asset.**
- **This role tends to be greater for more conservative portfolios, less for very aggressive portfolios.**
- **Role is based primarily on *diversification benefits* of real estate, somewhat sensitive to R.E. correlation w stocks & bonds.**
- **Optimal real estate share roughly matches actual real estate proportion of all investable assets in the economy.**
- **Optimal real estate share in theory is substantially greater than actual pension fund allocations to real estate.**
- **Optimal R.E. share can be reduced by adding assumptions and extensions to the classical model:**
  - Extra transaction costs, illiquidity penalties;
  - Long-term horizon risk & returns;
  - Net Asset-Liability portfolio framework;
  - Investor constrained to over-invest in owner-occupied house as investment.
- **But even with such extensions, optimal R.E. share often substantially exceeds existing P.F. allocations to R.E. (approx. 3% on avg.\*)**

# **Chapter 22**

## **Equilibrium Asset Pricing**

## 22.1.1 Practical Uses for Asset Price Theory

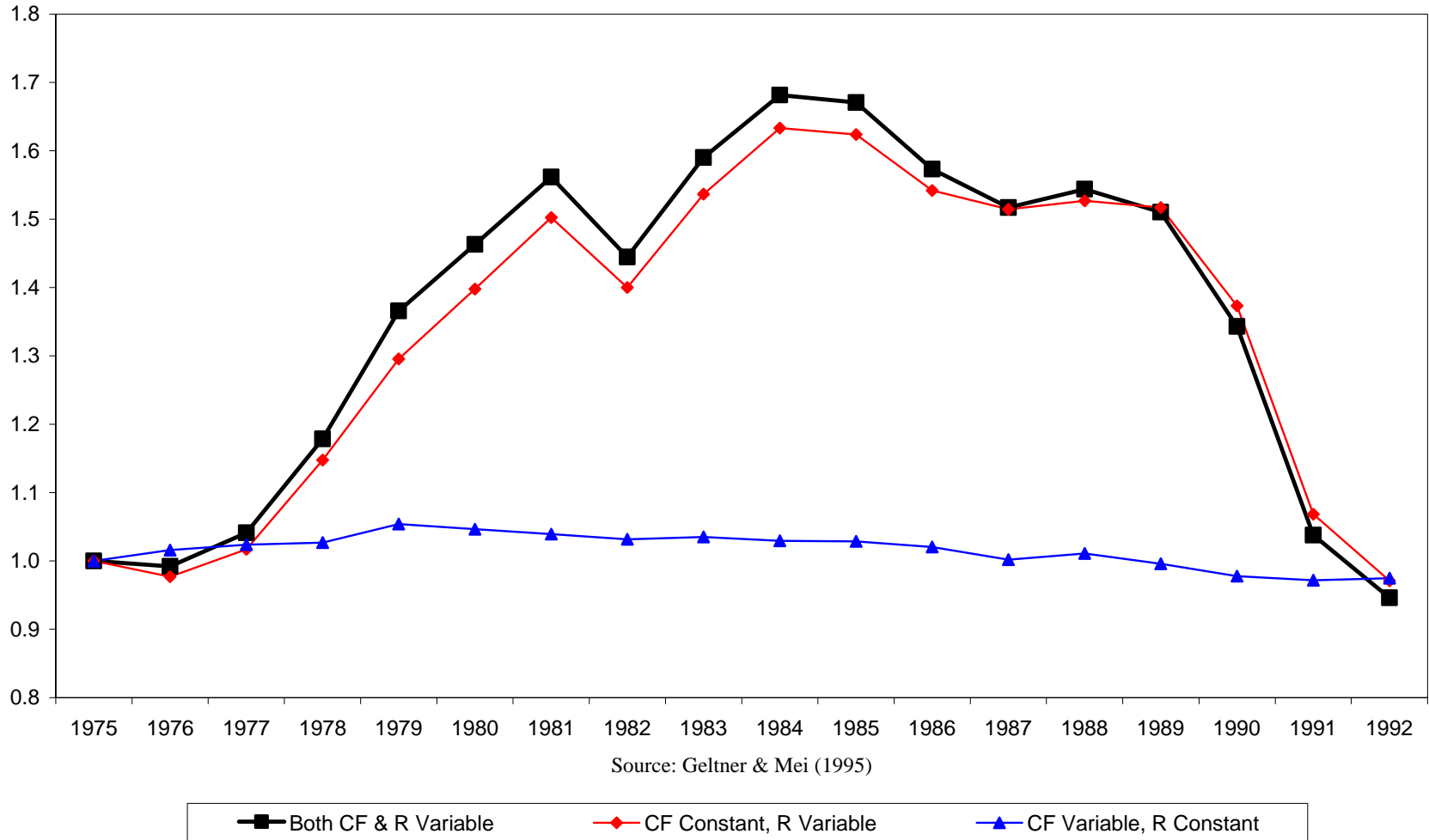
1. Help investors understand what are reasonable *ex ante* returns on investments in different asset classes or types of investment products. (Quantify the OCC – *Oppty Cost of Capital* or “hurdle rate”.)
2. Help identify specific types of assets or investment products (or “sectors” of the asset market) that are currently mispriced relative to long-run equilibrium.
3. Control for risk when evaluating portfolio returns or investment performance.

### **Asset models do two things:**

- **Identify “*risk*” as it matters in the capital markets, and;**
- **Quantify the market’s metric for such risk (as it matters in asset pricing).**

## 22.1.2 A Threshold Point: What Underlies Asset Risk?

**Simulated Historical Present Values**  
Using VAR-Forecasted Cash Flows (CF) & Returns (R) in Present Value Model



**Most of the volatility in asset prices does not derive from rational changes in future cash flow expectations**

## 22.1.3

### VIII. FROM PORTFOLIO THEORY TO EQUILIBRIUM ASSET PRICE MODELLING...

→ HOW ASSET MARKET PRICES ARE DETERMINED.

i.e.,

WHAT SHOULD BE “E[r]” FOR ANY GIVEN ASSET?...

RECALL RELATION BETW “PV” AND “E[r]”.

e.g., for perpetuity:  $PV = CF / E[r]$

*(A model of price is a model of expected return,  
and vice versa, a model of expected return is a model of price.)*

THUS, ASSET PRICING MODEL CAN IDENTIFY “MISPRICED” ASSETS (ASSETS WHOSE “E[r]” IS ABOVE OR BELOW WHAT IT SHOULD BE, THAT IS, ASSETS WHOSE CURRENT “MVs” ARE “WRONG”, AND WILL PRESUMABLY TEND TO “GET CORRECTED” IN THE MKT OVER TIME).

IF PRICE (HENCE E[r]) OF ANY ASSET DIFFERS FROM WHAT THE MODEL PREDICTS, THE IMPLICATION IS THAT THE PRICE OF THAT ASSET WILL TEND TO REVERT TOWARD WHAT THE MODEL PREDICTS, THEREBY ALLOWING PREDICTION OF SUPER-NORMAL OR SUB-NORMAL RETURNS FOR SPECIFIC ASSETS, WITH OBVIOUS INVESTMENT POLICY IMPLICATIONS.

## ***Quick & simple example...***

Suppose model predicts  $E[r]$  for \$10 perpetuity asset should be 10%.

This means equilibrium price of this asset should be \$100.

But you find an asset like this whose price is \$83.

This means it is providing an  $E[r]$  of 12% ( =  $10 / 83$  ).

Thus, if model is correct, you should buy this asset for \$83.

Because at that price it is providing a “supernormal” return,

and because we would expect that as prices move toward equilibrium the value of this asset will move toward \$100 from its current \$83 price.

(i.e., You will get your supernormal return either by continuing to receive a 12% yield when the risk only warrants a 10% yield, or else by the asset price moving up in equilibrium providing a capital gain “pop”.)

THE "SHARPE-LINTNER **CAPM**" (*in 4 easy steps!*)...  
(*Nobel prize-winning stuff here – Show some respect!*)

**1<sup>ST</sup>) 2-FUND THEOREM** SUGGESTS THERE IS A SINGLE COMBINATION OF RISKY ASSETS THAT YOU SHOULD HOLD, NO MATTER WHAT YOUR RISK PREFERENCES.  
THUS, ANY INVESTORS WITH THE SAME EXPECTATIONS ABOUT ASSET RETURNS WILL WANT TO HOLD THE SAME RISKY PORTFOLIO (SAME COMBINATION OR RELATIVE WEIGHTS).

**2<sup>ND</sup>)** GIVEN **INFORMATIONAL EFFICIENCY** IN SECURITIES MARKET, IT IS UNLIKELY ANY ONE INVESTOR CAN HAVE BETTER INFORMATION THAN THE MARKET AS A WHOLE, SO IT IS UNLIKELY THAT YOUR OWN PRIVATE EXPECTATIONS CAN BE SUPERIOR TO EVERY ONE ELSE'S. THUS, EVERYONE WILL CONVERGE TO HAVING THE SAME EXPECTATIONS, LEADING EVERYONE TO WANT TO HOLD THE **SAME PORTFOLIO**. THAT PORTFOLIO WILL THEREFORE BE OBSERVABLE AS THE **"MARKET PORTFOLIO"**, THE COMBINATION OF ALL THE ASSETS IN THE MARKET, IN VALUE WEIGHTS PROPORTIONAL TO THEIR CURRENT CAPITALIZED VALUES IN THE MARKET.



3<sup>RD</sup>) SINCE EVERYBODY HOLDS THIS SAME PORTFOLIO, THE **ONLY RISK THAT MATTERS** TO INVESTORS, AND THEREFORE THE ONLY RISK THAT GETS REFLECTED IN EQUILIBRIUM MARKET PRICES, IS THE **COVARIANCE WITH THE MARKET PORTFOLIO**. *(Recall that the contribution of an asset to the risk of a portfolio is the covariance betw that asset & the portf.)* THIS COVARIANCE, NORMALIZED SO IT IS EXPRESSED PER UNIT OF VARIANCE IN THE MARKET PORTFOLIO, IS CALLED "**BETA**".

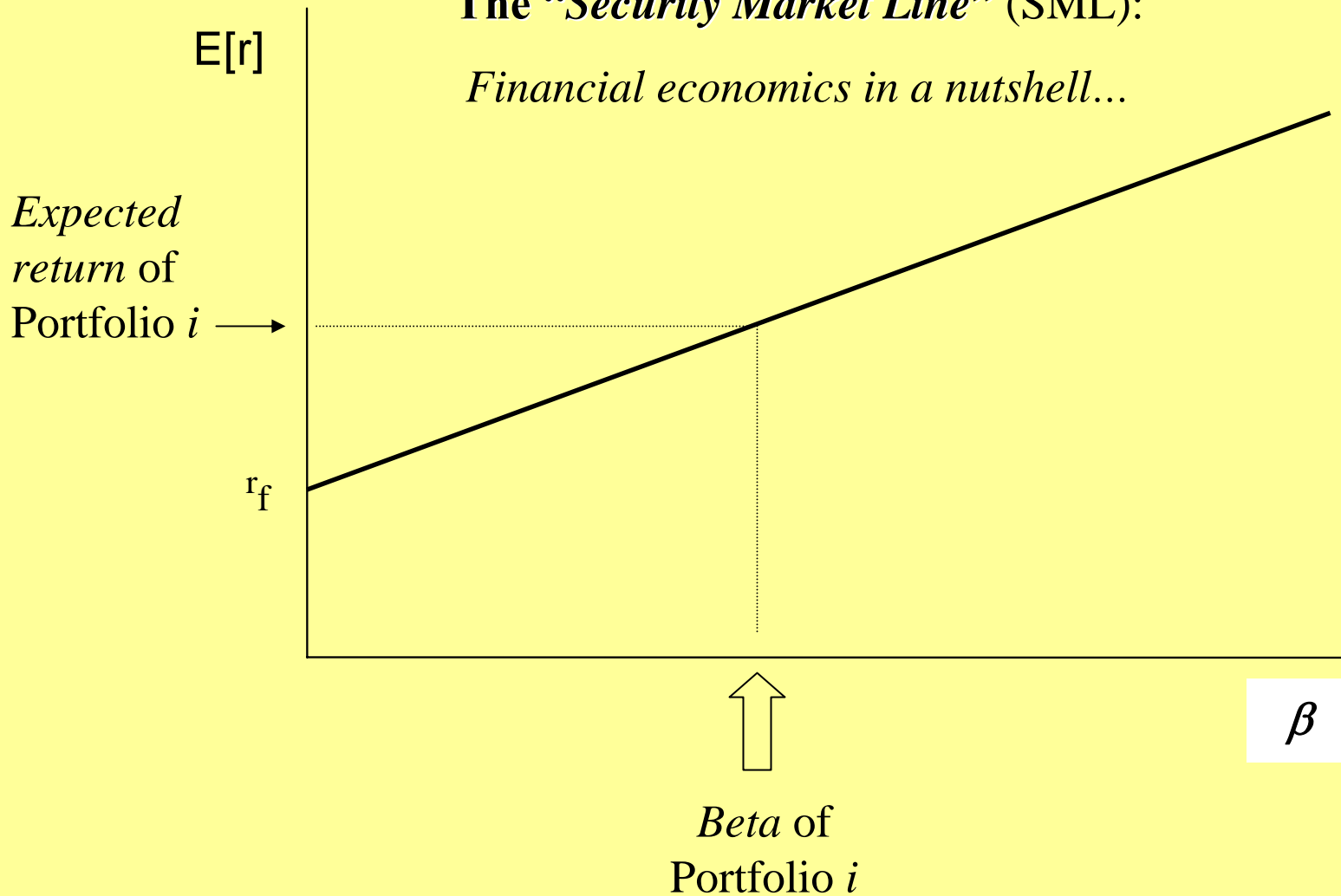
4<sup>TH</sup>) THEREFORE, IN EQUILIBRIUM, ASSETS WILL REQUIRE AN EXPECTED RETURN EQUAL TO THE RISKFREE RATE PLUS THE MARKET'S RISK PREMIUM TIMES THE ASSET'S BETA:

$$E[r_i] = r_f + RP_i = r_f + \beta_i(Er_M - r_f)$$

*Summary picture of the CAPM...*

**The “Security Market Line” (SML):**

*Financial economics in a nutshell...*



THE CAPM IS OBVIOUSLY A SIMPLIFICATION (of reality)...

*(Yes, I know that markets are not really perfectly efficient.*

*I know we don't all have the same expectations.*

*I know we do not all really hold the same portfolios.)*

BUT IT IS A POWERFUL AND WIDELY-USED MODEL. IT CAPTURES AN IMPORTANT PART OF THE ESSENCE OF REALITY ABOUT ASSET MARKET PRICING...

## 22.1.6 Strengths and Weaknesses in the Basic CAPM

### Strengths:

- Useful as normative (what *should be*) prescription (it makes sense).
- As positive (what *is*) description the classical (original) single-factor CAPM has some value (especially at broad-brush level, as we'll see later).
- Provides basic and elegant intuition that may at least partly explain why more complex models work better (e.g., maybe “Fama-French factors” proxy for types of systematic risk not quantified by *beta*).

### Weaknesses:

- Without “enhancements” (e.g., Fama-French factors), the basic single-factor CAPM is a pretty incomplete model of the expected returns of specific portfolios within an asset class.

## Section 22.2

### Applying the CAPM at the Asset Class Level

The basic single-factor CAPM does a pretty decent job of explaining the expected return to the real estate asset class as a whole, provided you:

- Correct the real estate returns for appraisal smoothing and lagging, and;
- Define the “market portfolio” to include all investible wealth, including real estate.

For the former purpose, you can accumulate the contemporaneous plus lagged covariances between the real estate index and the market portfolio. Or you can use “unsmoothed” or transactions-based real estate indexes.

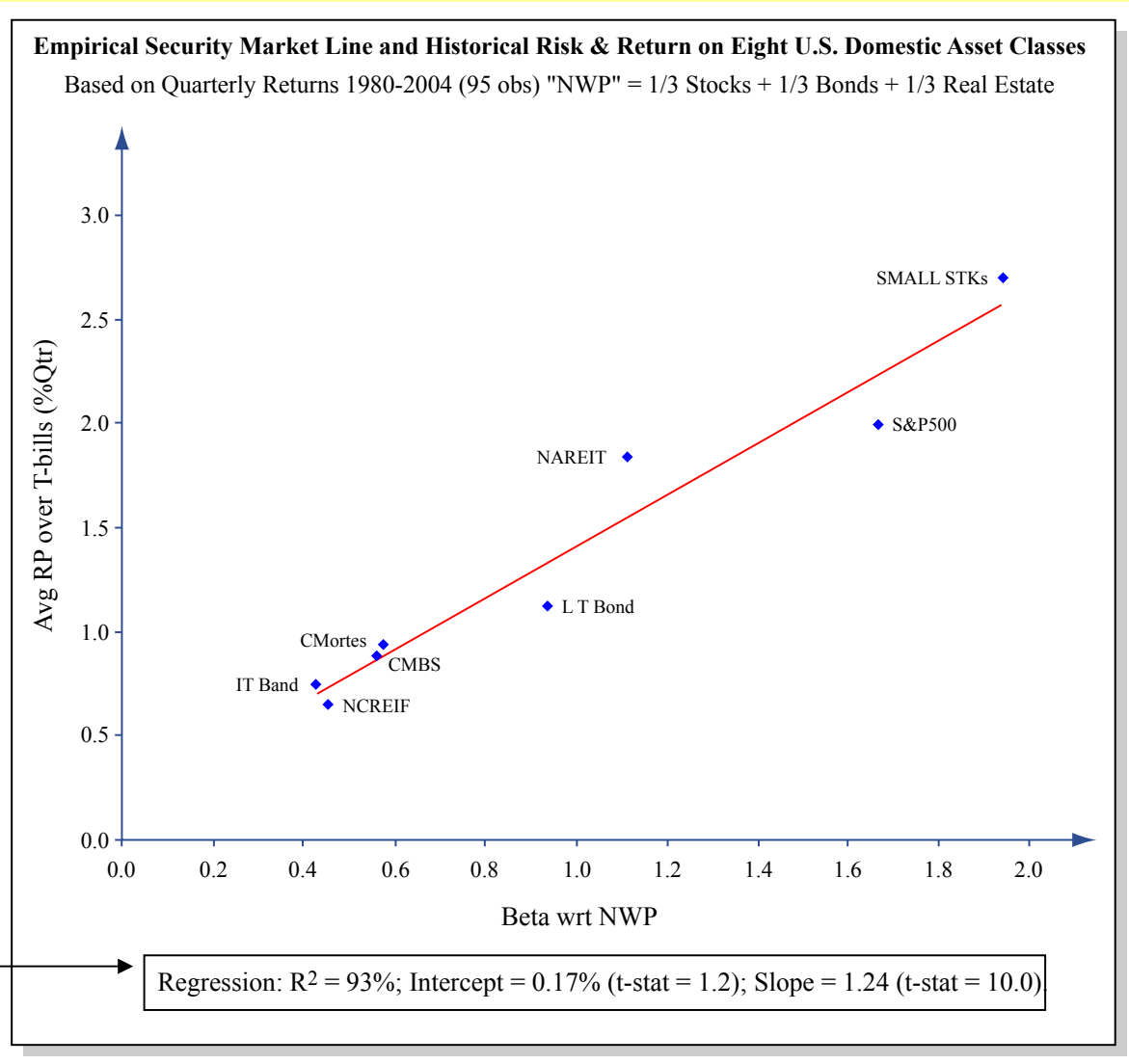
For the latter, you can define the market portfolio as a stylized “National Wealth Portfolio” consisting of one-third shares each of stocks, bonds, and real estate.

## 22.2.2

# Applying the Basic CAPM *ACROSS* asset classes

Correcting for smoothing, and defining *beta* wrt National Wealth...

“*CAPM works.*”



## 22.3.1

The simple 1-factor CAPM has trouble empirically within asset classes.

*Fama-French: CAPM by itself doesn't work very well within the stock market:*

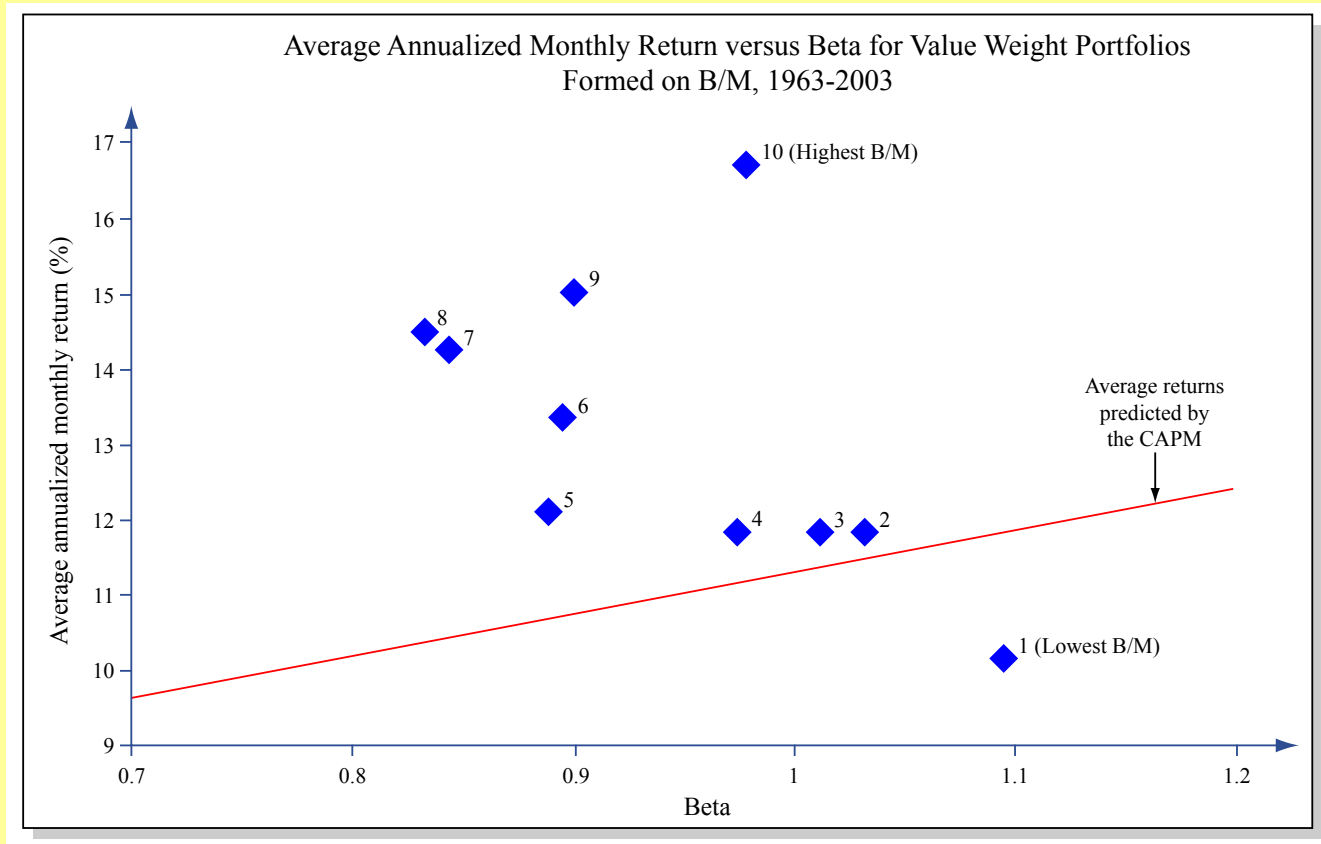


Figure by MIT OCW.

Enhance the basic model with additional factors that are more “tangible” than *beta*: (i) Stock’s Size (mkt cap), & (ii) Stock’s Book/Market Value Ratio. The market apparently associates these with “risk”.



Similarly, within the real estate asset class, *beta* does not explain the dispersion in long-run average total returns (nor does simple volatility).

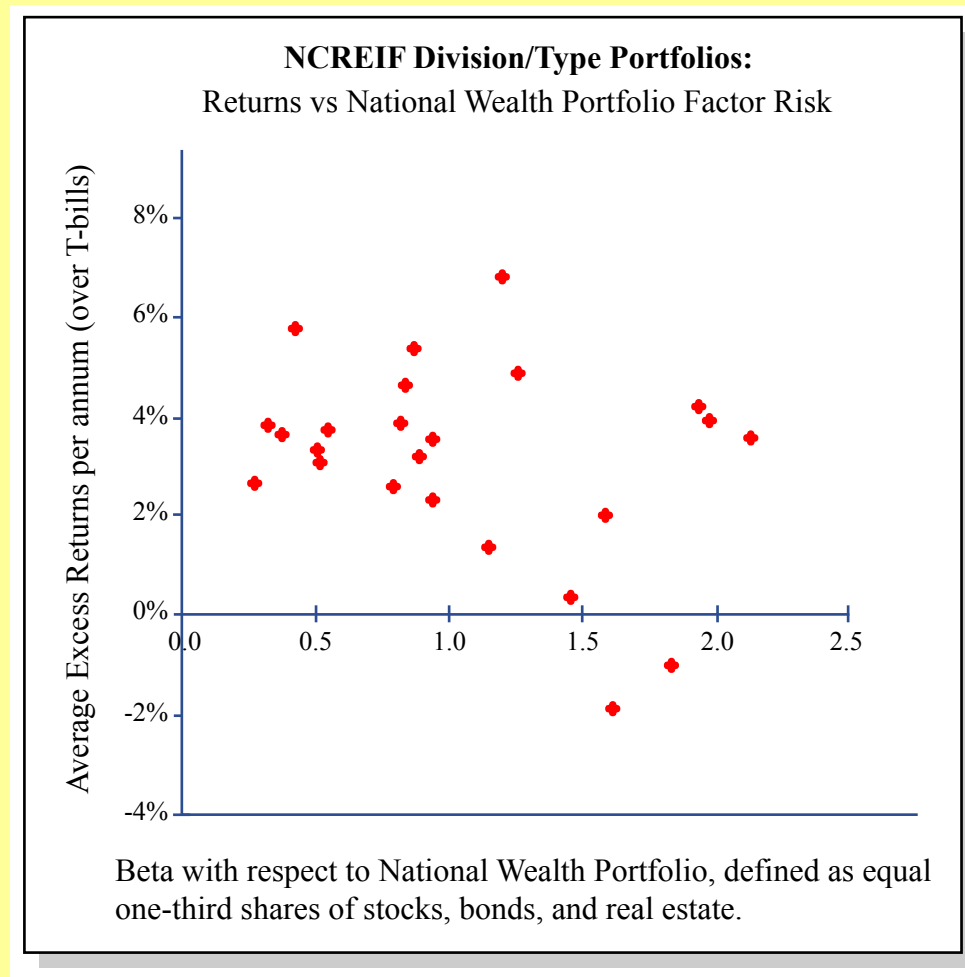
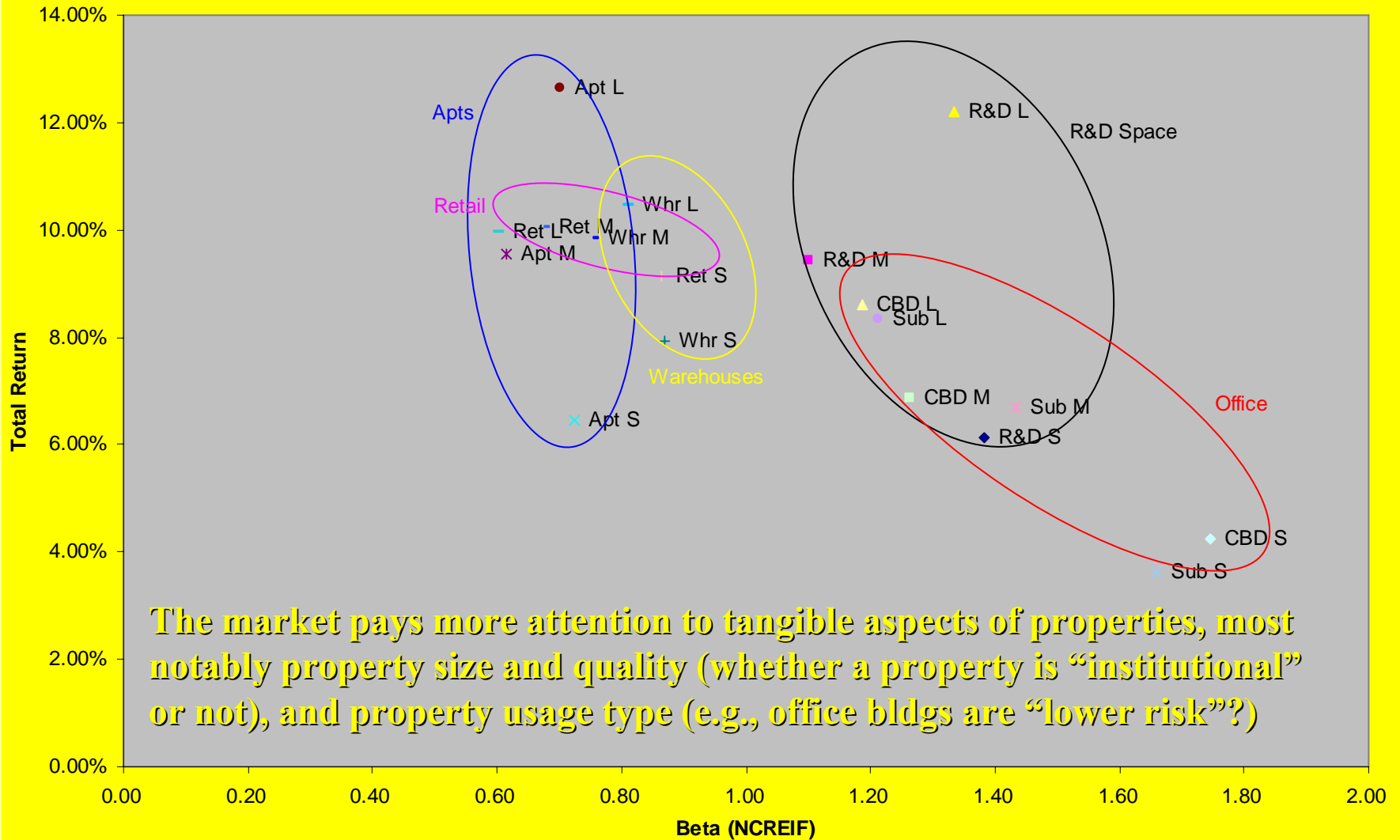


Figure by MIT OCW.

The market pays more attention to tangible aspects of properties, most notably property size and quality (whether a property is “institutional” or not), and property usage type (e.g., office bldgs are “lower risk”?)

# NCREIF portfolio avg returns and beta wrt NPI (1984-2003) by property size and type

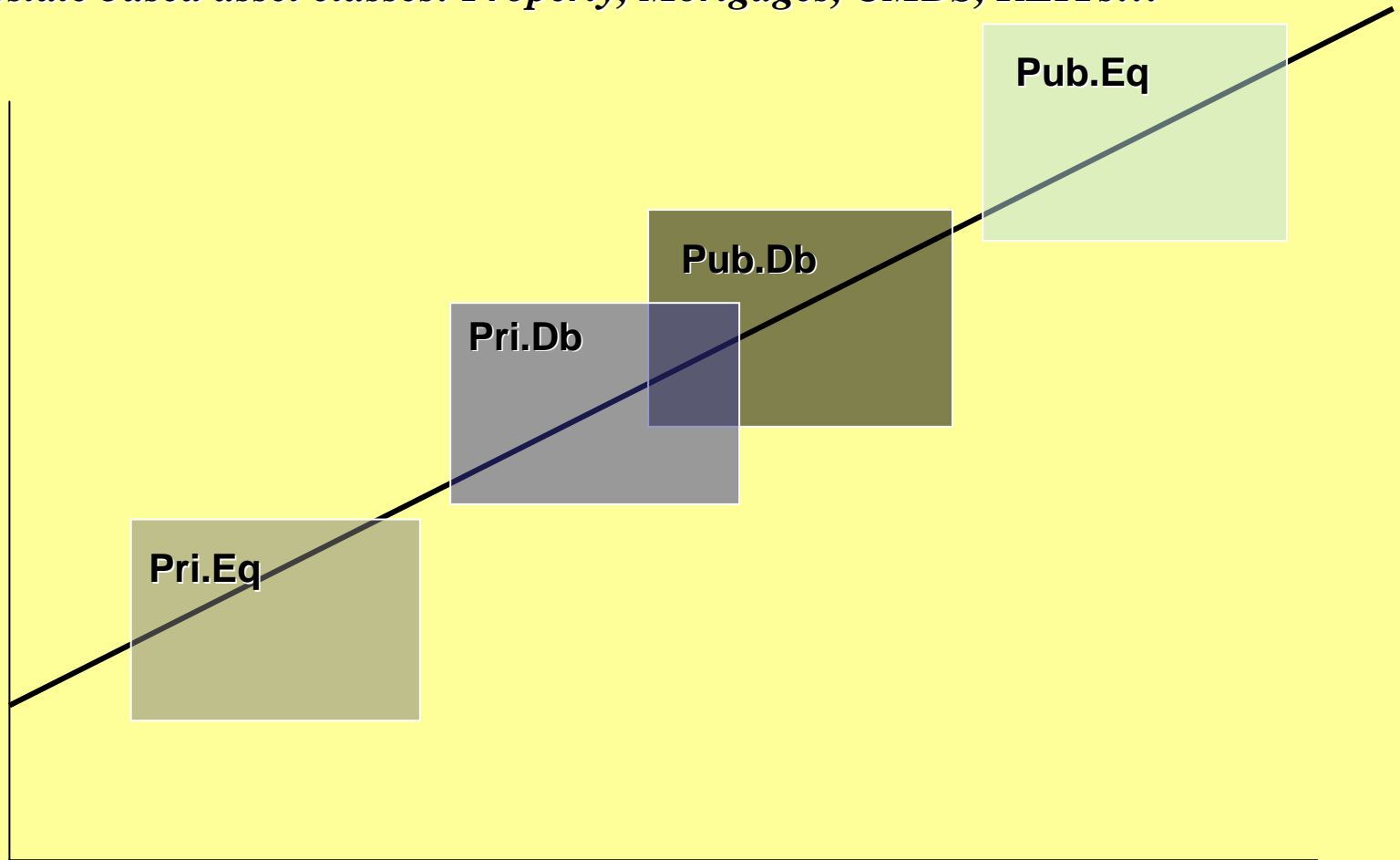
## Size - Total Return vs Beta (NCREIF)



The market pays more attention to tangible aspects of properties, most notably property size and quality (whether a property is “institutional” or not), and property usage type (e.g., office bldgs are “lower risk”?)

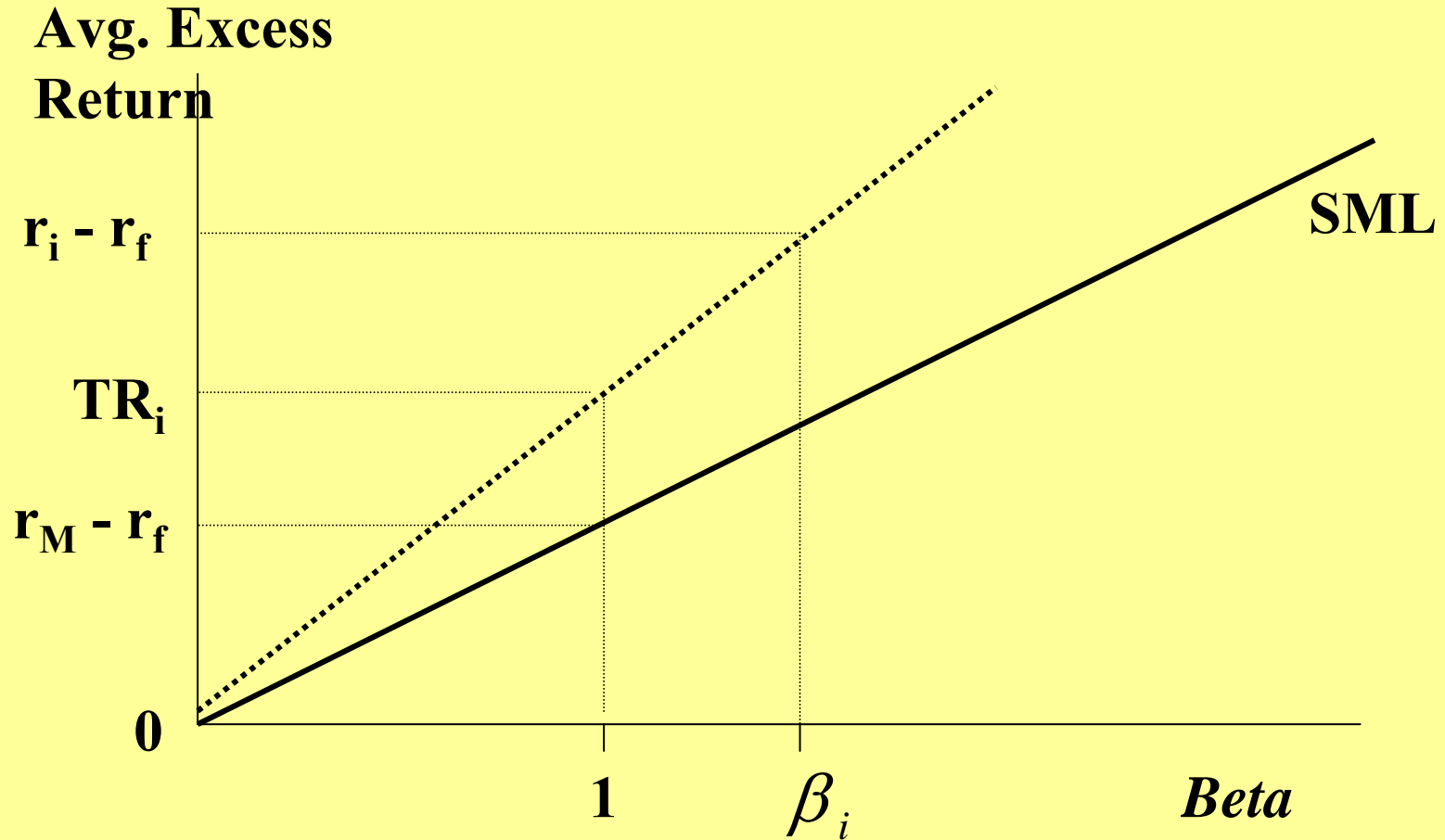
*The Capital Market does perceive (and price) risk differences*  
**ACROSS** asset classes . . .

*Real estate based asset classes: Property, Mortgages, CMBS, REITs...*



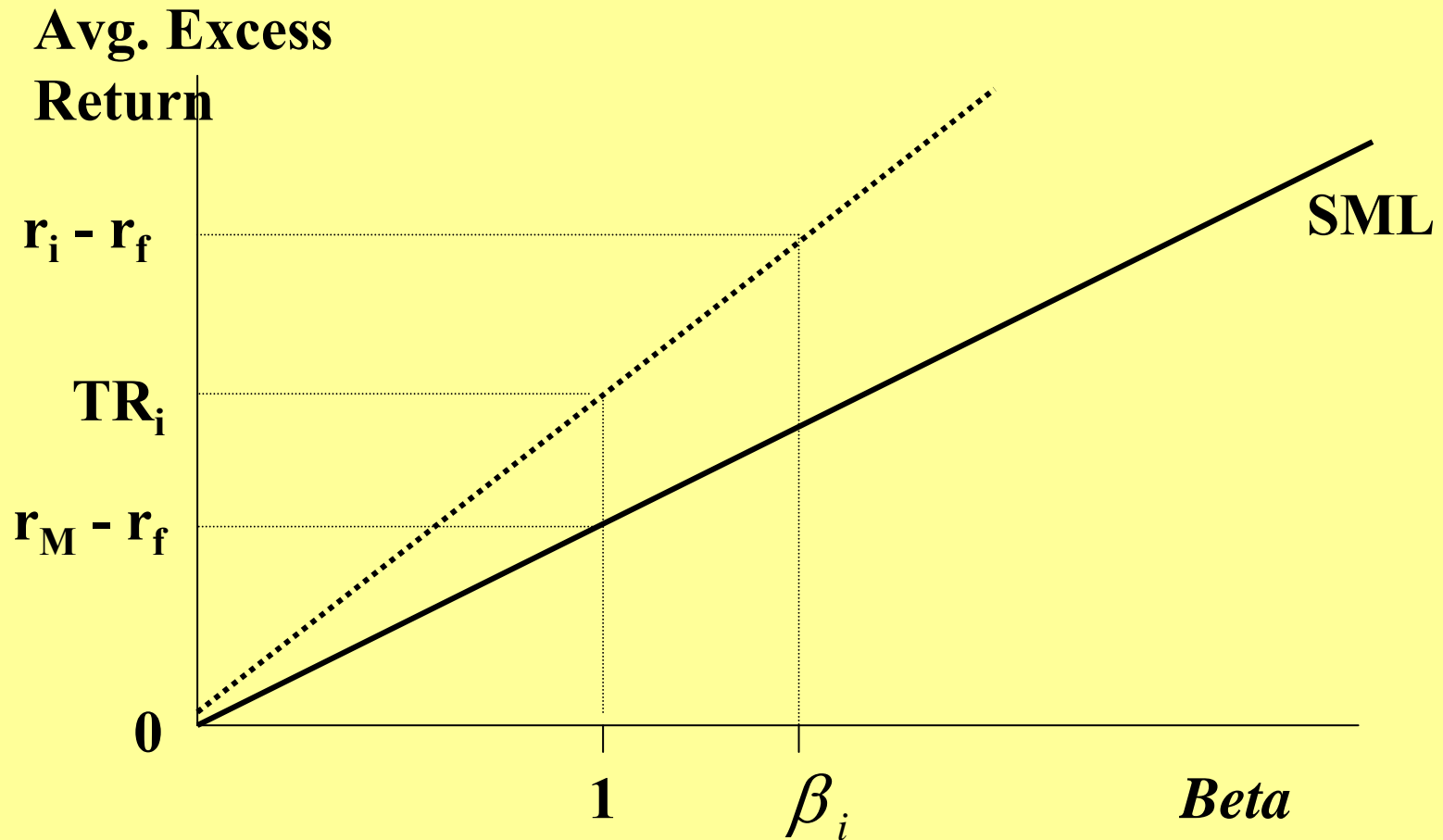
*National Wealth BETA*

*A CAPM-based method to adjust investment performance for risk: The Treynor Ratio...*

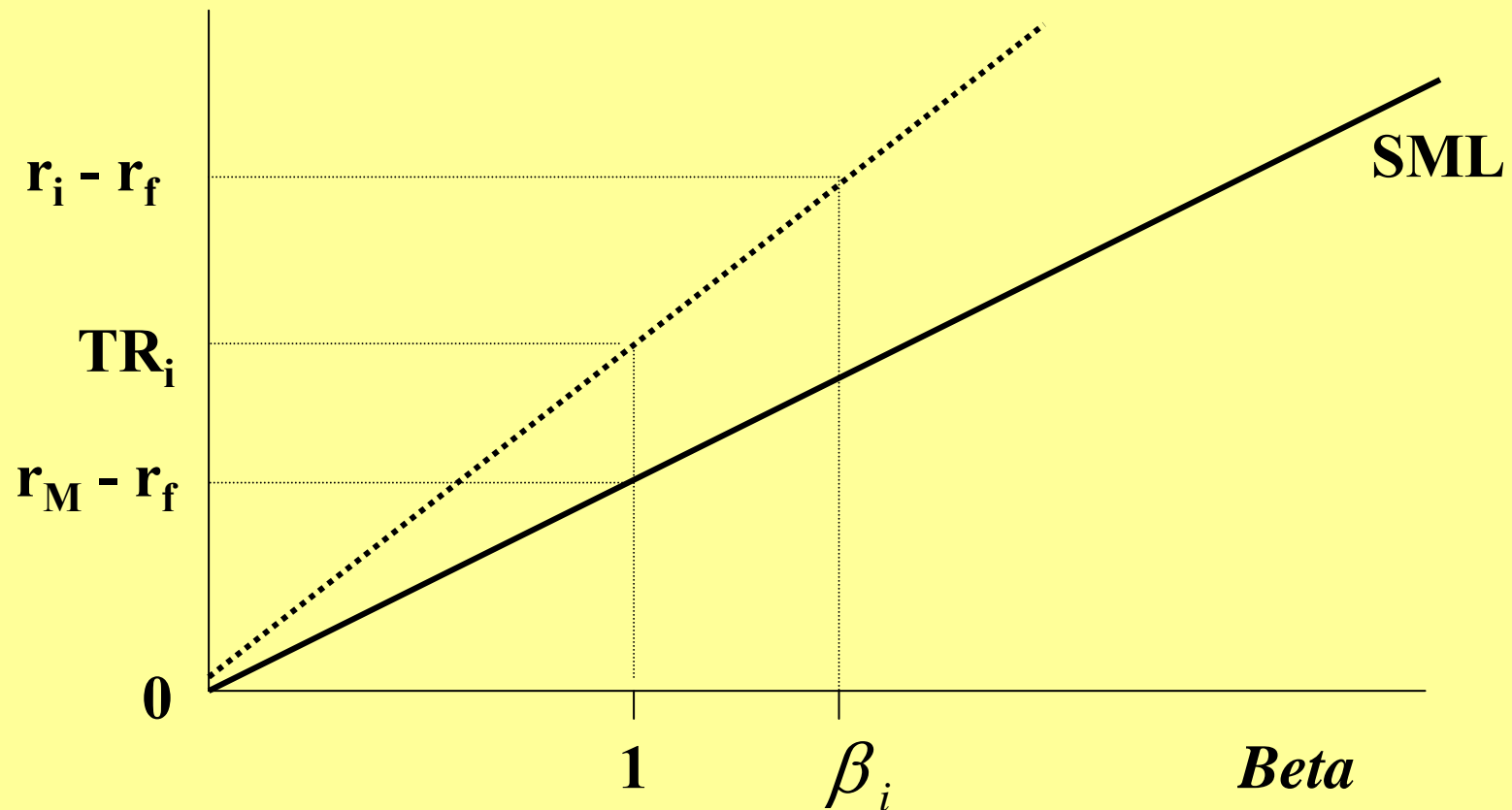


Based on "Risk Benchmark"

*The Treynor Ratio (or something like it) could perhaps be applied to managers (portfolios) spanning the major asset classes...*



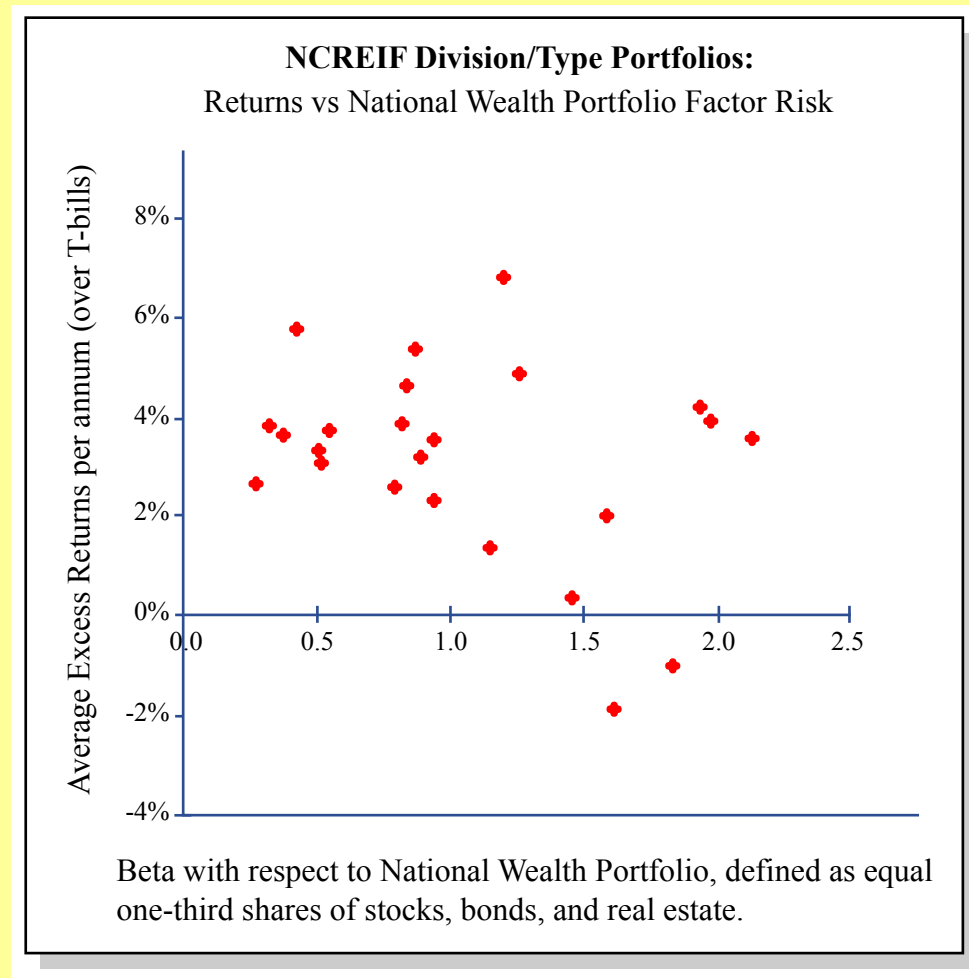
*The Beta can be estimated based on the “National Wealth Portfolio” ( = (1/3)Stocks + (1/3)Bonds + (1/3)RE ) as the mixed-asset “Risk Benchmark”...*



Based on “National Wealth Portfolio”

## 22.3.2: Go back to the *within the private real estate asset class* level of application of the CAPM...

Recall that we see little ability to systematically or rigorously distinguish between the risk and return expectations for different market segments *within* the asset class (e.g., Denver shopping ctrs vs Boston office bldgs):



**This holds implications for portfolio-level tactical investment policy:**

- **→ If all mkt segments effectively present the same investment risk, then those that present the highest expected returns automatically look like “good investments” (bargains) from a risk-adjusted ex ante return perspective.**

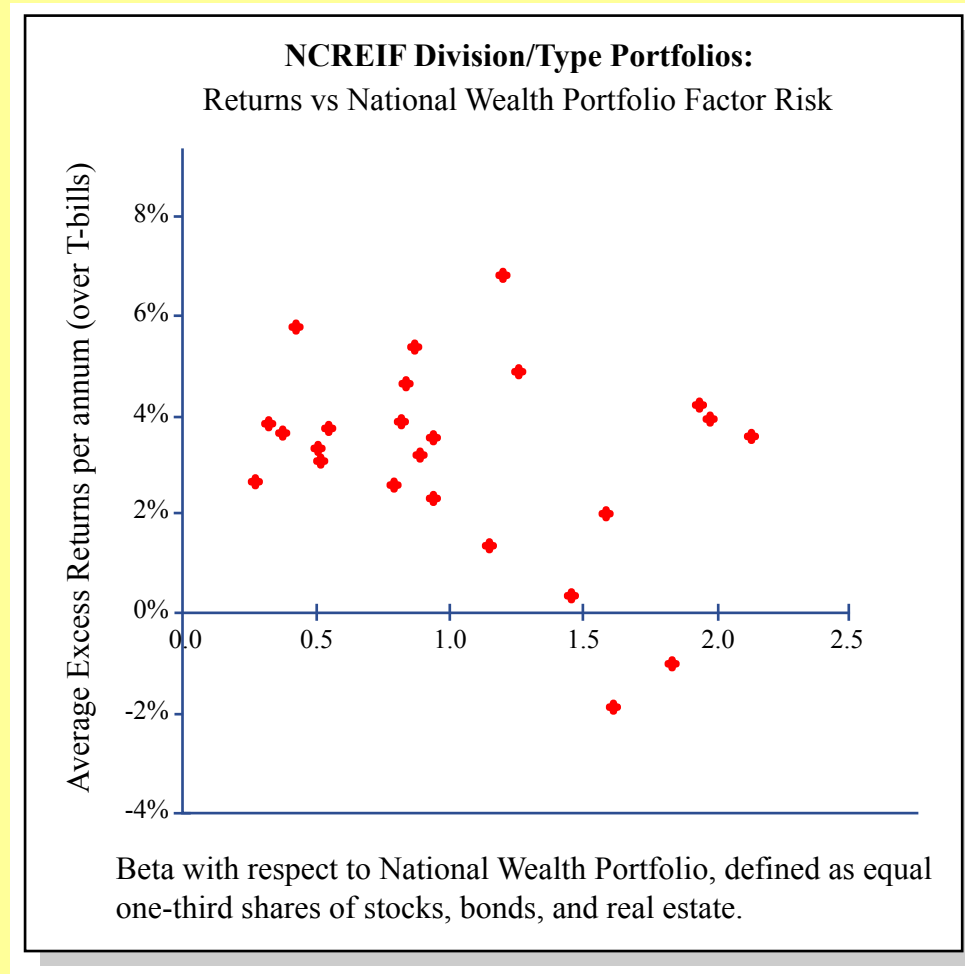


Figure by MIT OCW.

- **→ Search for markets where the combination of current asset yields (cap rates, “y”) and rental growth prospects (“g”) present higher expected total returns ( $E_r = y + E_g$ ).**



## Summarizing Chapter 22: *Equilibrium Asset Price Modelling & Real Estate*

- Like the MPT on which it is based, equilibrium asset price modeling (the CAPM in particular) has substantial relevance and applicability to real estate when applied at the broad-brush level ( *across asset classes* ).
- At the property level (unlevered), real estate in general tends to be a low-beta, low-return asset class in equilibrium, but certainly not riskless, requiring (and providing) some positive risk premium (ex ante).
- CAPM type models can provide some guidance regarding the relative pricing of real estate as compared to other asset classes (“*Should it currently be over-weighted or under-weighted?*”), and...
- CAPM-based risk-adjusted return measures (such as the Treynor Ratio) may provide a basis for helping to judge the performance of multi-asset-class investment managers (who can allocate across asset classes).\*
- Within the private real estate asset class, the CAPM is less effective at distinguishing between the relative levels of risk among real estate market segments, implying (within the state of current knowledge) a generally flat security market line.
- This holds implications for tactical portfolio investment research & policy within the private real estate asset class: → *Search for market segments with a combination of high asset yields and high rental growth opportunities: Such apparent “bargains” present favorable risk-adjusted ex ante returns.*