GENERIC STRUCTURES
IN OSCILLATING SYSTEMS I

Produced for the
MIT System Dynamics in Education Project

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Vensim Examples added in October 2001

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OSCILLATION

Celeste V. Chung[1]

INTRODUCTION

Bob, Ann, and Sam meet for dinner after a hard day at work. Bob, a high school physics teacher, tells them about his difficulty in explaining the pendulum problem to his class. Ann, a plant manager, does not understand his situation and has little interest in physics. Interrupting, she talks about her problems in regulating her inventory of widgets and her hiring/firing of factory workers.

Sam, a mutual friend, listens to both stories. On the surface, both problems seem to differ drastically. What similarities could there be between physics and manufacturing?

However, as Sam listened more closely, he began to notice similarities in the behavior of the two scenarios. Both seem to have fluctuating behavior. Sam recalled the system dynamics principles he learned in college. Forming a simple mental model, he pictured a two-level structure that could drive either example.

A two-level structure can display many modes of behavior. The following will focus on one specific mode of behavior− sustained oscillation.

A simple model demonstrating sustained oscillation contains two stocks, each influencing the flow to the other. The dependency of each flow on the opposite stock is such that the system never comes to rest.

[1] Celeste V. Chung is an undergraduate at the Massachusetts Institute of Technology, majoring in Management Science, June 1994. She is a participant of the System Dynamics in Education Project under Professor Jay W. Forrester.

Sam first explains the driving forces of oscillation in the pendulum model. He then transfers information from the pendulum model to the employment instability model.[2]

Through the pendulum and employment instability examples, you will learn about a simple two-level structure generating sustained oscillation. As you progress from the first example to the second, you may recognize a striking resemblance in the two models, with differences in only names and parameter values. The underlying structure causing the sustained oscillation remains the same for both models.

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[2] Note: This recurring systems principle, *transferable* or *generic structures*, is prevalent throughout Road Maps.

THE PENDULUM MODEL

Bob's high school physics class covers the following simple pendulum system, consisting of a mass attached to a rigid rod pivoting at the ceiling as shown in Figure #1. In a world with no external forces, such as friction, the force of gravity and tension are the sole forces acting upon the mass. If this pendulum system is initially in equilibrium[3] and no additional forces are exerted on it, the mass will stay in its equilibrium position forever.[4]
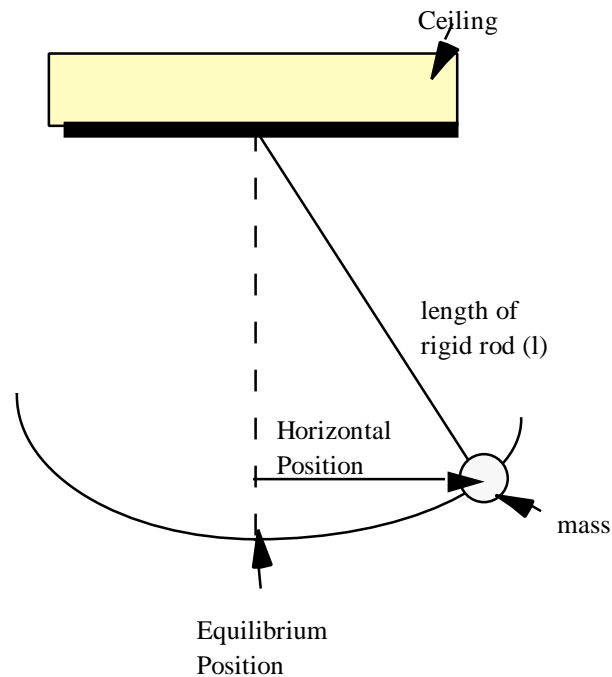

Figure #1: Mass and rigid rod pivoted at the ceiling

If the mass is pulled right or left, gravity provides a restoring force to drive the mass toward its equilibrium position. The tension force and the gravity, shown in Figure #2 on the next page, act on the mass to produce the motion. For this exercise, we will assume that the swing of the pendulum is small, around 10-20 cm for a 1 meter pendulum. For such small oscillations, we can assume the motion of the mass to be essentially horizontal. We will refer to the horizontal position of the mass with the variable X.

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[3] Equilibrium is a state of balance between opposing forces or actions.
[4] This is Newton's First Law — *A body at rest remains at rest and a body in motion remains in motion unless acted upon by an external force.*

Assuming that the equilibrium position is defined as X = 0, Figure #2 (a) shows the mass in a positive position. Gravity, the restoring force, impels the mass toward its equilibrium position. The mass overshoots to the negative position shown in Figure #2 (b). Here, the position is negative, and gravity again attempts to drive the mass back to the equilibrium position. The mass reaches equilibrium and overshoots to the positive position. Without any external forces such as friction, this back and forth or oscillatory motion would continue forever. For a more in-depth explanation, see Appendix 1.
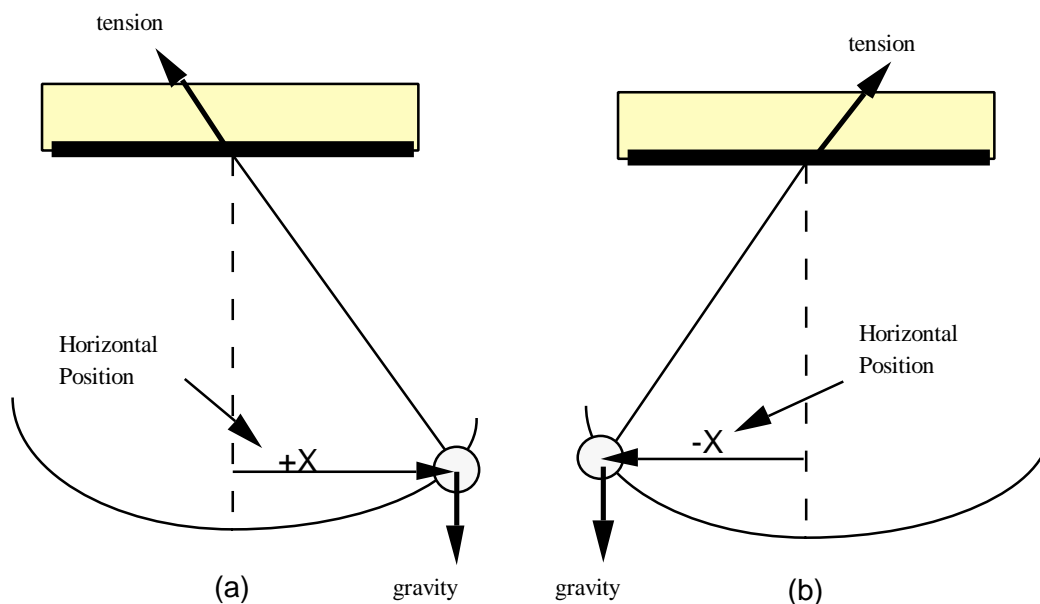

Figure #2: (a) Mass has positive position (displacement); (b) Mass has negative position

Sam's two level mental model of this simple pendulum system is modeled with the stock and flow diagram in Figure #3 on the following page. The **Position** stock represents the position mentioned earlier, with zero representing the **Desired Position**. Positive and negative positions represent the mass's horizontal displacement right and left of the vertical equilibrium line, respectively. The **Change in Position** is equal to the **Velocity**.
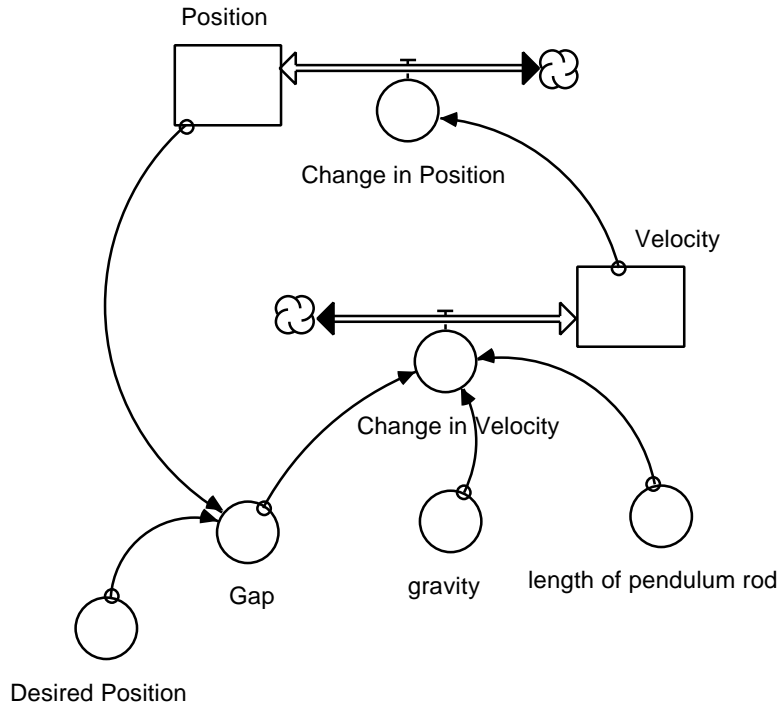
Position

Change in Position

Velocity

Change in Velocity

Gap   gravity   length of pendulum rod

Desired Position

Figure #3: Pendulum Model

The **Gap** is the difference between the Desired Position and the current Position. The **Gap**, along with the influence of **gravity** and the **length of pendulum rod**, determines the **Change in Velocity**. The effect of gravity and the length of the rod can be explained by using a bit of physics.

The **Change in Velocity** is:

$$\text{Change in Velocity} \quad = \left(\frac{\textbf{gravity}}{\textbf{length of pendulum rod}}\right) \textbf{Gap}^{5}$$

The **Change in Velocity** is the acceleration measured in meters per second per second. The sustained oscillatory behavior shown in Figure #4 is driven by the Change in Position's dependence on Velocity and the Change in Velocity's dependence on Position.

A derivation of the above equation is included in Appendix 1, but those of you who are unfamiliar with physics can still mentally justify the equation with a bit of intuition:

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[5]For those more interested in the physics aspect, please note that this is a simplified relationship for the purpose of this model.

Picture a pendulum with a meter long rod with a weight attached to the end. What happens if you pull the weight 20 cm to the right? The weight will swing back and forth slowly until it stops due to friction. If there were no friction, it would never stop. Picture that same pendulum but with only a 6 inch rod. What happens now if you pull the weight to the right? The weight still swings back and forth, but more quickly.

Picture the same pendulum on the moon. In the low gravity environment, the pendulum would swing much slower. If it were on the sun, the increased gravity would cause it to swing faster. You should be able to see that all of this information is summed up in the equation for **Change in Velocity.**
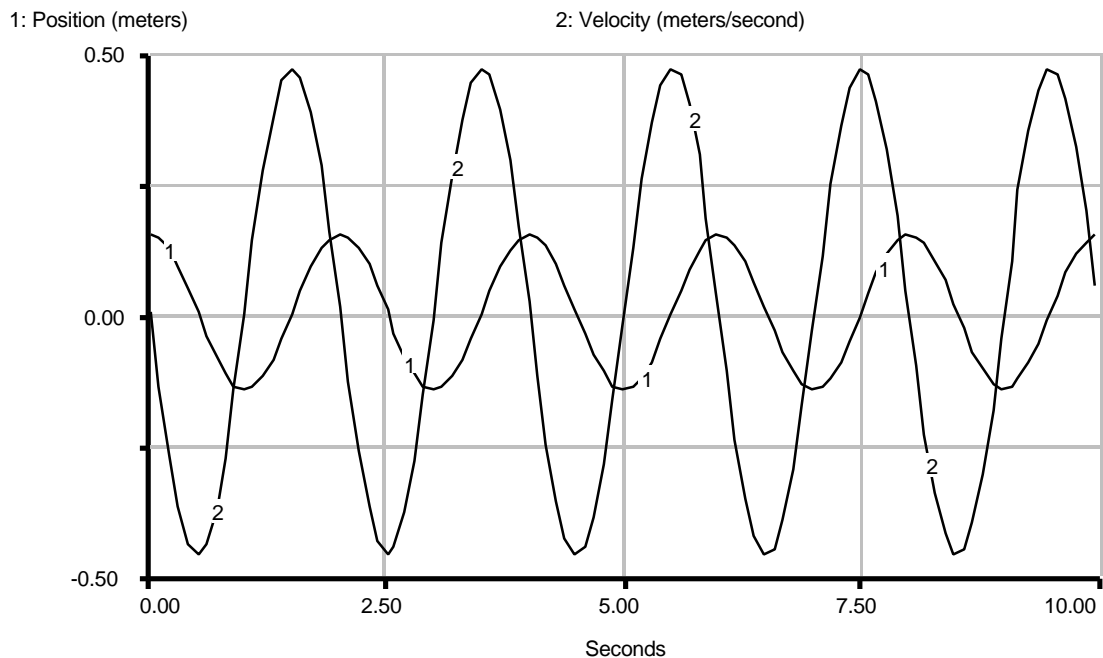

1: Position (meters)   2: Velocity (meters/second)

Figure #4: Sustained Oscillation of Position and Velocity

The two-level Pendulum Model is such that the largest Change in Position occurs each cycle at exactly the point where the Change in Velocity equals zero. When one stock is ready to come to rest, the other stock is moving at its fastest.[6] This is the underlying force driving the sustained oscillation.

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[6] <u>STELLA II Manual.</u> High Performance Systems, Inc. 45 Lyme Road. Hanover, NH. 03755. 1992. p. 99.

TRANSFERABLE STRUCTURES

The dinner is ending, and Bob, Ann, and Sam order coffee. At this point, Ann nods her head. She understands the physics and the underlying structure of the oscillating pendulum. However, she does not understand how a pendulum can have the same structure as her employee instability problem.

Sam explains the idea of transferable structures. Changing the parameter names and values transforms the Pendulum Model into the Employment Instability model as shown in Figure #5. The oscillatory behavior in the Employment Instability model takes months to complete each cycle instead of seconds. Thus, the parameter values must change to realistically represent the system.
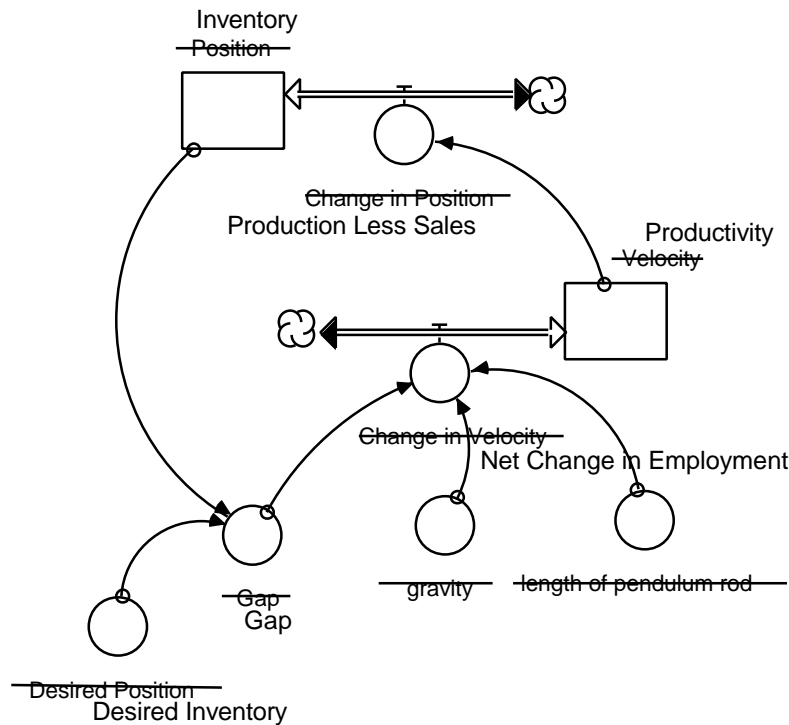

Figure #5: Changing the Parameter Names

Note that the Employment Instability model explained further in the next section has a few different converters not shown in this figure. Although the two models are not identical, the underlying structure is the same. The simplified generic structure which underlies both models is shown in Figure #6. Many

converters can be used to model each specific situation or story but the structure will still remain unchanged.
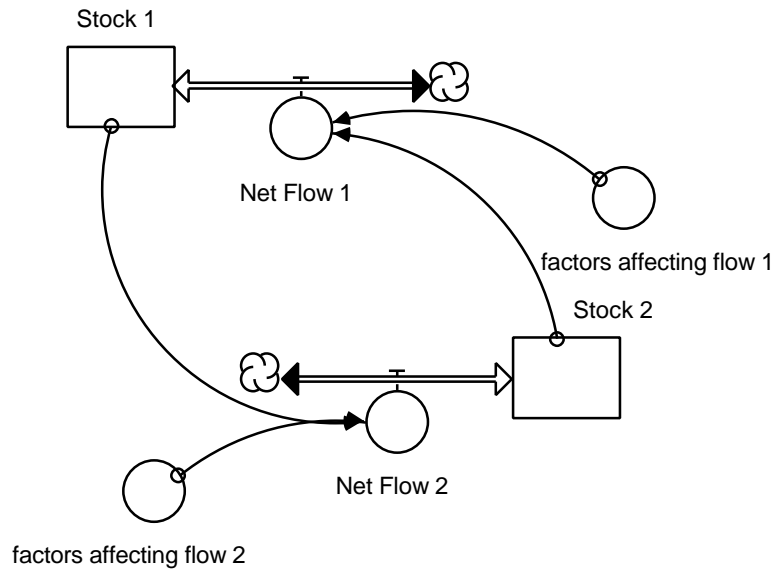

Figure #6: Generic Structure for Sustained Oscillation

Ann quickly protests. What could physics and manufacturing possibly have in common? Sam quickly reassures her that her particular problem is different, and thus the parameter names and values are specific to her situation. However, the underlying nature of the two situations is the same and thus the model structure is unchanged. By transferring what she learned from exploring the Pendulum Model, she can easily understand the driving forces of her system.

EMPLOYMENT INSTABILITY MODEL

Figure #7 shows the same underlying two-level structure as the Pendulum model. The parameter names were changed in Figure #5 from the Pendulum model to the Employment Instability model in Figure #7.
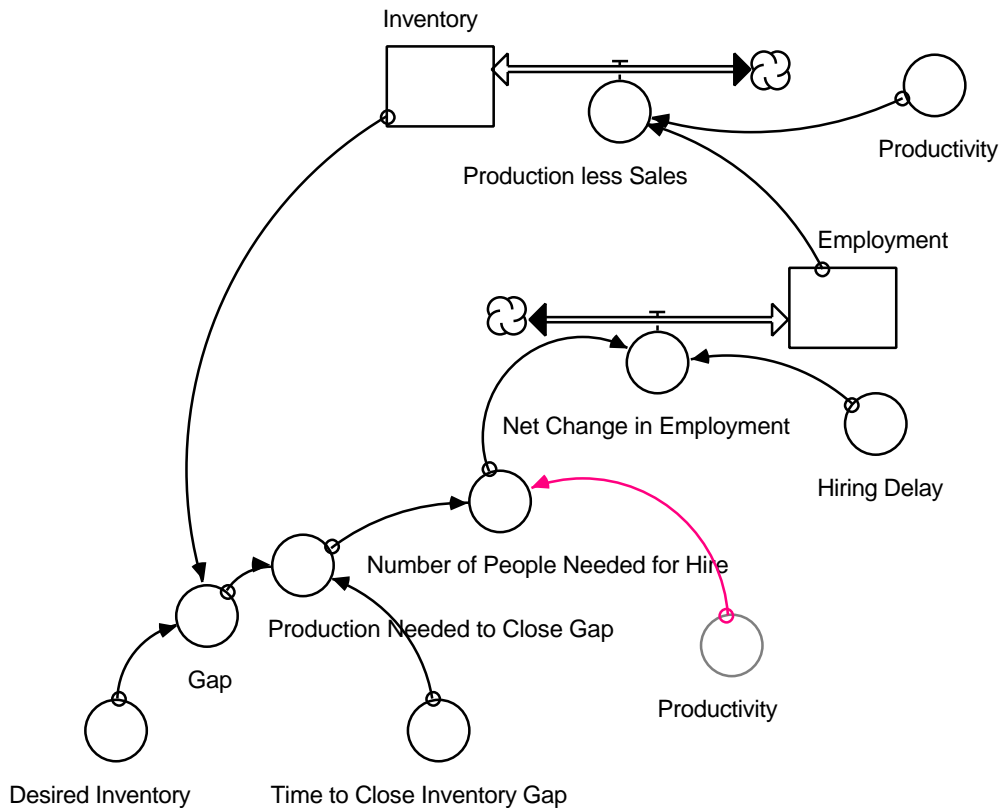

Figure #7: Employment Instability Model

What is the driving force behind Ann's Employment Instability problem? Let's first look at the **Inventory**. The Inventory is regulated by **Production less Sales**. **Sales** is constant at 20,000 Widgets per year. Each employee's **Productivity** is 100 Widgets per year, the number each person can make per year. Thus the **Production** is equal to **Productivity** * **Employment**, the number of employees. Production less Sales, which is the change in Inventory per year is

Production less Sales   =   Production   -   Sales
Production less Sales = (Productivity • Employment) - 20,000

Net Change in Employment is the net effect of all hiring and firing in the firm. The **Gap** is the discrepancy between **Desired Inventory** and I**nventory**. The **Time to Close Inventory Gap** is a parameter determined by Ann's firm. The **Time to Close Inventory Gap** and the **Gap** determine the **Production Needed to Close Gap**.

$$\textbf{Production Needed to Close Gap} \quad = \left[\frac{\textbf{Gap}}{\textbf{Time to Close Inventory Gap}} \right]$$

If the urgency to ship widgets is high, the time to close the inventory gap is low, and the production needed to close the gap is high. If there is a lot of time to close inventory gap, the production does not need to be as high.

The Production Needed to Close Gap and the Productivity determine the **Number of People Needed for Hire**. Since we know how many widgets we need to produce in a certain amount of time, we can determine how many people we need to hire by knowing the productivity.

$$\textbf{Number of People Needed for Hire} \quad = \frac{\textbf{Production Needed to Close Gap}}{\textbf{Productivity}}$$

Hence, the equation for **Net Change in Employment** is:

$$\textbf{Net Change in Employment} = \frac{\underline{\textbf{Number of People Needed for Hire}}}{\textbf{Hiring Delay}}$$

The **Hiring Delay** is the time it takes to hire and train new employees. In this case, it is 3 months or 1/4 years. This model produces the oscillatory behavior shown in Figure #8.
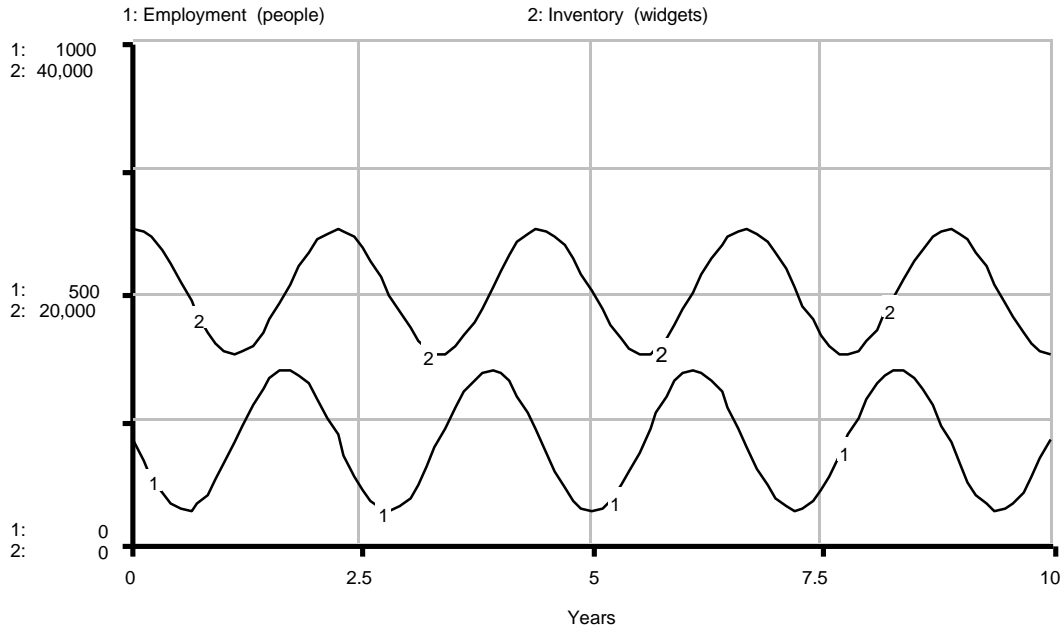
1: Employment (people)   2: Inventory (widgets)

Figure #8: Sustained Oscillation of Inventory and Employment
NOTE: Inventory and Employment are on different scales.

Note the similarity between this graph and the Pendulum Model graph. The shape of the behavior looks the same. However, the time scale differs greatly. The Employment Instability Model goes through one cycle in about two years while the Pendulum Model goes through one in about 2 seconds. The magnitudes of the stocks are also different.

CONCLUSION

Both Bob and Ann were stunned by Sam's explanation. While Ann's problems as a plant manager and Bob's difficulty as a physics teacher seemed completely different, they found that they had a lot in common.

They learned that this two-level generic structure drove the oscillating behavior in both of their systems. Sam taught them that by first understanding the underlying structure of this two-level model, both of them were able to transfer the knowledge from their own situation to the other. They utilized the system dynamics principle of *transferable structures.*

Sam not only taught them about transferable structures. He also showed that sustained oscillation can be generated by a simple two-level structure. The two-level models are such that in each cycle the highest rate of change of one flow occurs at the moment when the opposite flow equals zero. This is the underlying force driving the sustained oscillation.

After this short lesson on sustained oscillation and transferable structures, Ann and Bob returned to their respective jobs the following day. Both were able to use the newly developed models to simulate classroom experiments and test management strategies quickly and accurately. System dynamics gave them insight into their problems so they could begin forming effective manufacturing policies and physics lesson plans for the future.

APPENDIX

Appendix 1: A Further Physics Explanation

Figure #9 describes the arc motion in detail. The mass is initially pulled to the position shown in Figure #9(a). T is the tension force, the force from the rigid rod's attachment to the ceiling, and g is gravity. These two forces act together to produce a net restoring force represented by the bold arrow.   As the mass moves, the position X changes and thus the direction of the restoring force also changes to Figure #9(b).
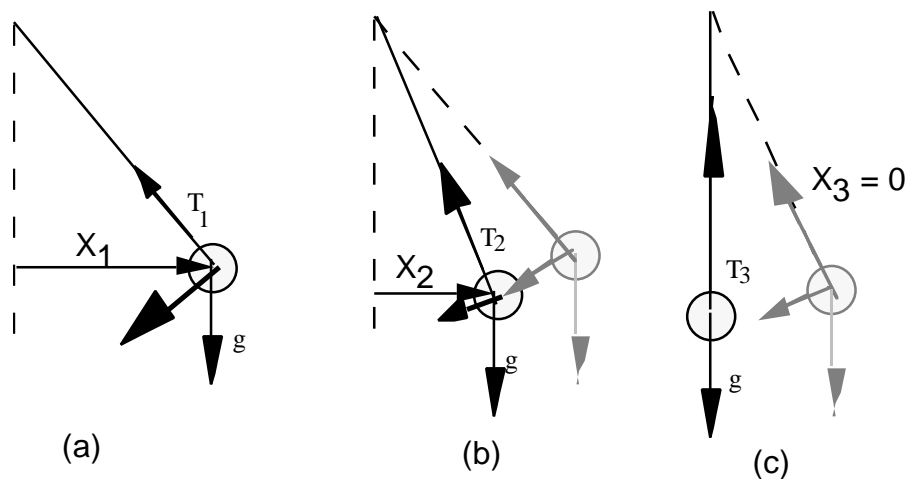

Figure #9: Mass with tension and gravity forces acting upon it.

Figure #9(b) shows the pendulum an instant later. The restoring force is more horizontal and smaller. This trend continues until the mass reaches the equilibrium position as shown in Figure #9(c). Here, the tension force and gravity exactly balance each other so the net restoring force is zero. However, the velocity has accumulated such that the momentum drives the mass horizontally and overshoots to the other side.

On the negative displacement side, the mass would similarly increase to its full negative position, as was previously shown in Figure #2(b). Again, gravity and tension work together to drive the mass back to the equilibrium position. The mass reaches equilibrium and overshoots to the positive position. Without any external forces, this back and forth or oscillatory motion would continue forever.
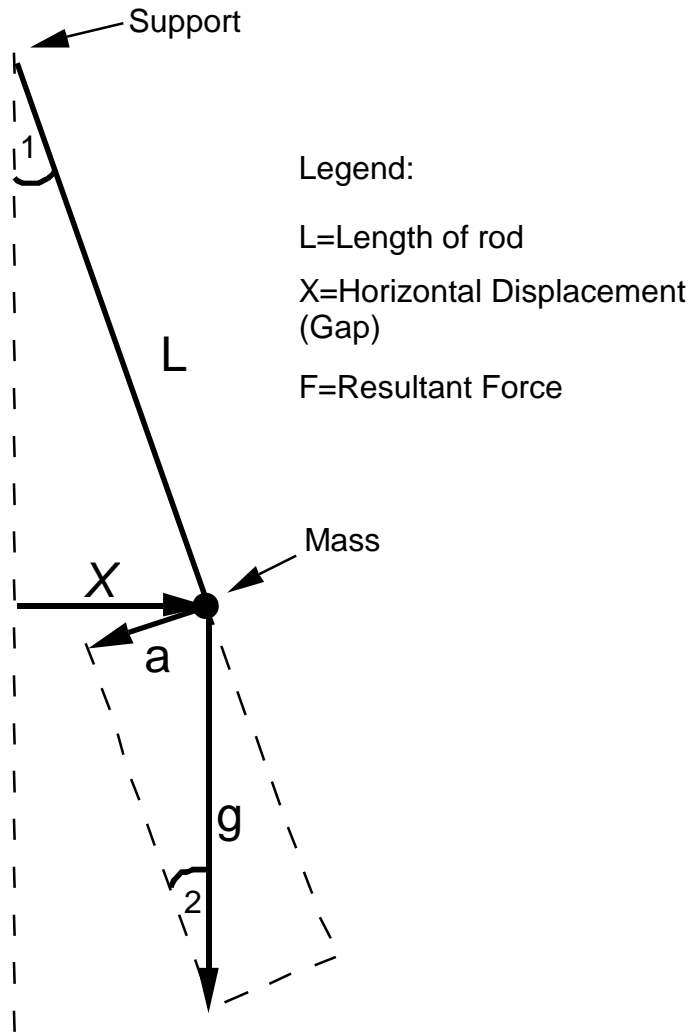
Derivation of motion equations:

Figure #10: Diagram of pendulum

Figure #10 shows a diagram of the pendulum. The gap *X* represents the horizontal displacement of the pendulum from the equilibrium position.   To simplify the physics of this system assume that the pendulum only has small oscillations. For our pendulum this means about 10-20 cm. For such small oscillations, the motion of the mass on the end of the pendulum is essentially horizontal.

The diagram in Figure #9 contains two triangles. One is formed by the pendulum's displacement *X* and its length L. The other triangle is formed by the acceleration of gravity *g* and the acceleration of the mass *a*. A bit of simple

trigonometry will show that angle 1 = angle 2; and the two triangles are similar. By similar triangles:

$$\frac{X}{L} ? \frac{a}{g} \tag{1}$$

This can be rearranged to:

$$a ? \frac{g}{L} ?X \tag{6}$$

In our model:

a = Change in Velocity

g = gravity

L = length of pendulum rod

X = Gap

So we get our equation for change in velocity:

Change in Velocity = (gravity/length of pendulum rod) * Gap

NOTE: Several simplifications are included in this derivation in the interest of making it understandable to someone with little or no physics experience. Those of you with more experience may wish to perform a more rigorous derivation on your own and compare the results with this simplified model.
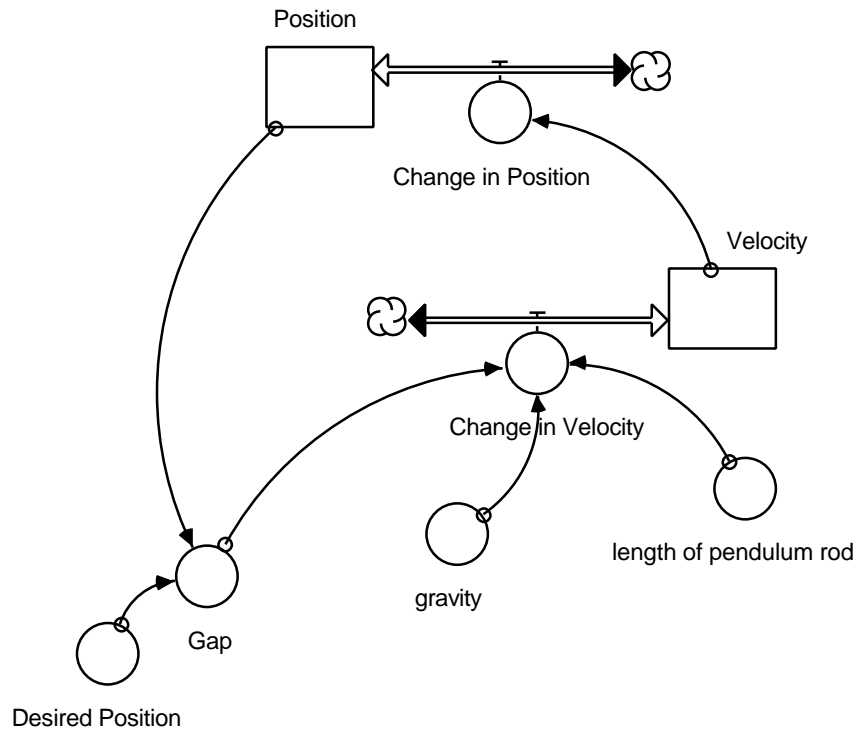
Appendix 2: The Pendulum Model

Figure #3: Pendulum Model

The Pendulum Model Equations

□   **Position**(t) = Position(t - dt) + (Change_in_Position) * dt
INIT Position = .15

DOCUMENT: The Position represents the horizontal displacement. Initially, the mass is displaced 15 centimeters.
UNITS: meters

INFLOWS:

○   **Change_in_Position** = Velocity
DOCUMENT: The Change in Position is the Velocity.
UNITS: meters/second

□   **Velocity**(t) = Velocity(t - dt) + (Change_in_Velocity) * dt
INIT Velocity = 0

DOCUMENT: UNITS: meters/second

INFLOWS:

○ **Change_in_Velocity** = (gravity/length_of_pendulum_rod)*Gap
DOCUMENT: UNITS: meters/second per second

○ **Desired_Position** = 0
DOCUMENT: This is the equilibrium position of the pendulum.
UNITS: meters

○ **Gap** = Desired_Position-Position
DOCUMENT: The gap is the difference in the Desired Equilibrium Position and
the current Position.
UNITS: meters

○ **gravity** = 9.8
DOCUMENT: UNITS: meters/second per second

○ **length_of_pendulum_rod** = 1
DOCUMENT: units: meters

Appendix 3: The Employment Instability Model

Inventory

Production less Sales

Productivity

Employment

Net Change in Employment

Hiring Delay

Number of People Needed for Hire

Production Needed to Close Gap

Gap

Productivity

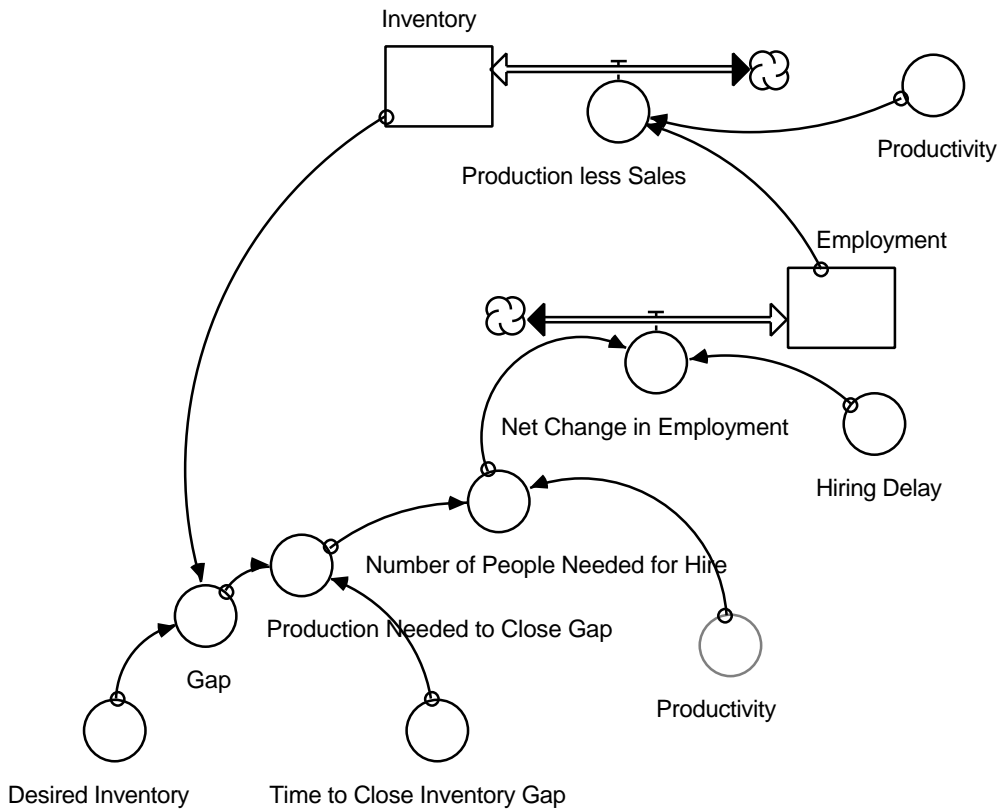Desired Inventory   Time to Close Inventory Gap

Figure #7: Employment Instability Model
The Employment Instability Model Equations

Employment = 200
UNITS: people

INFLOWS:

Net_Change_in_Employment =
            Number_of_People_Needed_for_Hire/Hiring_Delay
DOCUMENT: Hiring Less Firing is the number of people needed for hire
divided by the hiring delay.
UNITS: people / year

Inventory = 25000
DOCUMENT: Inventory is the amount of widgets on hand.
UNITS: widgets

INFLOWS:

○ **Production_less_Sales** = (Employment*Productivity) - 20000
DOCUMENT: Production less Sales is the change in inventory per year.
Each employee produces 100 widgets per year and 20,000 widgets are
sold.
UNITS: widgets/year

○ **Productivity** = 100
DOCUMENT: Productivity is the number of widgets each employee makes per
year.
UNITS: widgets/(person*year)

○ **Desired_Inventory** = 20,000
DOCUMENT: The desired inventory is 20,000 widgets, which is the yearly sales.
UNITS: widgets

○ **Gap** = Desired_Inventory-Inventory
DOCUMENT: The Gap is the difference in Desired Inventory and the current
Inventory.
UNITS: widgets

○ **Production_Needed_to_Close_Gap** =
                                      Gap/Time_to_Close_Inventory_Gap
DOCUMENT: The Production of Widgets needed to close the inventory gap is
the number of widgets needed divided by the time desired to close the inventory
gap.
UNITS: widgets/year

○ **Number_of_People_Needed_for_Hire** =
                          Production_Needed_to_Close_Gap/Productivity
DOCUMENT: The number of people needed for hire is the desired production
needed to close the inventory gap divided by the productivity, the number of
widgets each person can produce per year.
UNITS: people

○ **Time_to_Close_Inventory_Gap** = .5
DOCUMENT: Time to Close Inventory Gap is the desired time with which the
plant manager would like the inventory discrepancy to be fixed.
UNITS: years

○ **Hiring Delay** = 1/4
DOCUMENT: Time to adjust is the time required to hire and train new employees to work to full productivity. Here, the time is three months or 1/4 of the year.
UNITS: years

Vensim Examples:
Generating Structures in Oscillating Systems I
Lei Lei
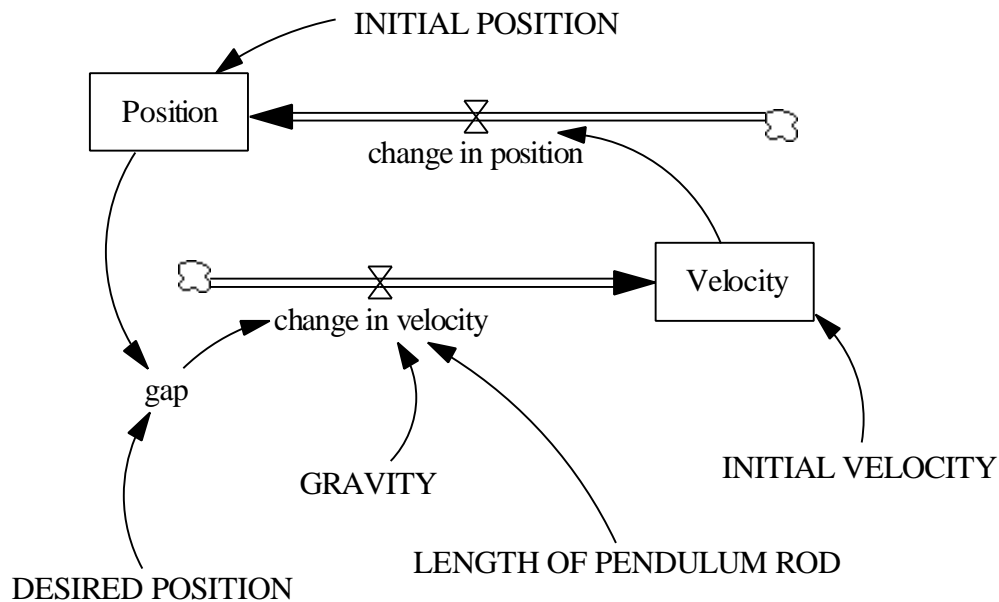Samitha Samaranayake
October 2001

The Pendulum Model

Figure 11: Vensim Equivalent of Figure 3: Pendulum Model

Documentation for Pendulum Model

(01)   change in position = Velocity
Units: meters/second
The Change in Position is the Velocity.

(02)   change in velocity = GRAVITY*gap/LENGTH OF PENDULUM ROD
Units: meters/(second*second)

(03)   DESIRED POSITION = 0
Units: meters
This is the equilibrium position of the pendulum.

(04)   FINAL TIME = 10
Units: second

The final time for the simulation.

(05)   gap = DESIRED POSITION-Position
        Units: meters
        The gap is the difference in the Desired Equilibrium Position and the current
        Position.

(06)   GRAVITY = 9.8
        Units: meters/(second*second)

(07)   INITIAL POSITION = 0.15
        Units: meters

(08)   INITIAL TIME = 0
        Units: second
        The initial time for the simulation.

(09)   INITIAL VELOCITY = 0
        Units: meters/second

(10)   LENGTH OF PENDULUM ROD = 1
        Units: meters

(11)   Position = INTEG (change in position, INITIAL POSITION)
        Units: meters
        The Position represents the horizontal displacement. Initially, the mass is
        displaced by initial position.

(12)   SAVEPER = 1
        Units: second
        The frequency with which output is stored.

(13)   TIME STEP = 0.03125
        Units: second
        The time step for the simulation.

(14)   Velocity = INTEG (change in velocity, INITIAL VELOCITY)
        Units: meters/second

As you would notice when you simultate the model, the graph for position shows
exapanding oscilations. This behaviour is contrary to what is expected. The Euler
integration method causes integration errors that result in these expanding oscilations.
The Runge-Kutta integration method can be used to solve this problem. The following
section explains how to detect such an error and correct it using Runga-Kutta integration.
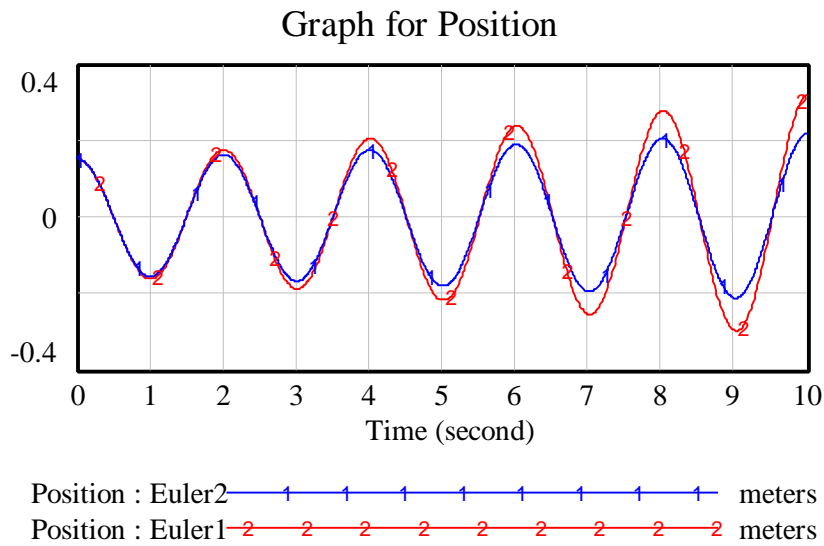
Graph for Position

Position : Euler2 ———†——†——†——†——†——†——†——†— meters
Position : Euler1 —2——2——2——2——2——2——2——2——2— meters

Figure 12: Behaviour of the system under different time steps with Eular integration

The time step used when simulating a model should be lowered until two successive time steps give the same behaviour. This shows that the time step used is small enough to eliminate integration errors. The two smallest time steps provided by Vensim are used in the above example. The graph still shows a difference in the behaviour of the model. Therefore, we can deduce that the integration is still faulty. Runga-Kutta integration can be used in such instances to get a accurate integration.
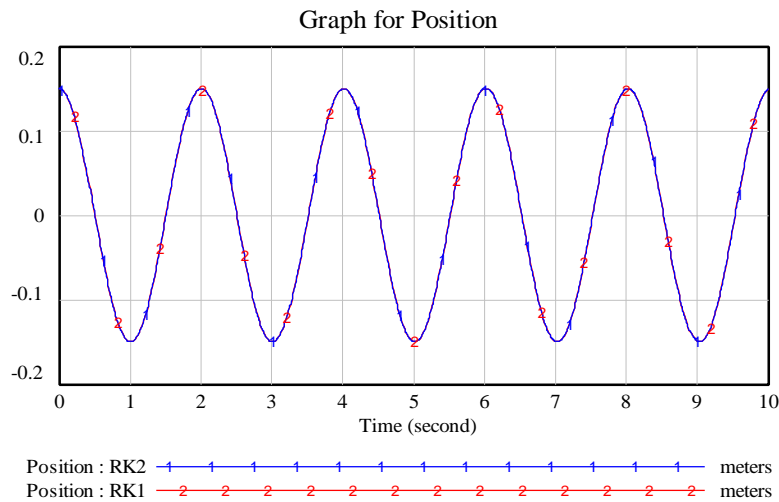
Graph for Position

Position : RK2 —†——†——†——†——†——†——†——†——†——†— meters
Position : RK1 —2——2——2——2——2——2——2——2——2——2— meters

Figure 13: Behaviour of the system under different time steps with Runga-Kutta integration

When using the Runga-Kutta integration method the behavious of the system does not change with the time step. Therefore, we can assume that there are no integration errors.

To change from Euler Intergration to Runga-Kutta Integration in Vensim, press the "Simulation Control" button, which is left of the "Set" button in the menu at the top of the main page. In th new window, under "Integration Technique" check the "RK4-Auto" option, then "Simulate" to exit.
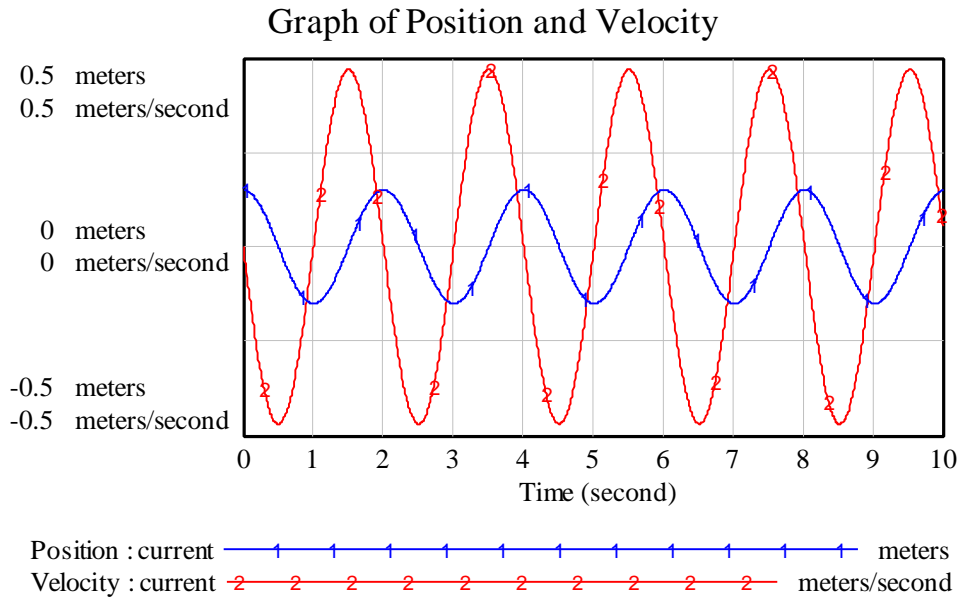

Figure 14: Vensim Equivalent of Simulation for Position and Velocity
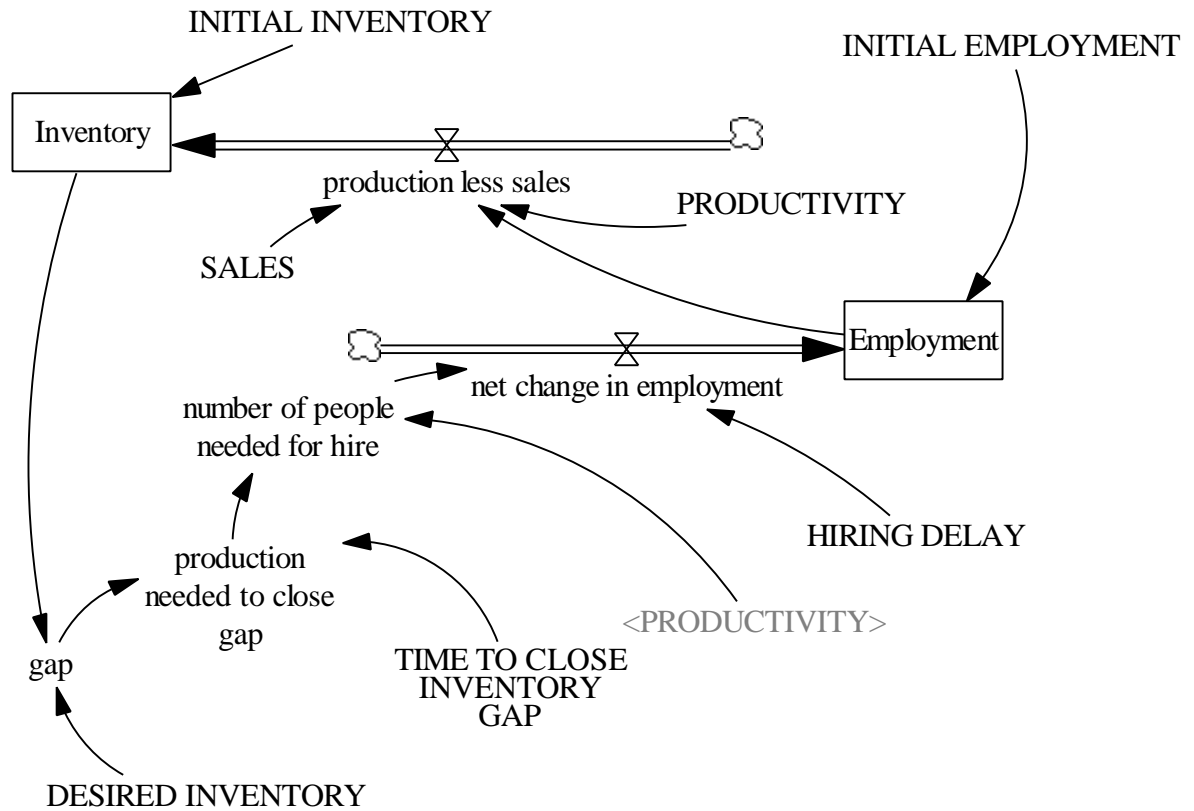
The Employment Instability Model

Figure 15: Vensim Equivalent of Figure 7: Employment Instability Model

Documentation for Employment Instability Model

(01)   DESIRED INVENTORY = 20000
        Units: widgets
        The desired inventory is 20,000 widgets, which is the desired sales.

(02)   Employment = INTEG (net change in employment, INITIAL EMPLOYMENT)
        Units: people

(03)   FINAL TIME = 10
        Units: year
        The final time for the simulation.

(04)   gap = DESIRED INVENTORY-Inventory
        Units: widgets
        The Gap is the difference in Desired Inventory and the current Inventory.

(05)   HIRING DELAY = 0.25
        Units: year
        Time to adjust is the time required to hire and train new employees to work to full
        productivity. Here, the time is three months or 1/4 of the year.

(06)   INITIAL EMPLOYMENT = 200
        Units: people

(07)   INITIAL INVENTORY = 25000
        Units: widgets

(08)   INITIAL TIME = 0
        Units: year
        The initial time for the simulation.

(09)   Inventory = INTEG (production less sales, INITIAL INVENTORY)
        Units: widgets
        Inventory is the number of widgets on hand.

(10)   net change in employment = number of people needed for hire/HIRING DELAY
        Units: people/year
        Hiring less firing is the number of people needed for hire divided by the hiring
        delay.

(11)   number of people needed for hire =
        production needed to close gap/ PRODUCTIVITY
        Units: people
        The number of people needed for hire is the desired production needed to close
        the inventory gap divided by the productivity, the number of widgets each person
        can produce per year.

(12)   production less sales = (Employment* PRODUCTIVITY)-SALES
        Units: widgets/year
        Production less Sales is the change in inventory per year. Each employee
        produces a number of widgets per year equal to the productivity and the number
        of widgets sold is equal to the sales.

(13)   production needed to close gap = gap/TIME TO CLOSE INVENTORY GAP
        Units: widgets/year
        The Production of Widgets needed to close the inventory gap is the number of
        widgets needed divided by the desired Time to Close the Inventory Gap.

(14)   PRODUCTIVITY = 100
        Units: widgets/(people* year)
        Productivity is the number of widgets each employee makes per year.

(15)   SALES = 20000
        Units: widgets/year

(16)   SAVEPER = TIME STEP
        Units: year
        The frequency with which output is stored.

(17)   TIME STEP = 0.0625
        Units: year
        The time step for the simulation.

(18)   TIME TO CLOSE INVENTORY GAP = 0.5
        Units: year
        Time to Close Inventory Gap is the desired time with which the plant manager
        would like the inventory discrepancy to be fixed.
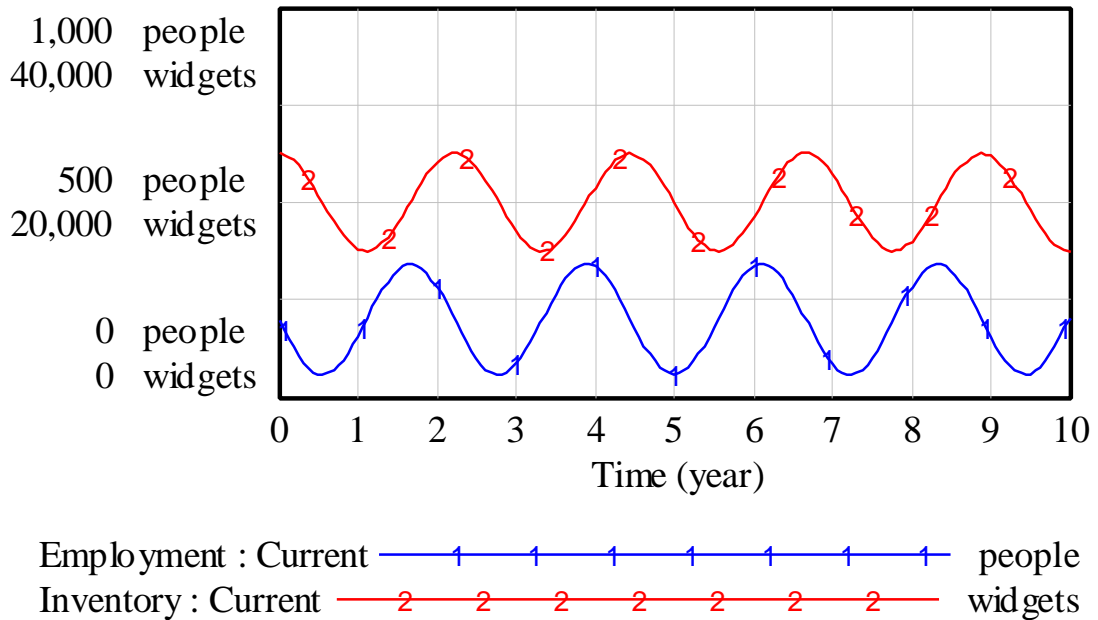
Graph of Inventory and Employment

Figure 16: Vensim Equivalent of Figure 8: Sustained Oscillation of Inventory and Employment