Guided Study Program in System Dynamics

System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #20
Saturday, April 17, 1999

Reading Assignment:

Please read the following paper:

- *Mistakes and Misunderstandings: DT Error, by Lucia Breierova (D-4695)*

Please read the following:

- Principles of Systems,[2] *by Jay W. Forrester, Section 2.3*

Exercises:

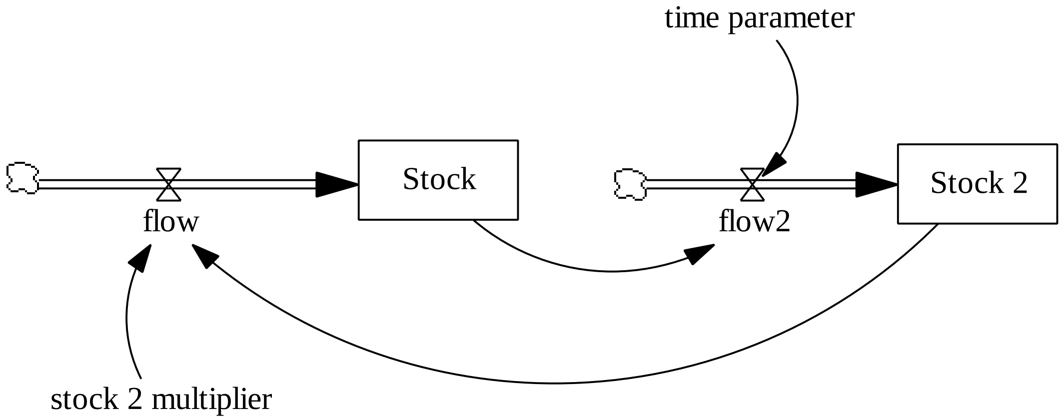
1. Mistakes and Misunderstandings: DT Error

Read this paper carefully. You do not have to answer any questions for this paper but if you can think of an instance when you made the same mistake, feel free to share the lesson gained with us.

2. Understanding Oscillatory Systems

This is the fourth in a series of exercises designed to help your understanding of oscillatory systems. Build the following model in Vensim PLE and then complete the exercises. Make sure that the time step is smaller than 0.0625 (or small enough that the value of DT no longer has a substantial effect on the result of the simulation). The time horizon over which you simulate the model should be at least 16 units of time.

[2] Forrester, Jay W., 1968. *Principles of Systems*, (2nd. ed.). Waltham, MA: Pegasus Communications. 391 pp.



*flow = Stock 2 * stock 2 multiplier*
Units: unit/Time

flow2 = Stock / time parameter
Units: unit/Time

Stock = INTEG (flow, -1)
Units: unit

Stock 2 = INTEG (flow2, 0)
Units: unit

stock 2 multiplier = -1
Units: 1/Time

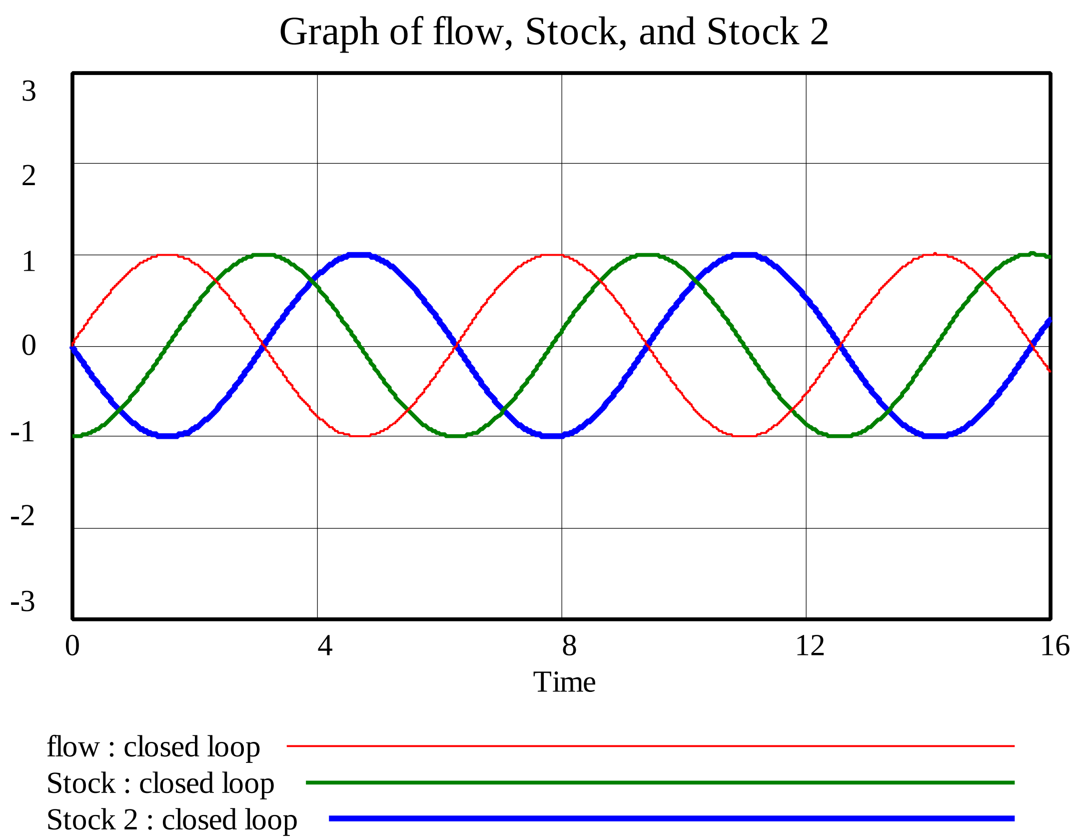
time parameter = 1
Units: Time

TIME STEP = 0.001
Units: Time

Notice that "flow2" is not an outflow from "Stock"; only the information about the value of "Stock" is used as the input to "flow2." Similarly, the information about the value of "Stock2" is used as the input to "flow."

Please notice that we have modified the model slightly from the one presented in the assignment. As given in the assignment, the equations were not complete with units, and a "time parameter" was missing in the equation for "flow2." The modified model includes units of measure and is dimensionally consistent. The analysis of oscillatory behavior in the following exercises is not affected by the changes in the model.

A. Simulate the model.[3] *In your assignment solutions document, include graphs of the behavior of the "flow," "Stock," and "Stock2." Explain the behavior that you observe and relate it to your conclusions from the previous exercises on oscillatory systems.*



Note that a very small time step has been used to simulate this model. An explanation is included in the appendix, and simulations with larger time steps are also included.

The system exhibits oscillatory behavior. Recall the previous exercise on oscillatory systems, where the two levels were connected sequentially, not in a feedback loop. In that exercise, we noted that "Stock 2" lagged one half of a period behind the "flow" and that, if the input period were chosen to have a specific value, the value of "Stock 2" at any time was the negative of the value of "flow." At other input periods, the value of "Stock 2" at any time was equal to some negative multiple of the value of "flow." Therefore, the negative value of "Stock 2" can be substituted for the original external input in the equation of "flow," thereby closing the loop consisting of the two stocks.

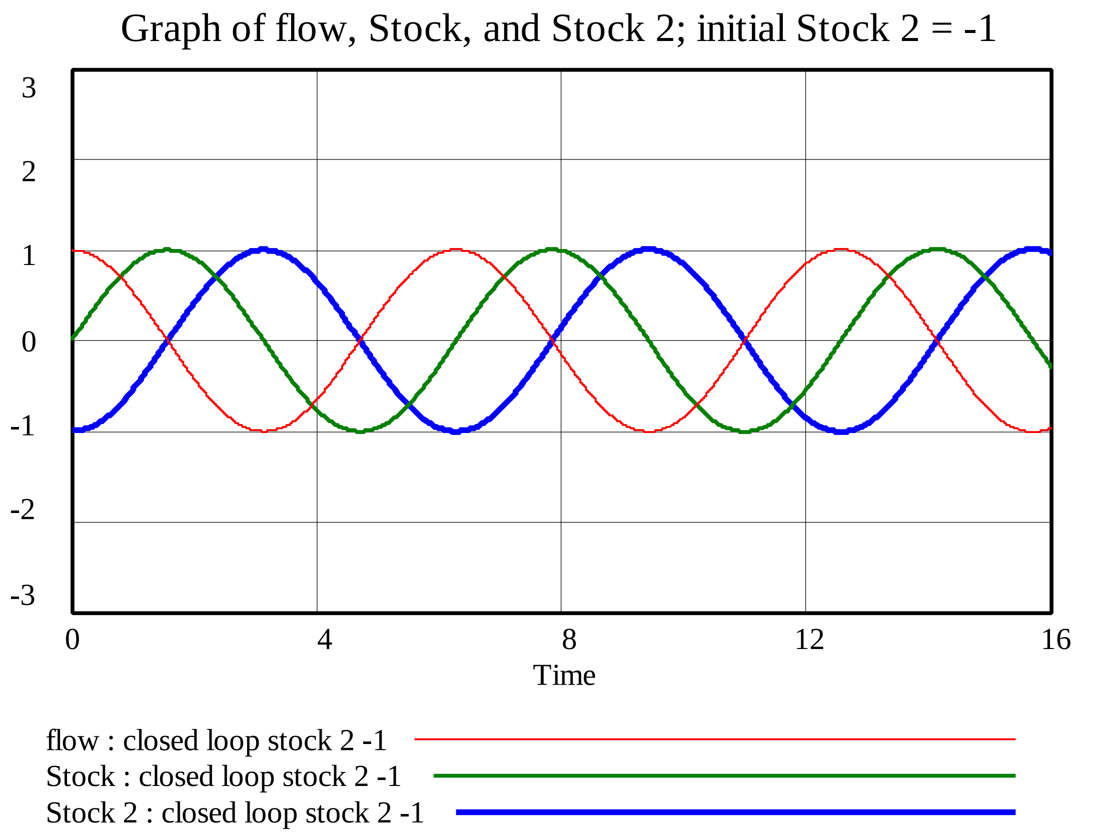
[3] When simulating this model, you might notice that the oscillations are slightly expanding. This is due to the computation process of the First-Order Euler Method, which is normally used in system dynamics simulations. Higher-order integration methods exist to perform these calculation, but those methods lead to other subtle problems under certain circumstances. To adjust for expanding oscillation, try repeatedly reducing the time step by a half until you believe the results are no longer significantly affected by the DT value. Also see the Appendix.

The output of "Stock 2," being identical to the original input, then sustains the oscillation indefinitely.

Note also that the model consists of only one feedback loop that contains two level equations (integrations). There is no cross loop from either level back to any part of the system. If such a system contains an initial imbalance, it will oscillate continuously without the oscillation growing or diminishing. A sinusoidal oscillation can exist because of the "phase shift" caused by the level equations in the loop. The "phase shift means that "Stock" is a sinusoid that lags 90 degrees, or one quarter of a period, behind the "flow," and "Stock 2" lags another quarter of a period behind the "Stock." Consequently, "Stock 2" lags 180 degrees, or one half of a period, behind "flow."

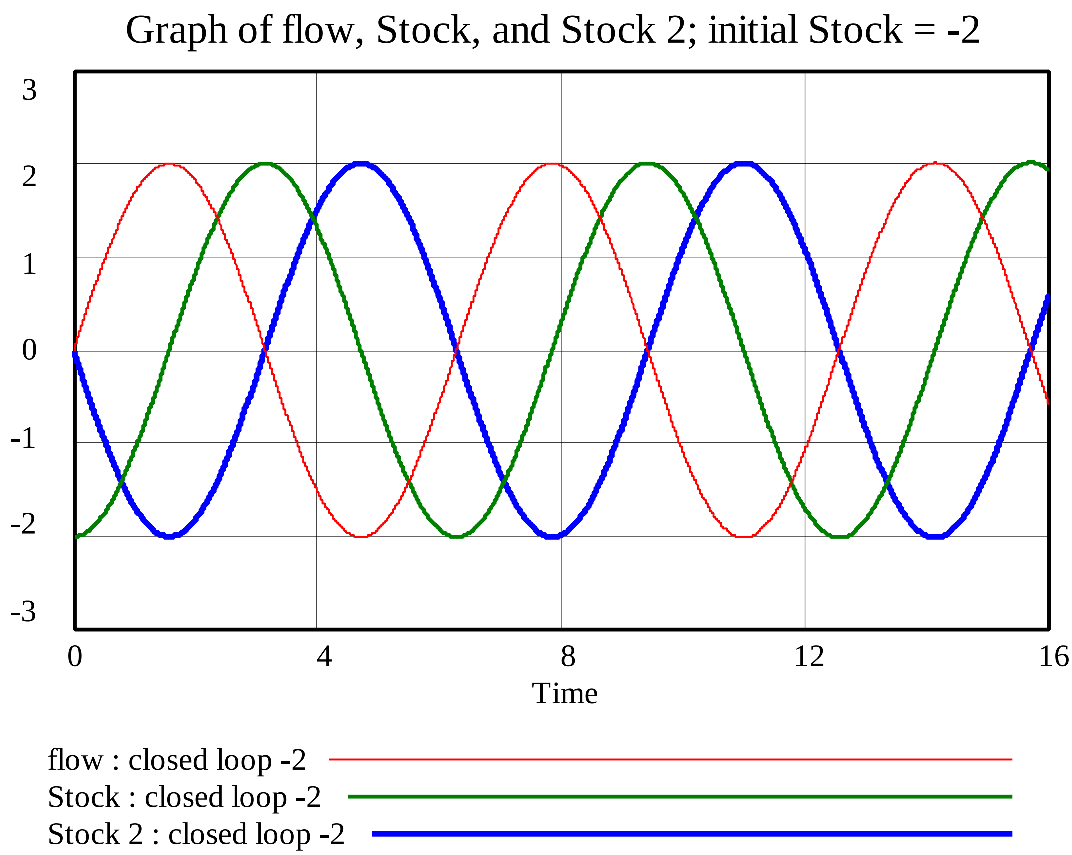
In such a system consisting of a single loop with two levels, the amplitude of oscillation depends on an initial imbalance in the system. As the system corrects that imbalance, an oscillation is initiated, which then continues. The amplitude is dependent on the degree of imbalance. With initial values for the two levels that represent system equilibrium, there will be no oscillation. In this example, the initial imbalance is due to the initial value of "Stock" being equal to –1.

B. Create a new dataset and simulate the model with initial value of "Stock" equal to 0 and initial value of "Stock 2" equal to –1. In your assignment solutions document, include graphs of the behavior of the "flow," "Stock," and "Stock2" in this simulation. Explain the behavior that you observe and compare the simulation to that from part A.



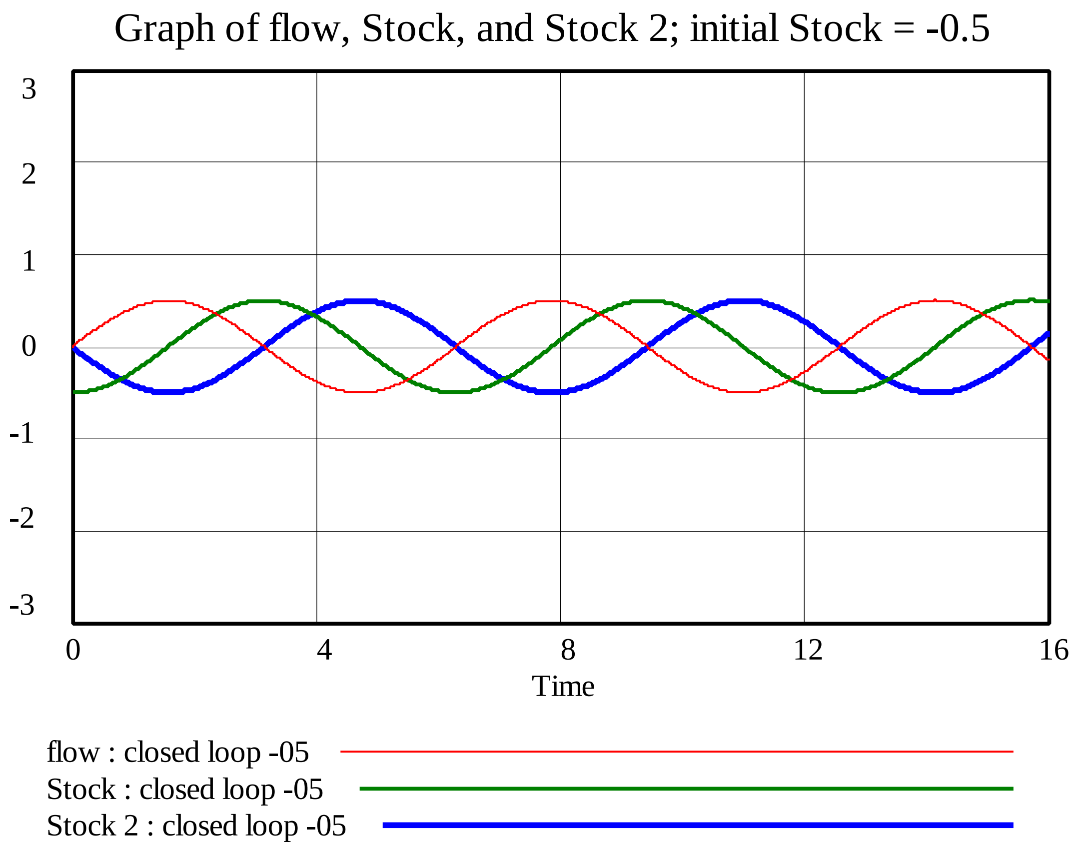
The system again generates oscillatory behavior with the same period and amplitude as in part A. In this case, the initial imbalance is due to the initial value of "Stock 2" being equal to –1. Notice that the behavior is the same as the behavior after the first quarter of a period in part A, at which point "Stock 2" was equal to –1 and "Stock" was equal to 0. Hence, the entire system behavior is simply shifted by one quarter of a period. That is, this graph is the same as the graph from part A, but it is shifted one quarter of a period to the left. Notice that we still observe the same "phase shift."

C. Repeat part B with initial value of "Stock" equal to –2 and initial value of "Stock 2" equal to 0.



The system again generates oscillatory behavior with the same period as in part A, but with twice the amplitude. In this case, the initial imbalance is due to the initial value of "Stock" being equal to –2. The imbalance is twice as large as in part A, so the amplitude is doubled. As in part A, the "Stock" lags one quarter of a period behind the "flow," and "Stock 2" lags one quarter of a period behind the "Stock."

D. Repeat part B with initial value of "Stock" equal to –0.5 and initial value of "Stock 2" equal to 0.

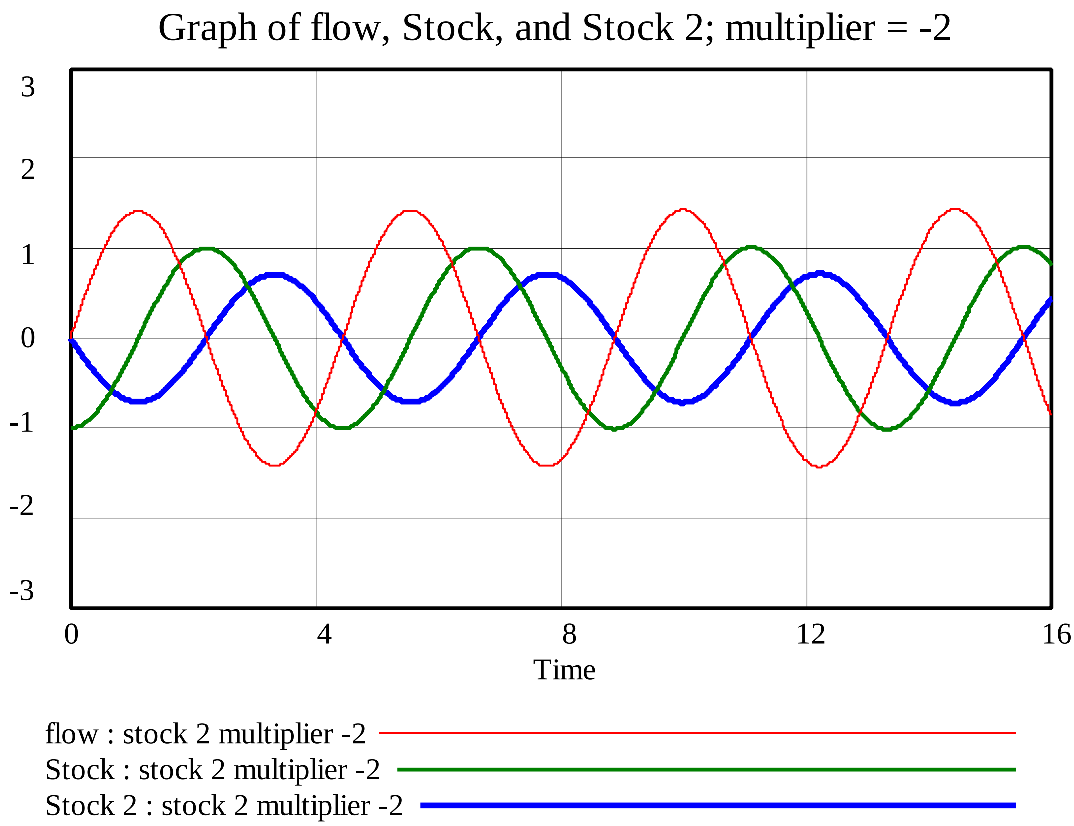


The system again generates oscillatory behavior with the same period as in part A, but with half the amplitude. In this case, the initial imbalance is due to the initial value of “Stock” being equal to –0.5. The imbalance is half of that in part A, so the amplitude is halved. Again, we observe the same phase shifts between the “flow,” “Stock,” and “Stock 2.”

E. What conclusions can you make about the behavior of an oscillatory system as the initial value of “Stock” and “Stock 2” changes?

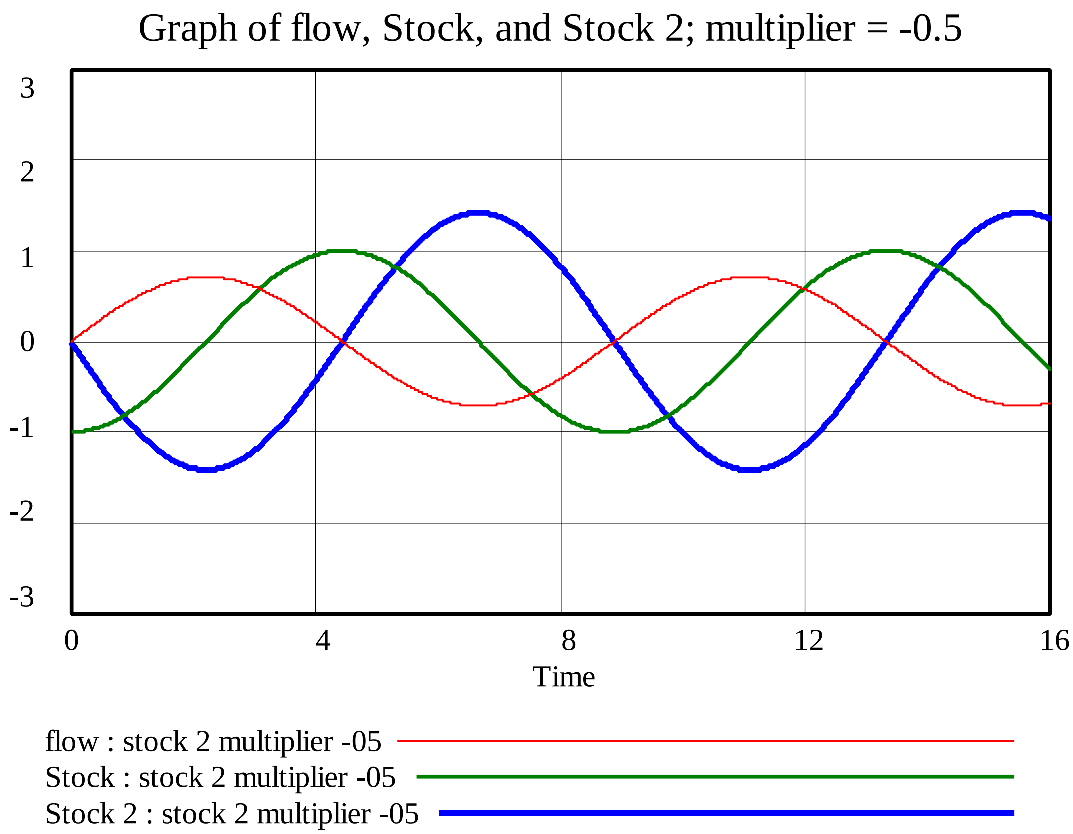
As the initial value of “Stock” and “Stock 2” changes, the degree of initial imbalance changes, and the amplitude changes. The period of oscillations is not affected by changes in the initial values. The same phase shift is always observed no matter what the initial values are. Notice that there is nothing particular about the initial values being negative; similar results can be obtained for simulations in which the initial value of one of the stocks is positive.

F. Repeat part B with initial value of “Stock” equal to –1, initial value of “Stock 2” equal to 0, and “stock 2 multiplier” equal to –2.



The three variables oscillate with the same period, but with different amplitudes. The amplitude of "Stock" is still determined by its initial value. Because the value of "flow" at any time is now negative twice the value of "Stock 2," "Stock" grows and falls at a faster rate, and crosses the time axis more frequently than in part A. Hence, the period of oscillations is shorter than in part A. The amplitude of "Stock" is the same as the amplitude of "Stock," "flow," and "Stock 2" in part A, but the amplitude of "Stock 2" is smaller than the amplitude of "Stock." The amplitude of "flow" is larger than the amplitude of "Stock"; it equals twice the amplitude of "Stock 2." The same relative phase shift is still observed.

G. Repeat part B with initial value of "Stock" equal to –1, initial value of "Stock 2" equal to 0, and "stock 2 multiplier" equal to –0.5.



The three variables oscillate with the same period, but with different amplitudes. The amplitude of “Stock” is still determined by its initial value. Because the value of “flow” at any time is now negative one half the value of “Stock 2,” “Stock” grows and falls at a slower rate, and crosses the time axis less frequently than in part A. Hence, the period of oscillations is longer than in part A. The amplitude of “Stock” is the same as the amplitude of “Stock,” “flow,” and “Stock 2” in part A, but the amplitude of “Stock 2” is larger than the amplitude of “Stock.” The amplitude of “flow” is smaller than the amplitude of “Stock,” and equals one half the amplitude of “Stock 2.” The same relative phase shift is still observed.

H. What conclusions can you make about the behavior of an oscillatory system as the “stock 2 multiplier” changes?

As the “stock 2 multiplier” changes, the period of oscillations and amplitude of “Stock 2” and “flow” change. Increasing the magnitude of “stock 2 multiplier” (that is, making it more negative) shortens the period of oscillation, decreases the amplitude of “Stock 2,” and increases the amplitude of “flow.” Decreasing the magnitude of “stock 2 multiplier” (that is, making it less negative) lengthens the period of oscillation, increases the amplitude of “Stock 2,” and decreases the amplitude “flow.” Changing the “stock 2 multiplier” has no effect on the relative phase shifts.

3. Principles of Systems

Please read section 2.3 of Principles of Systems and do the workbook exercises for this section (located at the end of the book). The material in this chapter is very important and you should make sure you understand it. Please let us know if you have any questions. You do not need to submit anything for this reading assignment.

Please note that the model presented in this section looks similar to the model used in Exercise 2 of this assignment, but it generates damped oscillations. The model from section 2.3 contains a material flow “receiving rate” between the stock “Goods on Order” and the stock “Inventory.” That is, the amount that leaves “Goods on Order” through the “receiving rate” is added to the “Inventory.” In the model from Exercise 2, on the other hand, “flow 2” does not subtract from “Stock 1” but only uses the information about the value of “Stock 1” to determine the inflow to “Stock 2.” This structural difference between the two models results in the different behaviors that they generate.

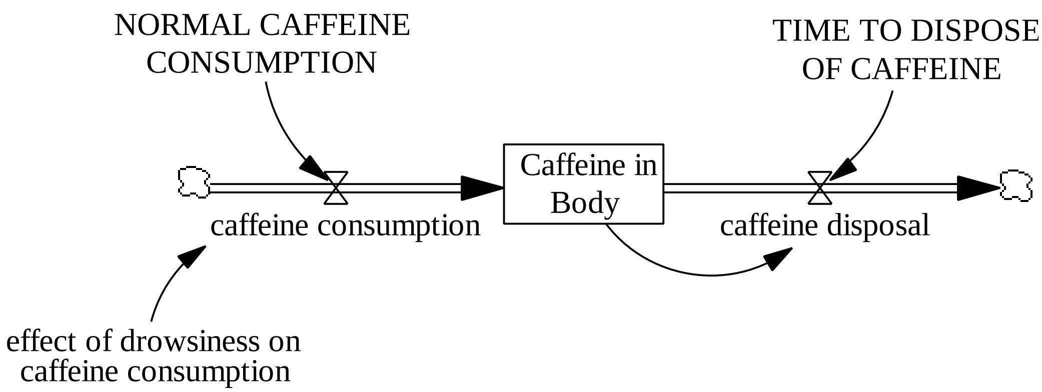
4. Modeling Exercise

Helen is addicted. Every day Helen visits a local coffeehouse, a subsidiary of a famous café chain with a French name that no one can pronounce correctly. There she buys tall steaming cups of Java brew coffee that she slowly drinks over the course of the day to fight off the drowsiness caused by many sleepless nights spent programming in the computer room. In this exercise we will study the effects of Helen’s addiction to caffeine.

Initially Helen has 50 mg of caffeine in her body. Every day she usually consumes 200 mg of caffeine. If she is feeling particularly drowsy, however, she will consume more. Helen feels the effect of the caffeine for an average of 6 hours before her body disposes of it.

A. From the above description, create a stock-and-flow diagram of the level of caffeine in Helen’s body. For now, define the effect of drowsiness on caffeine consumption in such a way that the rate of caffeine consumption will equal Helen’s usual consumption. In your assignment solutions document, include the model diagram and documented equations.

Model diagram:



<u>Model equations:</u>

caffeine consumption = NORMAL CAFFEINE CONSUMPTION * effect of drowsiness on caffeine consumption
Units: mg caffeine/Day
The amount of caffeine that Helen consumes every day.

caffeine disposal = Caffeine in Body / TIME TO DISPOSE OF CAFFEINE
Units: mg caffeine/Day
The amount of caffeine of which Helen's body disposes every day.

Caffeine in Body = INTEG (caffeine consumption – caffeine disposal, 50)
Units: mg caffeine
The amount of caffeine in Helen's body.

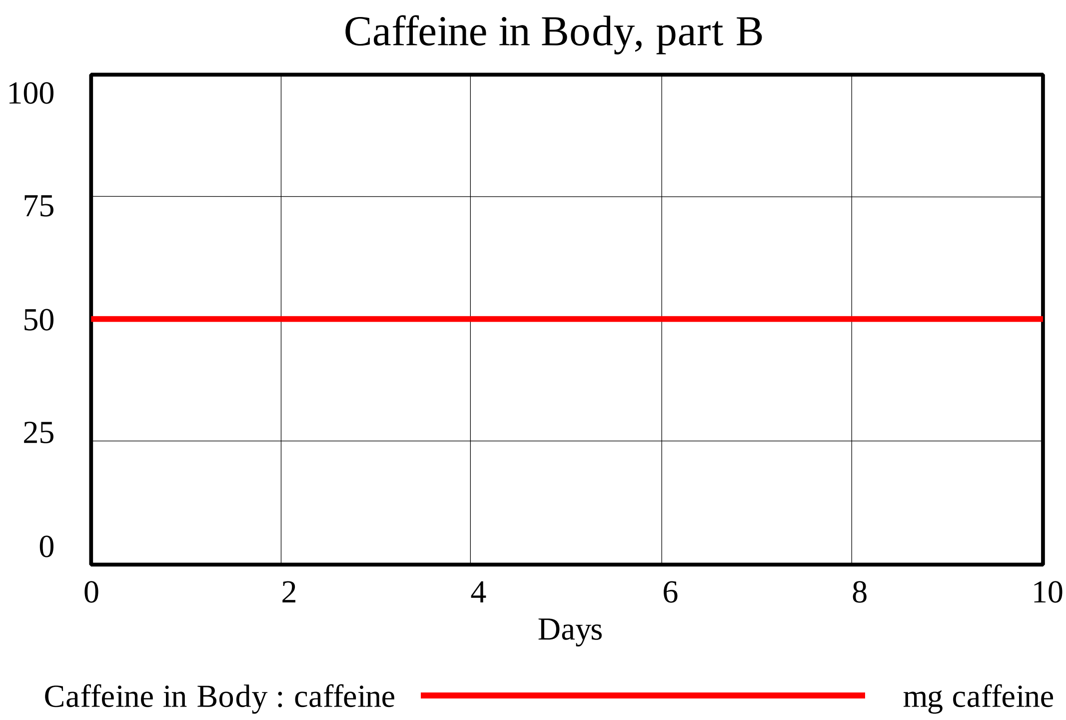
effect of drowsiness on caffeine consumption = 1
Units: dmnl
The effect of drowsiness on Helen's consumption of caffeine.

NORMAL CAFFEINE CONSUMPTION = 200
Units: mg caffeine/Day
The normal amount of caffeine that Helen consumes every day.

TIME TO DISPOSE OF CAFFEINE = 0.25
Units: Day
The amount of time it takes Helen's body to dispose of caffeine.

B. Draw a reference mode for the behavior of the stock. Simulate the model over a period of ten days. In your assignment solutions document, include a graph of the behavior of the level of caffeine in Helen's body.

Because drowsiness does not yet have any effect on consumption, Helen's body is in an equilibrium state where she is capable of disposing all the coffee she consumes daily. Every day she consumes 200 milligrams (mg) of caffeine and disposes of 200 mg of caffeine. The caffeine in Helen's body will stay constant over time at its initial value of 50 mg:

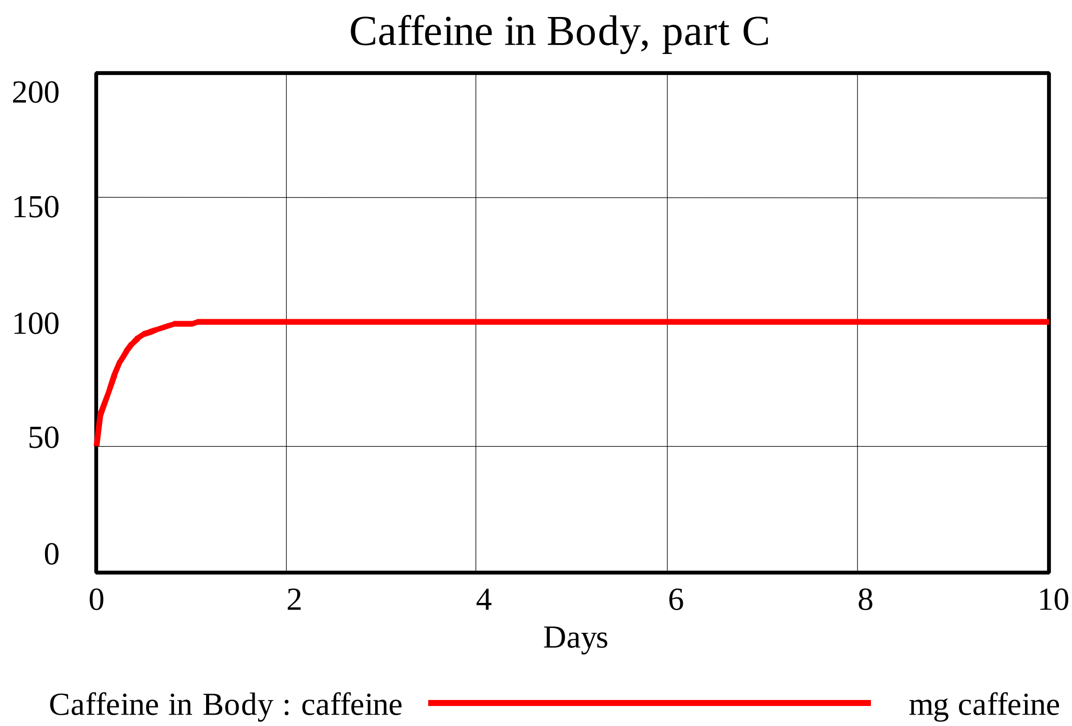


C. Helen is having a hectic semester because of a difficult class that she is taking: the Laboratory in Software Engineering (coincidentally, she programs in Java). She now consumes twice her usual amount of coffee. Draw a reference mode for the new behavior of the stock. Simulate the model. In your assignment solutions document, include a graph of the model behavior in this scenario. Did the model generate the behavior that you predicted?

If Helen consumes twice as much coffee per day as previously, her body will have to work much harder to dispose of caffeine. Because there is only one feedback loop in the system (a negative feedback loop involving caffeine disposal), the system will grow asymptotically to equilibrium. The equilibrium point occurs when the inflow and outflow are equal:

inflow = outflow
caffeine consumption = caffeine disposal
400 mg of caffeine /day = Caffeine in Body / TIME TO DISPOSE OF CAFFEINE
400 mg of caffeine /day = Caffeine in Body / 0.25 day
Caffeine in Body = 100 mg of Caffeine

The following figure shows asymptotic growth to the stock equilibrium value of 100 mg of caffeine:

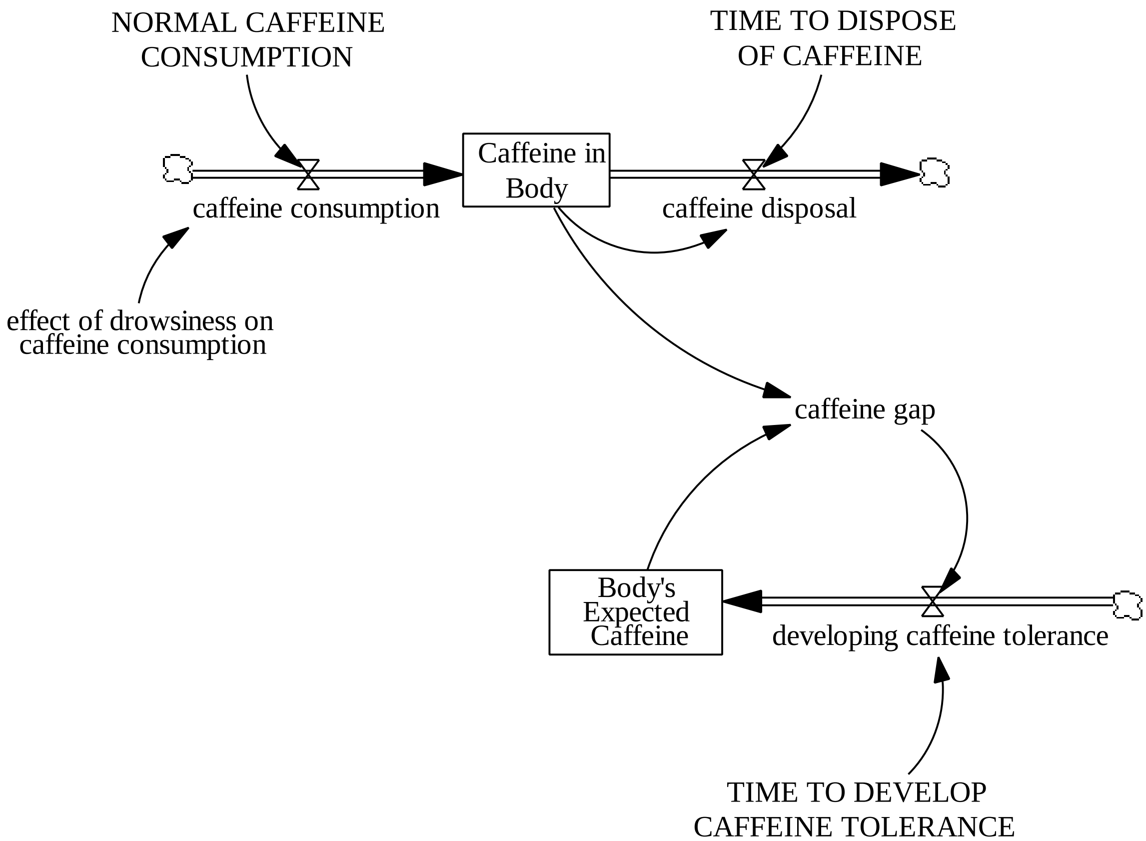


Over time, Helen has developed a tolerance for caffeine. Helen's body expects a certain level of caffeine. If the level of caffeine in Helen's body changes, her body will adapt over time and begin to expect a new level of caffeine. After about five days, Helen's body will develop a tolerance for her new caffeine habits.

D. From the above description, add a stock and a flow to the model to account for the amount of caffeine Helen's body expects at any point in time.[4] *Set the initial value of the stock equal to the initial value of the level of caffeine in Helen's body. In your assignment solutions document, include the modified model diagram and documented equations.*

Model diagram:

[4] Formulate the exponential smoothing process without using the SMOOTH function that Vensim PLE provides.



<u>Model equations:</u>

Body’s Expected Caffeine = INTEG (developing caffeine tolerance, 50)
Units: mg caffeine
The amount of caffeine that Helen’s body expects.

caffeine consumption = NORMAL CAFFEINE CONSUMPTION * effect of drowsiness on caffeine consumption
Units: mg caffeine/Day
The amount of caffeine that Helen consumes every day.

caffeine disposal = Caffeine in Body / TIME TO DISPOSE OF CAFFEINE
Units: mg caffeine/Day
The amount of caffeine of which Helen’s body disposes every day.

caffeine gap = Caffeine in Body – Body’s Expected Caffeine
Units: mg caffeine
The difference between the actual amount of caffeine in Helen’s body and the amount that her body expects.

Caffeine in Body = INTEG (caffeine consumption – caffeine disposal, 50)
Units: mg caffeine
The amount of caffeine in Helen’s body.

developing caffeine tolerance = caffeine gap / TIME TO DEVELOP CAFFEINE TOLERANCE
Units: mg caffeine/Day
The rate at which Helen's body's expected amount of caffeine changes.

effect of drowsiness on caffeine consumption = 1
Units: dmnl
The effect of drowsiness on Helen's consumption of caffeine.

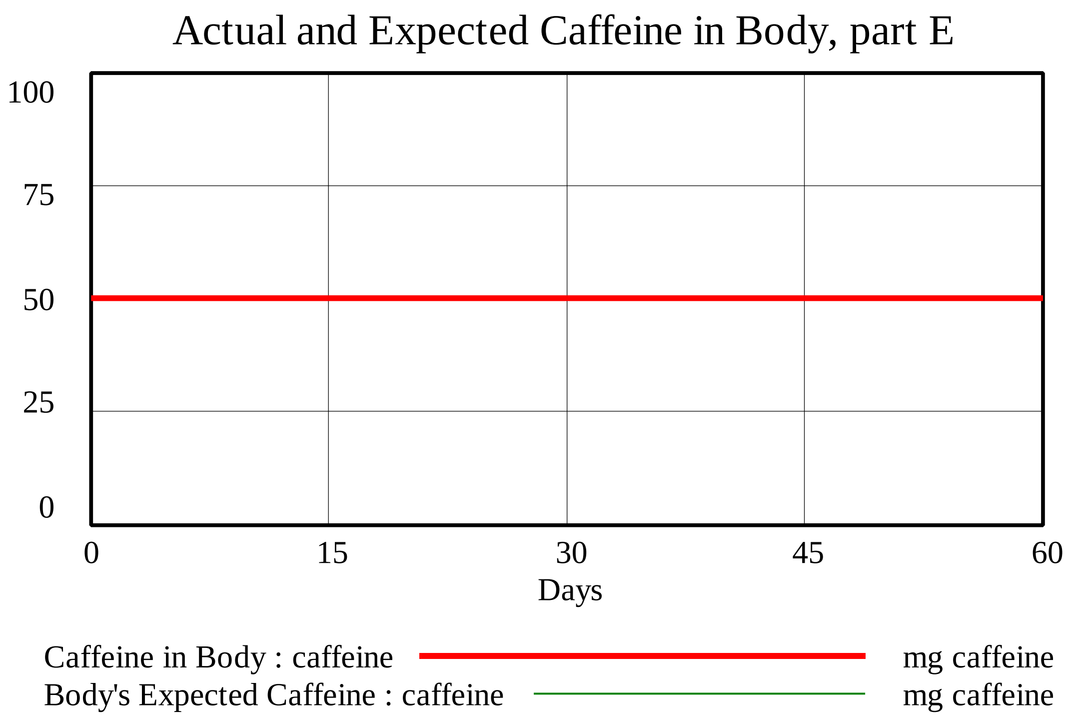
NORMAL CAFFEINE CONSUMPTION = 200
Units: mg caffeine/Day
The normal amount of caffeine that Helen consumes every day.

TIME TO DEVELOP CAFFEINE TOLERANCE = 5
Units: Day
The time it takes Helen's body to adapt and begin to expect a new level of caffeine.

TIME TO DISPOSE OF CAFFEINE = 0.25
Units: Day
The amount of time it takes Helen's body to dispose of caffeine.

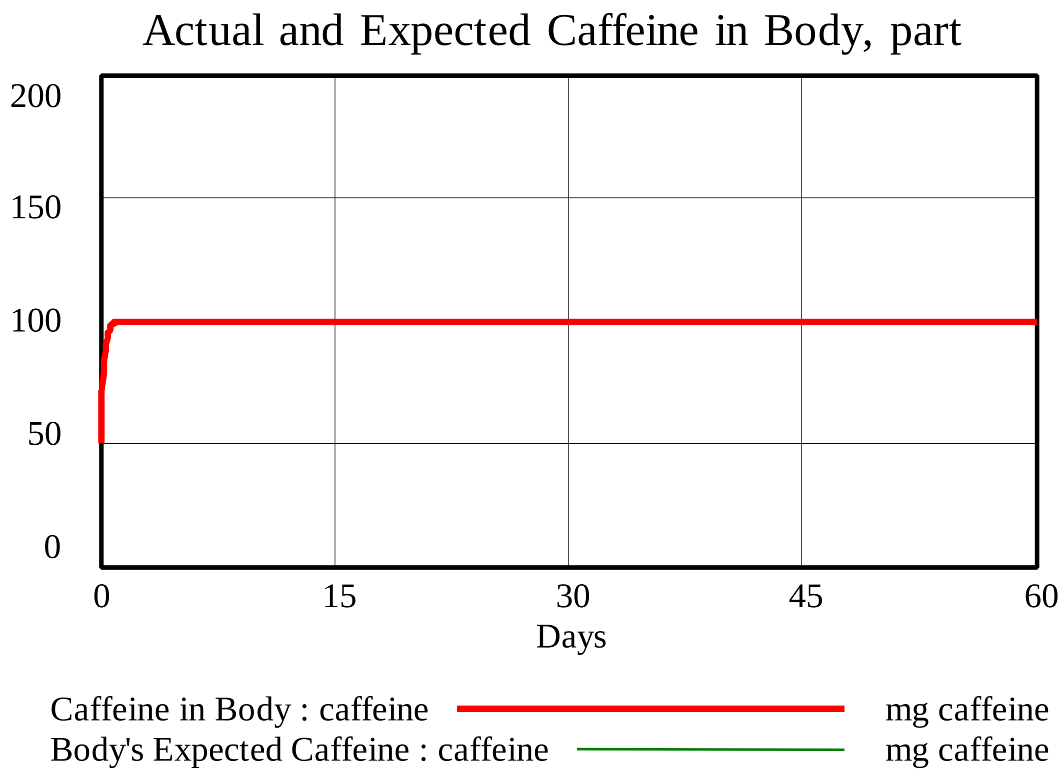
E. Draw reference modes over sixty days for the behavior of expected level of caffeine in Helen's body under the normal workload scenario (when Helen drinks her normal amount of coffee). Simulate the model. In your assignment solutions document, include a graph of model behavior. Did the model generate the behavior that you predicted? Why or why not?

If Helen consumes 200 mg of caffeine per day, the system starts out in equilibrium. Both "Caffeine in Body" and "Body's Expected Caffeine" stocks start out at 50 mg of caffeine. Expected and actual levels of caffeine in the body are therefore equal and Helen does not develop further tolerance to caffeine. Both stocks are in equilibrium:



F. Draw reference modes over sixty days for the behavior of expected level of caffeine in Helen's body under the heavy workload scenario (when Helen drinks twice her usual amount of coffee). Simulate the model. In your assignment solutions document, include a graph of model behavior in this scenario. Did the model generate the behavior that you predicted? Why or why not?

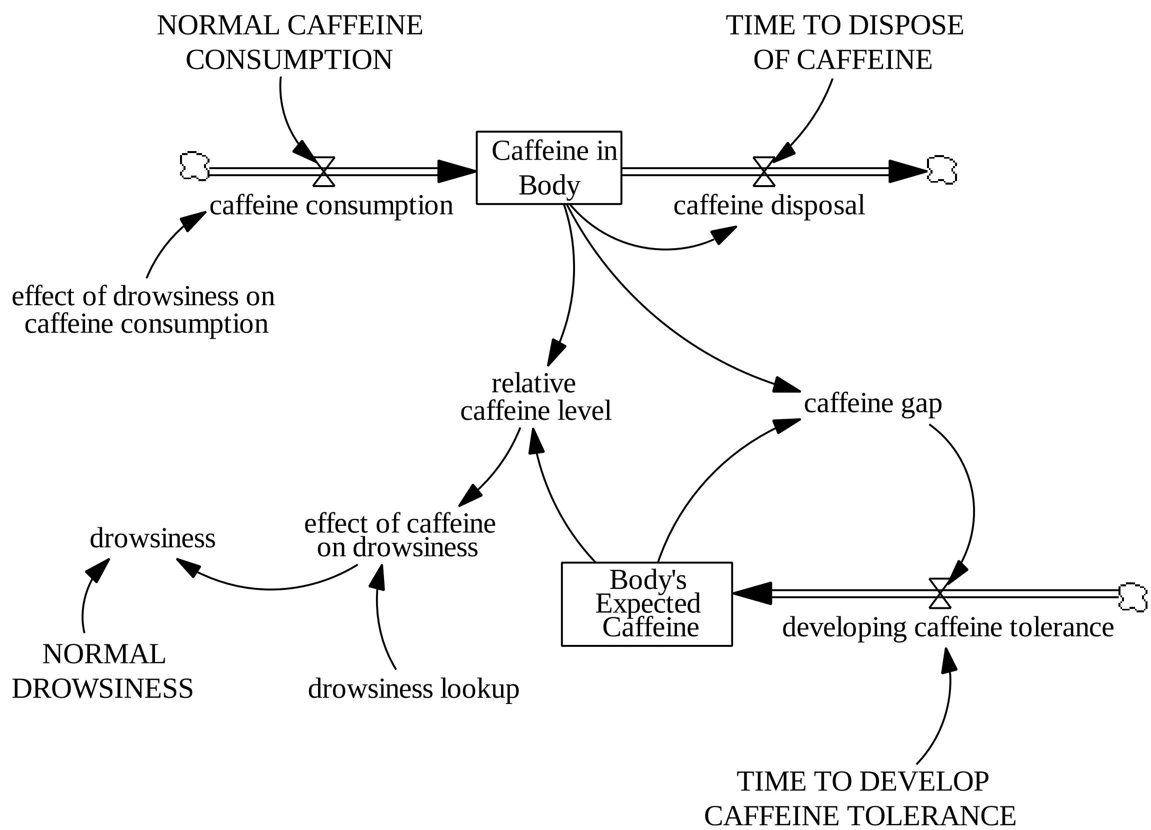
If Helen consumes 400 mg of caffeine per day, the amount of Helen's "Caffeine in Body" quickly rises, driving up her tolerance ("Body's Expected Caffeine") after a time delay. Eventually Helen's "Body's Expected Caffeine" also reaches 100 mg of caffeine, but grows more slowly due to the time it takes to develop tolerance:



Helen drinks coffee to ward off drowsiness. Normally, Helen yawns approximately ten times a day. When she has a relatively high level of caffeine, she is less drowsy; she yawns less frequently. When she has a relatively low level of caffeine, she feels drowsy and yawns more frequently. Specifically, when her body has one and a half times the expected level of caffeine, she only yawns once or twice a day. When her body has half the expected level of caffeine, she yawns approximately eighteen times a day.

G. Add auxiliary variables and lookup function to your model to account for Helen's drowsiness.
Hint: drowsiness has units of yawns/day.

<u>Model diagram:</u>



<u>Modified model equations:</u>

drowsiness = NORMAL DROWSINESS * effect of caffeine on drowsiness
    Units: yawn/Day
    Helen’s actual drowsiness, measured by the number of times she yawns per day.

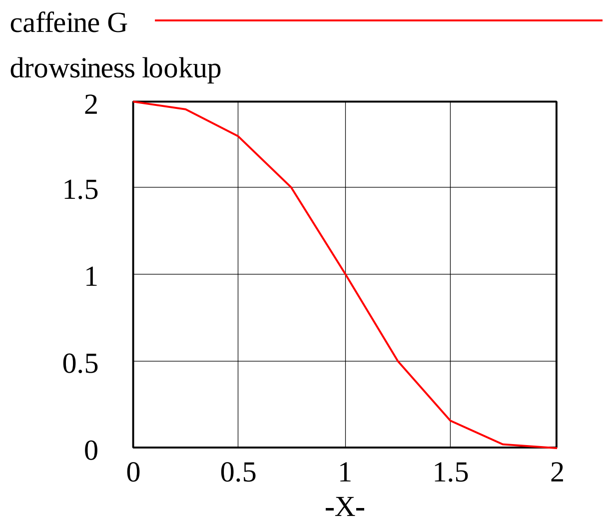
drowsiness lookup([(0,0) - (2,2)], (0,2), (0.25,1.95), (0.5,1.8), (0.75,1.5), (1,1), (1.25,0.5), (1.5,0.15), (1.75,0.02), (2,0))
    Units: dmnl
    The lookup function for the effect of caffeine on drowsiness.

effect of caffeine on drowsiness = drowsiness lookup(relative caffeine level)
    Units: dmnl
    The effect of the relative amount of caffeine in Helen’s body on her drowsiness.

NORMAL DROWSINESS = 10
    Units: yawn/Day
    The normal number of times per day that Helen yawns.

relative caffeine level = Caffeine in Body / Body’s Expected Caffeine
    Units: dmnl
    The ratio of the actual amount of caffeine in Helen’s body to the expected amount.

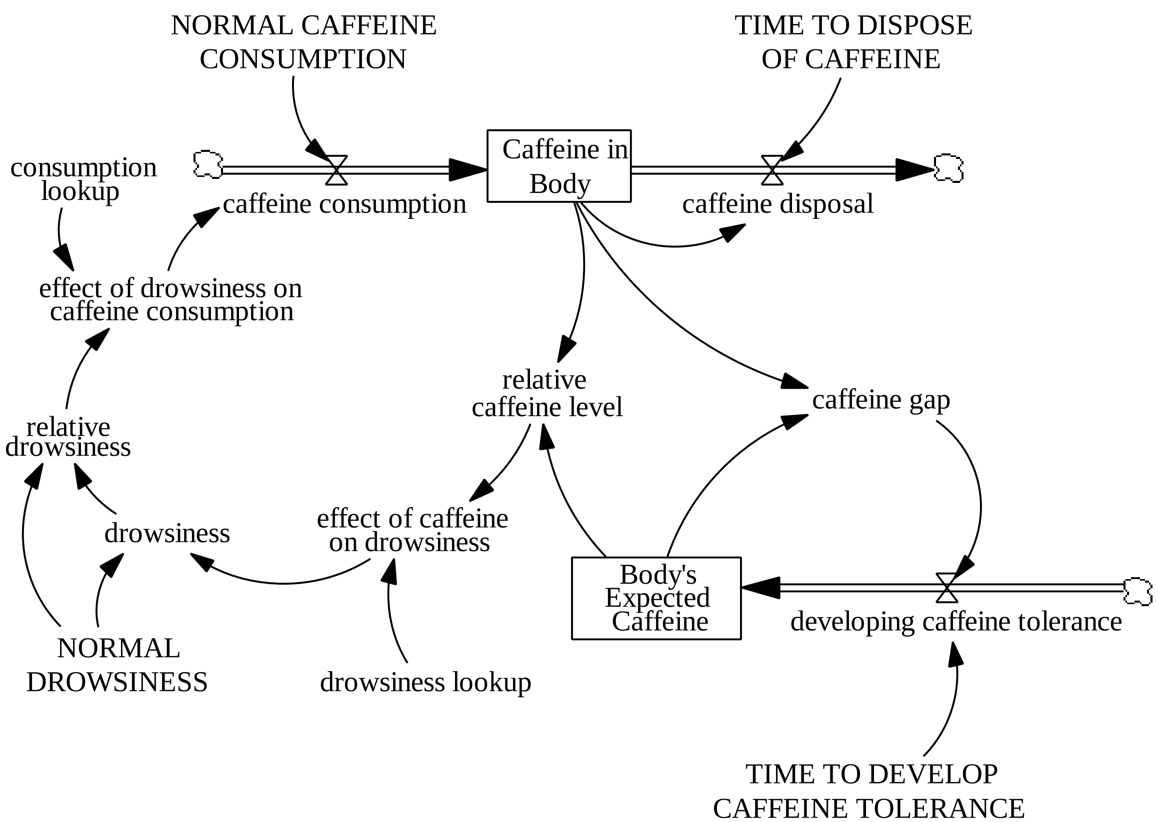
<u>Graph of the lookup function:</u>



The drowsier Helen feels, the more caffeine she consumes. For example, when she feels twice as drowsy as usual, she drinks twice and a half as much coffee. If she feels half as drowsy, she drinks one third as much coffee. If she feels four times as drowsy, she will consume five times as much coffee. She will never consume more then five times her usual amount of coffee, however, because if she does, she will start shaking and be unable to type.

H. Add a second lookup function to the model and close the feedback loop. In your assignment solutions document, include the modified model diagram, documented equations, and graphs of the lookup functions. How many feedback loops are now in the model? Describe each feedback loop.

<u>Model diagram:</u>



<u>Model equations:</u>

Body's Expected Caffeine = INTEG (developing caffeine tolerance, 50)
Units: mg caffeine
The amount of caffeine that Helen's body expects.

caffeine consumption = NORMAL CAFFEINE CONSUMPTION * effect of drowsiness on caffeine consumption
Units: mg caffeine/Day
The amount of caffeine that Helen consumes every day.

caffeine disposal = Caffeine in Body / TIME TO DISPOSE OF CAFFEINE
Units: mg caffeine/Day
The amount of caffeine of which Helen's body disposes every day.

caffeine gap = Caffeine in Body – Body's Expected Caffeine
Units: mg caffeine
The difference between the actual amount of caffeine in Helen's body and the amount that her body expects.

Caffeine in Body = INTEG (caffeine consumption – caffeine disposal, 50)
Units: mg caffeine
The amount of caffeine in Helen's body.

consumption lookup ([(0,0) - (5,5)], (0,0), (0.5,0.33), (1,1), (1.5,1.7), (2,2.5), (2.5,3.4), (3,4.1), (3.5,4.75), (4,5), (5,5))
Units: dmnl
The lookup function for the effect of drowsiness on consumption.

developing caffeine tolerance = caffeine gap / TIME TO DEVELOP CAFFEINE TOLERANCE
Units: mg caffeine/Day
The rate at which Helen's body's expected amount of caffeine changes.

drowsiness = NORMAL DROWSINESS * effect of caffeine on drowsiness
Units: yawn/Day
Helen's actual drowsiness, measured by the number of times she yawns per day.

drowsiness lookup ([(0,0) - (2,2)], (0,2), (0.25,1.95), (0.5,1.8), (0.75,1.5), (1,1), (1.25,0.5), (1.5,0.15), (1.75,0.02), (2,0))
Units: dmnl
The lookup function for the effect of caffeine on drowsiness.

effect of caffeine on drowsiness = drowsiness lookup(relative caffeine level)
Units: dmnl
The effect of the relative amount of caffeine in Helen's body on her drowsiness.

effect of drowsiness on caffeine consumption = consumption lookup(relative drowsiness)
Units: dmnl
The effect of drowsiness on Helen's consumption of caffeine.

NORMAL CAFFEINE CONSUMPTION = 200
Units: mg caffeine/Day
The normal amount of caffeine that Helen consumes every day.

NORMAL DROWSINESS = 10
Units: yawn/Day
The normal number of times per day that Helen yawns.

relative caffeine level = Caffeine in Body / Body's Expected Caffeine
Units: dmnl
The ratio of the actual amount of caffeine in Helen's body to the expected amount.

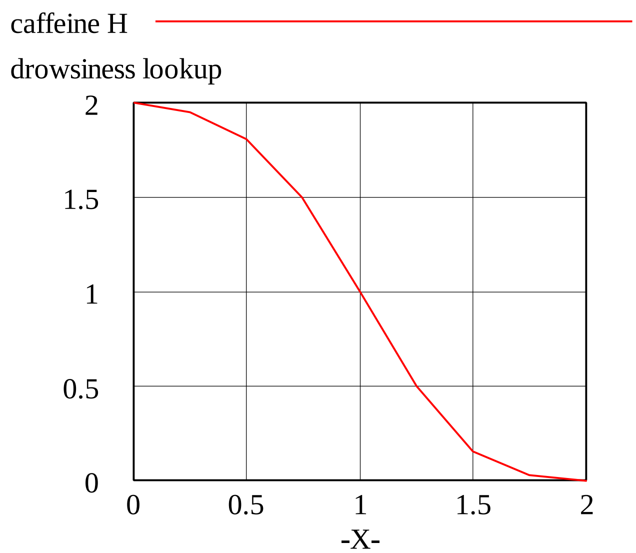
relative drowsiness = drowsiness / NORMAL DROWSINESS
Units: dmnl
The ratio of Helen's current to normal drowsiness.

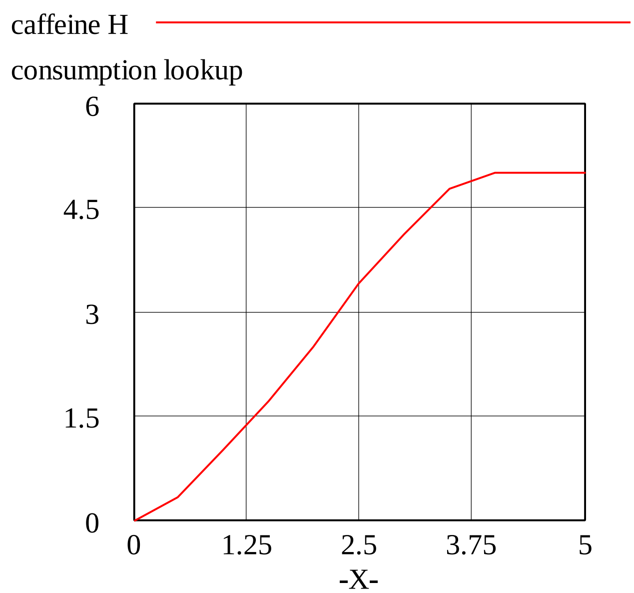
TIME TO DEVELOP CAFFEINE TOLERANCE = 5

Units: Day
The time it takes Helen's body to adapt and begin to expect a new level of caffeine.

TIME TO DISPOSE OF CAFFEINE = 0.25
Units: Day
The amount of time it takes Helen's body to dispose of caffeine.

<u>Graphs of lookup functions:</u>



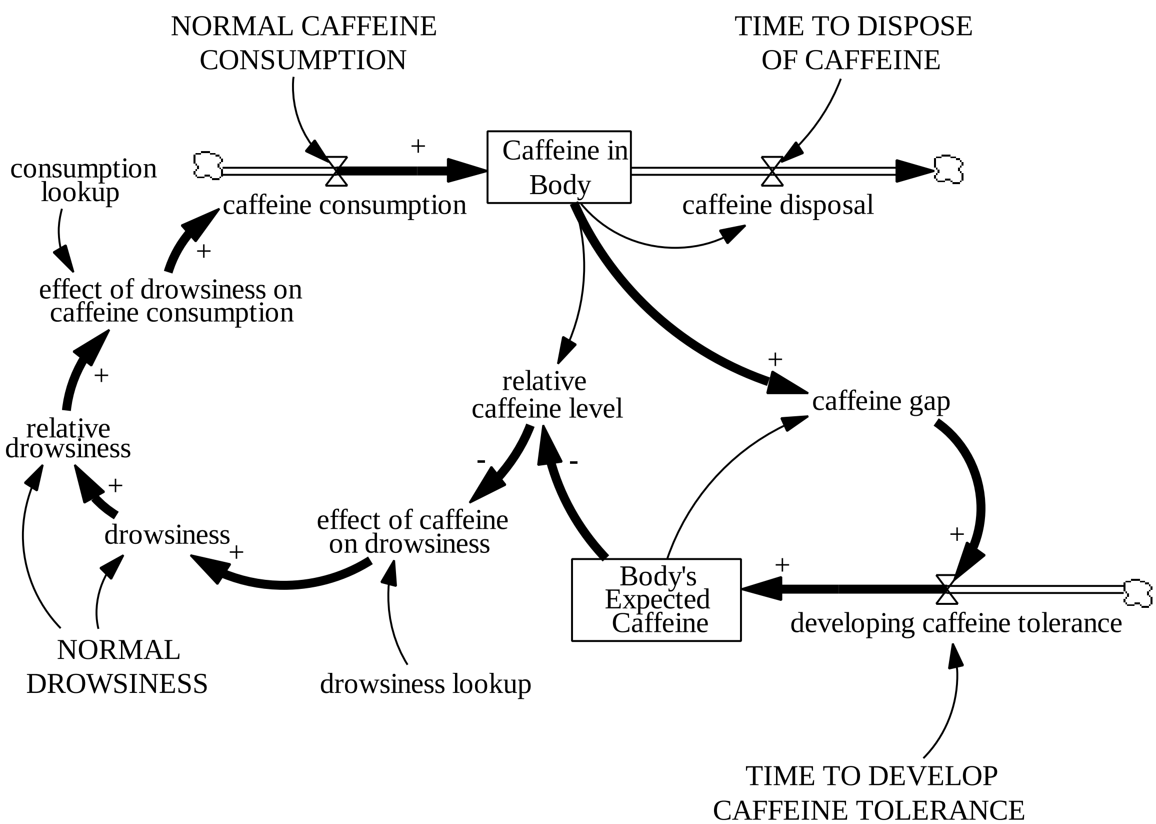


Four feedback loops are embedded within the addiction model:

Loop 1: Negative feedback loop from "Caffeine in Body" to "caffeine disposal." A higher level of caffeine in Helen's body causes her body to dispose of caffeine faster, decreasing the level of caffeine in Helen's body at a faster rate, which results in a lower level of "Caffeine in Body."

Loop 2: Negative feedback loop from "Body's Expected Caffeine" stock to "caffeine gap" to "developing caffeine tolerance" back to the stock. As the expected caffeine level in the body increases, the gap between expected and actual caffeine level decreases, decreasing the development of tolerance to caffeine, which in turn increases "Body's Expected Caffeine" at a slower rate.

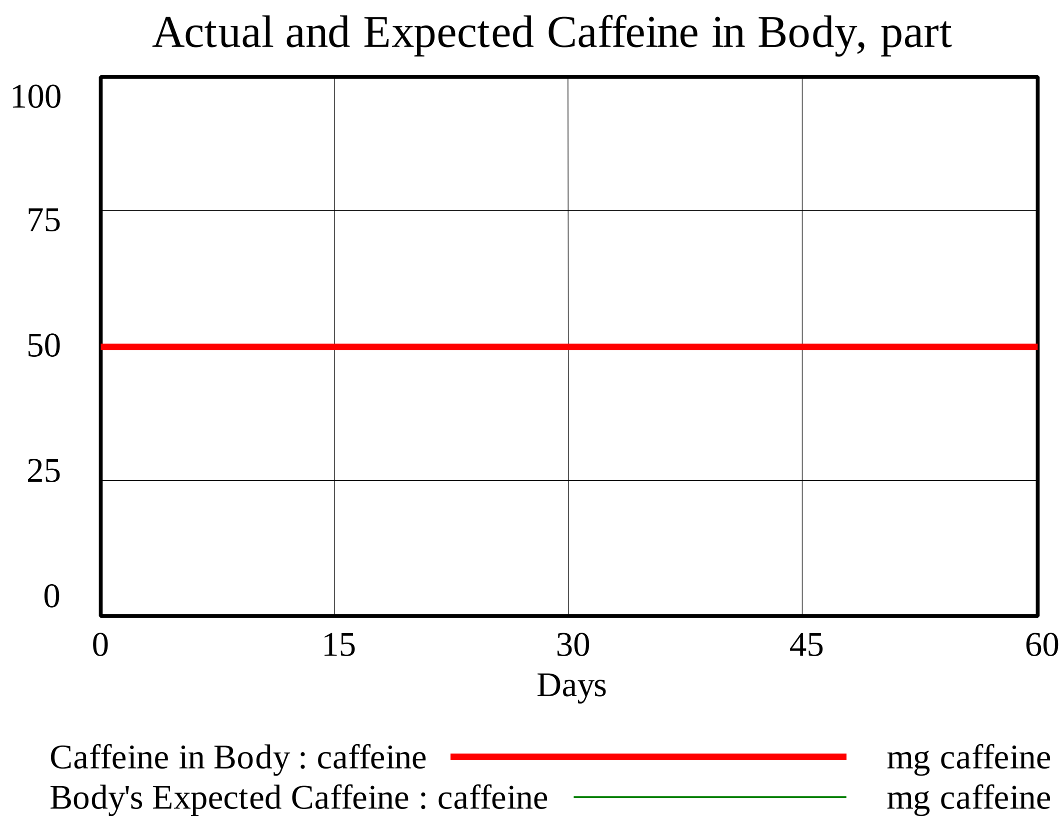
Loop 3: Positive feedback loop involving both stocks. The larger the amount of "Caffeine in Body," the greater the "caffeine gap," the greater the change in tolerance. Over time, "Body's Expected Caffeine" rises, lowering the "relative level of caffeine," and increasing Helen's drowsiness. As Helen gets more and more drowsy, she consumes more and more caffeine, increasing her "Caffeine in Body." The loop is highlighted in the figure below:



Loop 4: Negative feedback loop from "Caffeine in Body" to "drowsiness" to "caffeine consumption" and back to "Caffeine in Body." As Helen consumes more and more coffee, the caffeine builds up in her body, leading her to be less drowsy and therefore to consume less coffee.

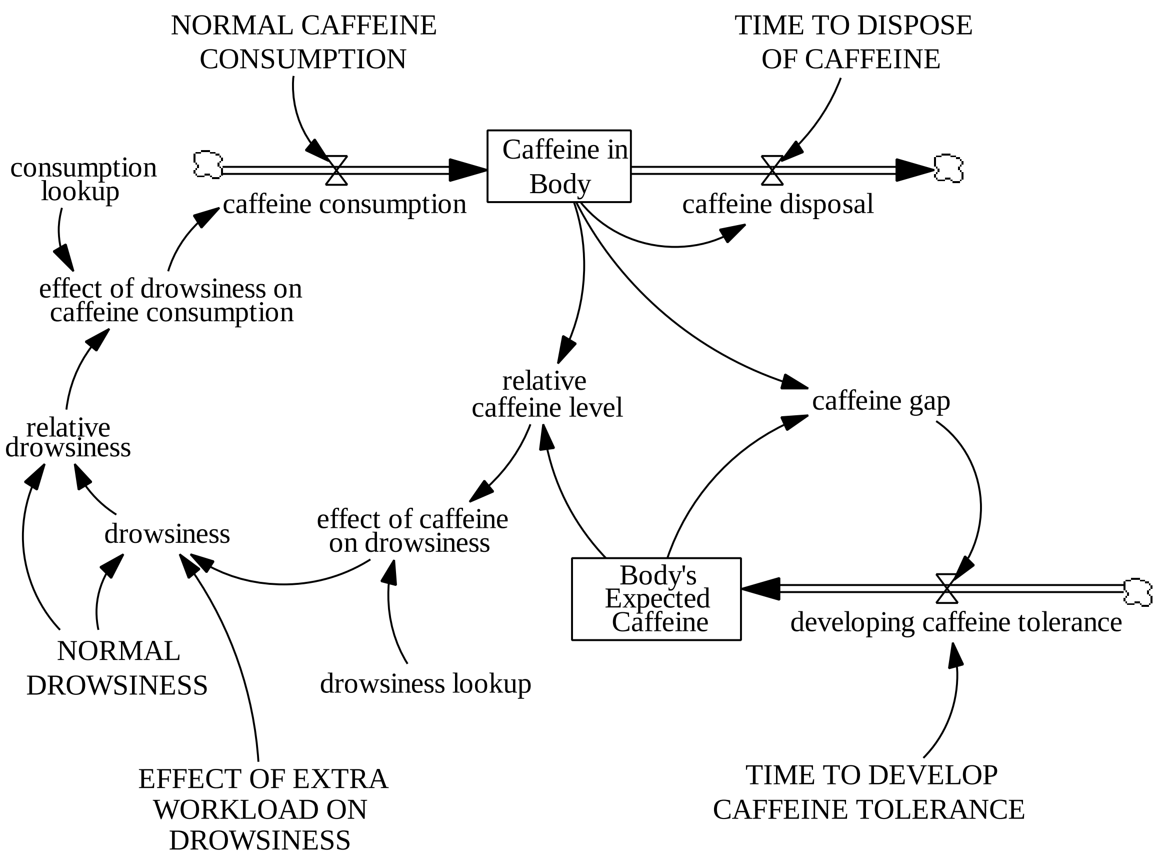
I. Draw reference modes for the behavior of the two stocks under the normal workload scenario. Simulate the model. In your assignment solutions document, include graphs of the model behavior. Does the model produce the behavior that you expected? Why or why not?

Under the normal workload scenario, there is no imbalance between the two stocks and therefore no "caffeine gap" initially. The system is never driven out of equilibrium:



J. Now that the drowsiness loop is in place, the second scenario can be implemented more realistically. Create a parameter called "effect of extra workload on drowsiness." The parameter will have no effect until the tenth day, when the final project is assigned and the extra workload doubles Helen's current drowsiness. Draw reference modes for the behavior of the two stocks in the model in this scenario. Simulate the model. In your assignment solutions document, include the modified model diagram, documented equations, and graphs of the model behavior. Does the model produce the behavior that you expected? Why or why not?

<u>Model diagram:</u>



<u>Modified model equations:</u>

drowsiness = NORMAL DROWSINESS * effect of caffeine on drowsiness * EFFECT OF EXTRA WORKLOAD ON DROWSINESS

Units: yawn/Day

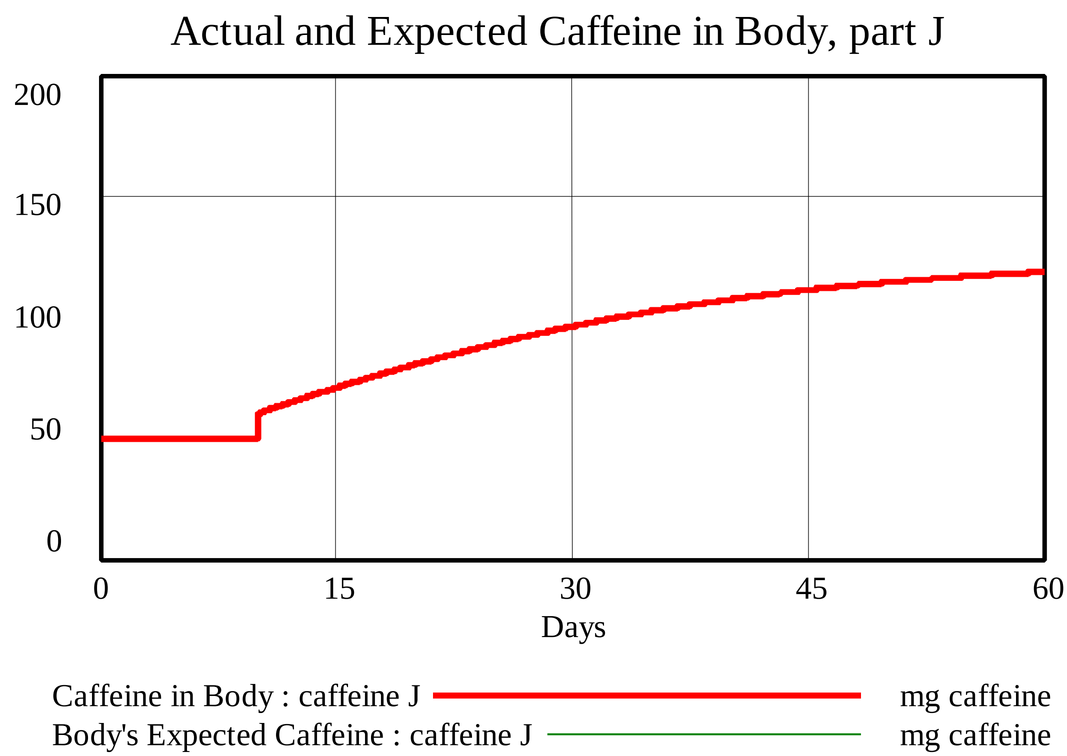
Helen’s actual drowsiness, measured by the number of times she yawns per day.

EFFECT OF EXTRA WORKLOAD ON DROWSINESS = 1 + STEP(1,10)

Units: dmnl

The effect of Helen’s extra workload on the number of times she yawns per day.

The increased drowsiness is reflected by doubling the “EFFECT OF EXTRA WORKLOAD ON DROWSINESS” on day 10 by using a STEP function, resulting in the following model behavior:

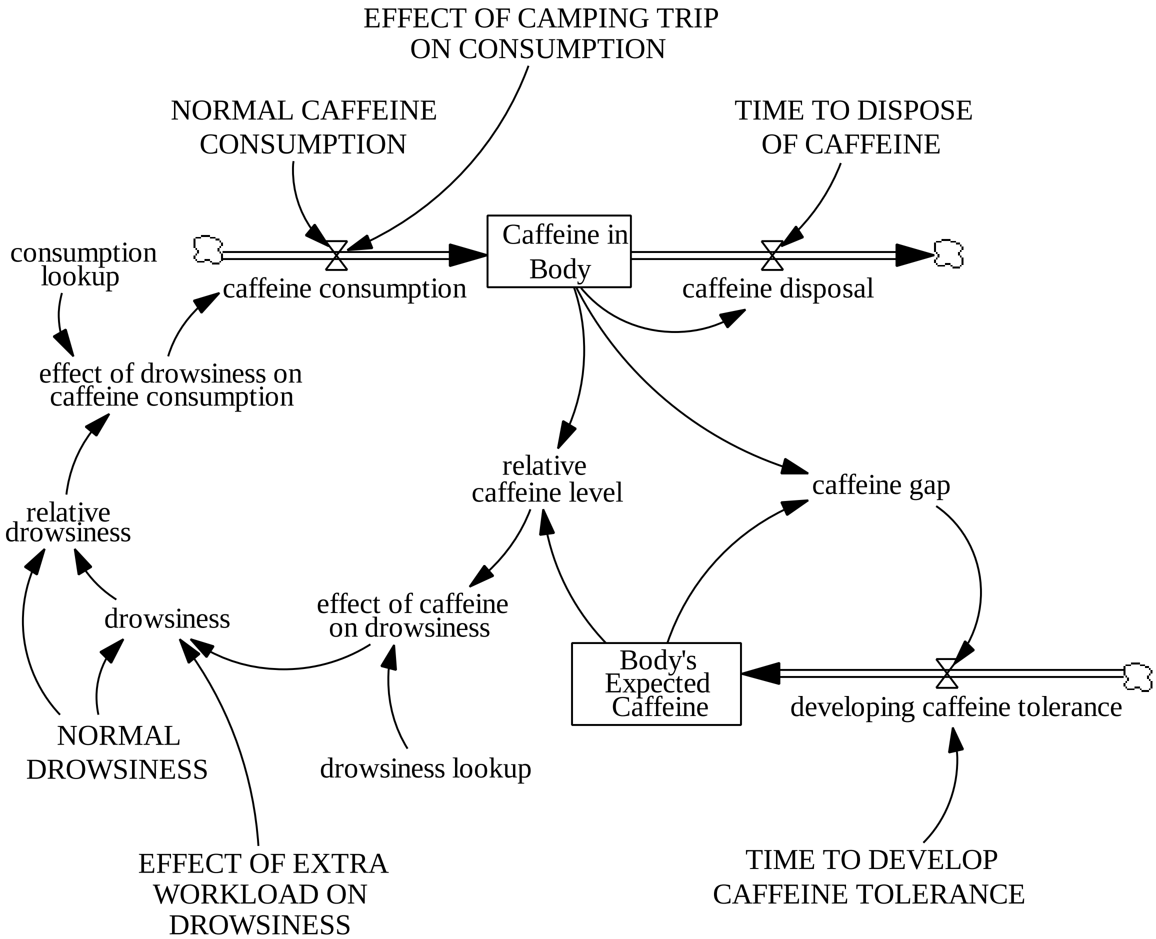


Such behavior should have been expected, as the negative feedback loops within the system dominate the positive feedback and drives both stocks towards equilibrium. A sudden increase in drowsiness is immediately reflected in the "Caffeine in Body" stock but takes a while to propagate to the "Body's Expected Caffeine" stock. Notice, however, that the equilibrium values of the two stocks have both increased (remember that in order for the system to be at equilibrium the two stocks have to be equal).

K. After graduation, Helen decides to go camping in the mountains for a few weeks with her friends. She forgets to bring coffee with her and is unable to find any as she hikes up the rocky trails. Create a parameter called "effect of camping trip on consumption." Draw reference modes for the two stocks in the model and the variable drowsiness. Simulate the model over a period of ten days. Assume that Helen leaves Boston to go camping on the second day. In your assignment solutions document, include the modified model diagram, documented equations, and graphs of model behavior. Does the model produce the behavior that you expected? Why or why not?[5]

Model diagram:

[5] For this scenario, make sure to change the "Effect of extra workload on drowsiness" back to 1, to study one scenario at a time. Obviously, when Helen is on her camping trip, she is not subject to her increased workload.



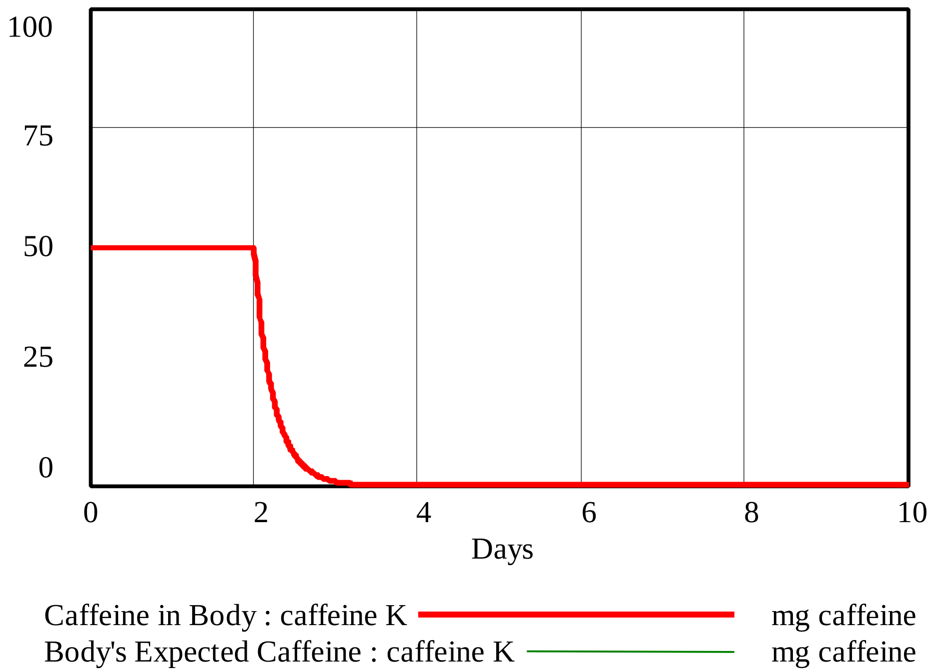
<u>Modified model equations:</u>

caffeine consumption = NORMAL CAFFEINE CONSUMPTION * effect of drowsiness on caffeine consumption * EFFECT OF CAMPING TRIP ON CONSUMPTION
Units: mg caffeine/Day
The amount of caffeine that Helen consumes every day.

EFFECT OF CAMPING TRIP ON CONSUMPTION = 1 – STEP(1,2)
Units: dmnl
The effect of the camping trip on Helen’s consumption of caffeine.

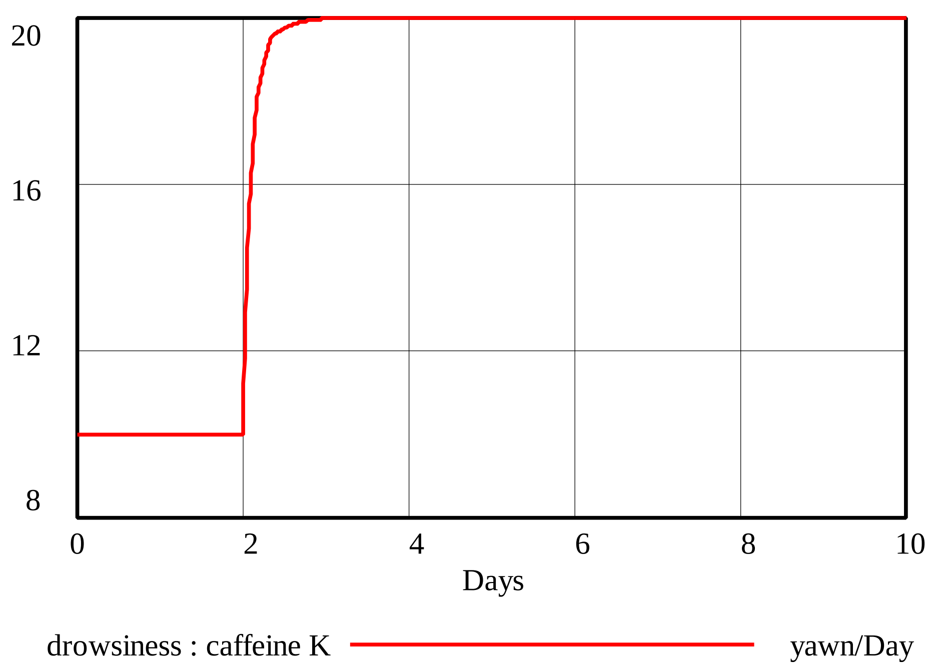
Actual caffeine in the body drops when Helen goes on the camping trip because her body is only disposing of, not consuming any caffeine. Expected caffeine, on the other hand, takes longer to go down because of a time delay. In fact, at the end of the 10 days, Helen’s body is still expecting more coffee than it gets, so the goal-gap structure is still driving the system towards equilibrium.

Actual and Expected Caffeine in Body, part K



The camping trip takes Helen off caffeine completely, suddenly increasing her drowsiness because her body cannot get the amount of caffeine it expects:

Drowsiness, part K

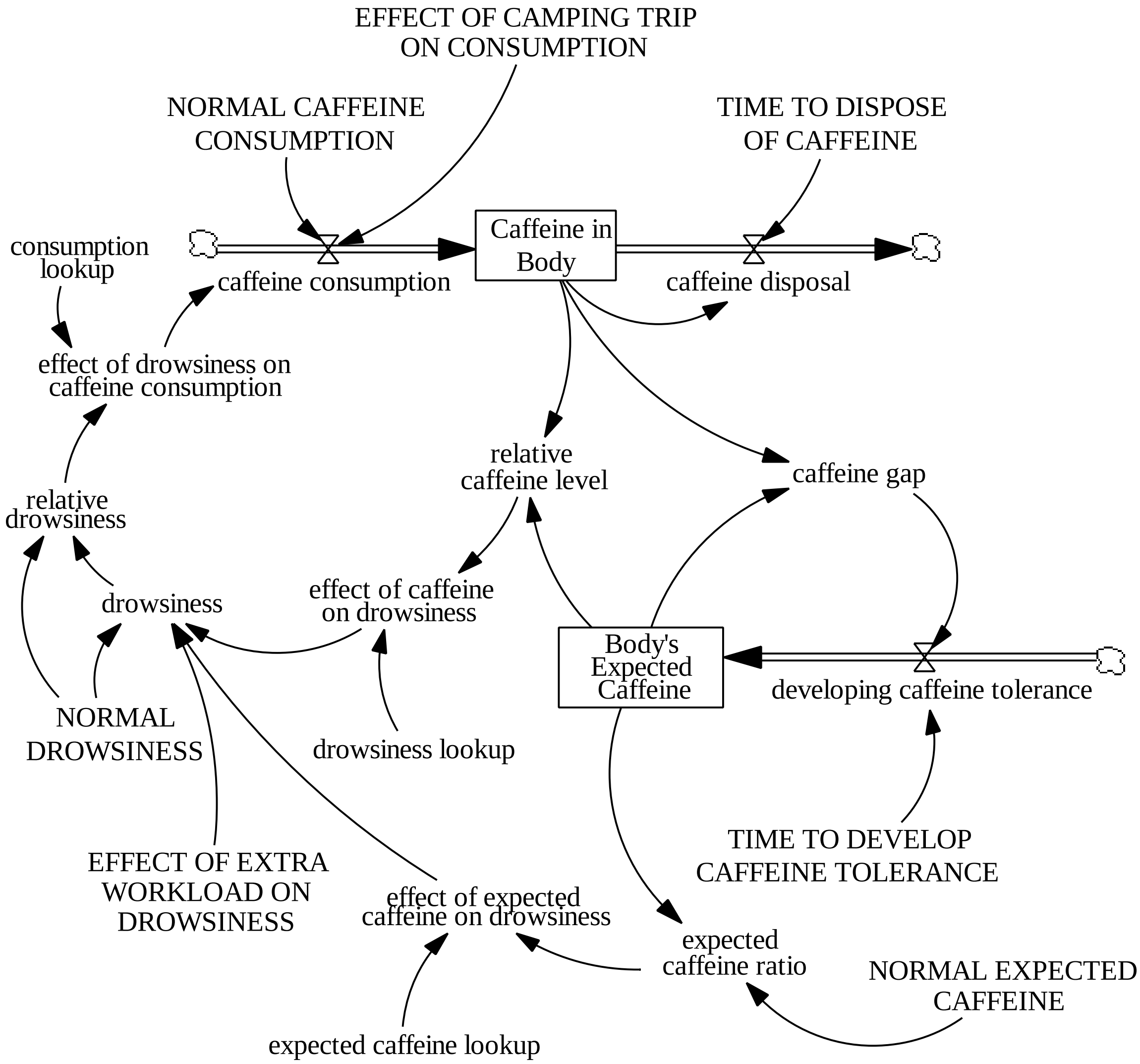


The behavior generated by the model is not realistic. Although both "Caffeine in Body" and "Body's Expected Caffeine" approach zero, the decline of "Body's Expected Caffeine" is much slower because of the longer time constant. Hence, "relative caffeine level" also approaches zero. Even for very long time periods, "Body's Expected Caffeine" remains more than twice the "Caffeine in Body." Because of the "drowsiness lookup" function, the model thus produces an equilibrium where Helen yawns 20 times a day even though her body expects almost no caffeine. This behavior is true if our assumption is correct, that is, if "drowsiness" depends on the "relative caffeine level," no matter what the "Body's Expected Caffeine" is. This is unrealistic. If the "Body's Expected Caffeine" is extremely small,[6] then even if "Caffeine in Body" is much lower and hence the "relative caffeine level" is close to zero, "drowsiness" should be close to "NORMAL DROWSINESS." The current model does not, however, depend on the true level of caffeine at all, only on the relative amounts.

To improve the model, one could define a normal value for the expected level of caffeine in the body, at which one's drowsiness is at the normal level. One could then formulate a lookup function that takes in the ratio of Helen's expected caffeine level to the normal expected caffeine level. The output of the lookup function would be such that when Helen's body expects less caffeine than the normal expected amount (for example, when her "Body's Expected Caffeine" drops to very low values during the camping trip), she is less drowsy than usual. The effect of this new lookup function would then balance the effect of the "drowsiness lookup," resulting in a lower level of "drowsiness" at low quantities of caffeine than in the current model. A possible improved model is as follows:

<u>Model diagram:</u>

[6] Also note that with the current formulation of the model, if "Body's Expected Caffeine" equals zero, the model will not work because of division by zero in the "relative caffeine level." Although this is a possible limitation of the model, the purpose of this model was to study the effects of caffeine after one has built up some caffeine tolerance, not the actual process of becoming addicted that could start with "Body's Expected Caffeine" at zero.



<u>Modified model equations:</u>

drowsiness = NORMAL DROWSINESS * effect of caffeine on drowsiness * EFFECT OF EXTRA WORKLOAD ON DROWSINESS * effect of expected caffeine on drowsiness
Units: yawn/Day
Helen’s actual drowsiness, measured by the number of times she yawns per day.

effect of expected caffeine on drowsiness = expected caffeine lookup (expected caffeine ratio)
Units: dmnl
The effect of expected caffeine on Helen’s drowsiness.

expected caffeine lookup ([(0,0) - (1,1)], (0,0.3), (0.25,0.65), (0.5,0.85), (0.75,0.95), (1,1))
Units: dmnl

The lookup function for the effect of expected caffeine level on drowsiness. The lookup function assumes that even if “Body’s Expected Caffeine” is zero and hence “expected caffeine ratio” is zero, Helen will still be somewhat drowsy. The lookup function also assumes that if “Body’s Expected Caffeine” is higher than “NORMAL EXPECTED CAFFEINE,” Helen will be as drowsy as usual.

expected caffeine ratio = Body’s Expected Caffeine / NORMAL EXPECTED CAFFEINE

Units: dmnl

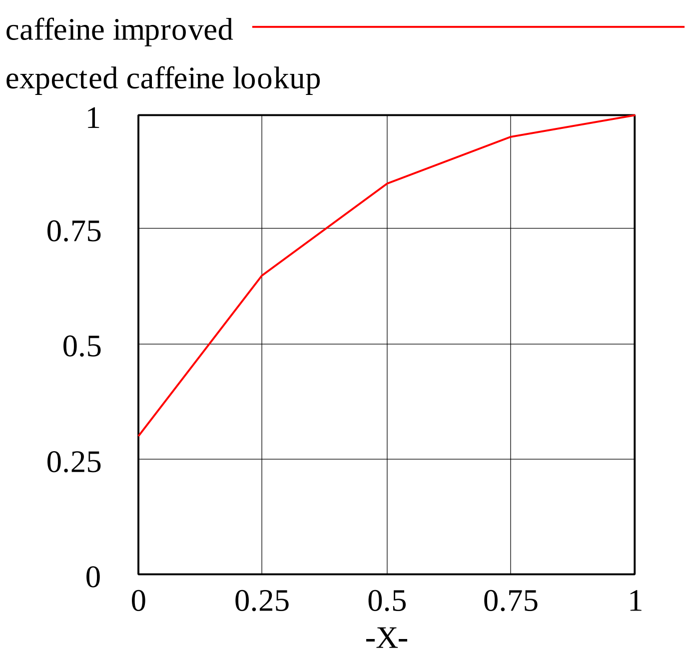
The ratio of Helen’s body’s expected caffeine level to a normal expected caffeine level.

NORMAL EXPECTED CAFFEINE = INITIAL (Body's Expected Caffeine)[7]

Units: Mg caffeine

The normal expected level of caffeine.

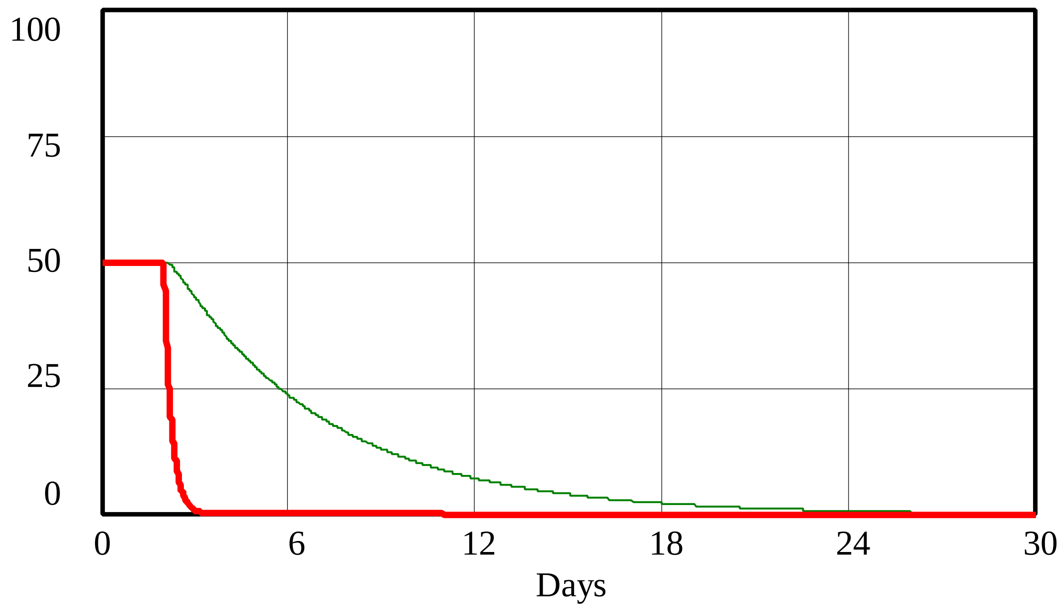
Graph of lookup function:



Model behavior:

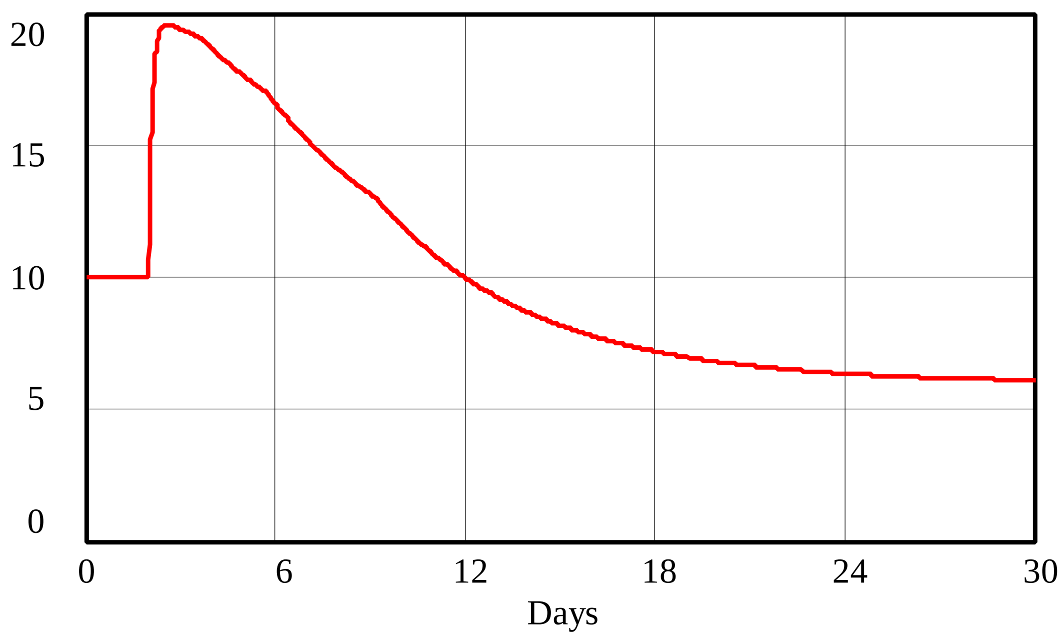
[7] The “INITIAL” feature is used instead of CONSTANT when the value of some reference variable needs to equal the initial value of a stock throughout the simulation. Thus, when you change the initial value of the stock, you need not update that constant’s value. Open the equation’s window for “NORMAL EXPECTED CAFFEINE.” On the left side of the window, pull down the menu titled “Type” and chose “Initial.” Then type “Body’s Expected Caffeine” into the equation box.

Actual and Expected Caffeine in Body, improved model



Caffeine in Body : caffeine improved — Mg caffeine
Body's Expected Caffeine : caffeine improved — Mg caffeine

Drowsiness, improved model



drowsiness : caffeine improved — yawn/Day

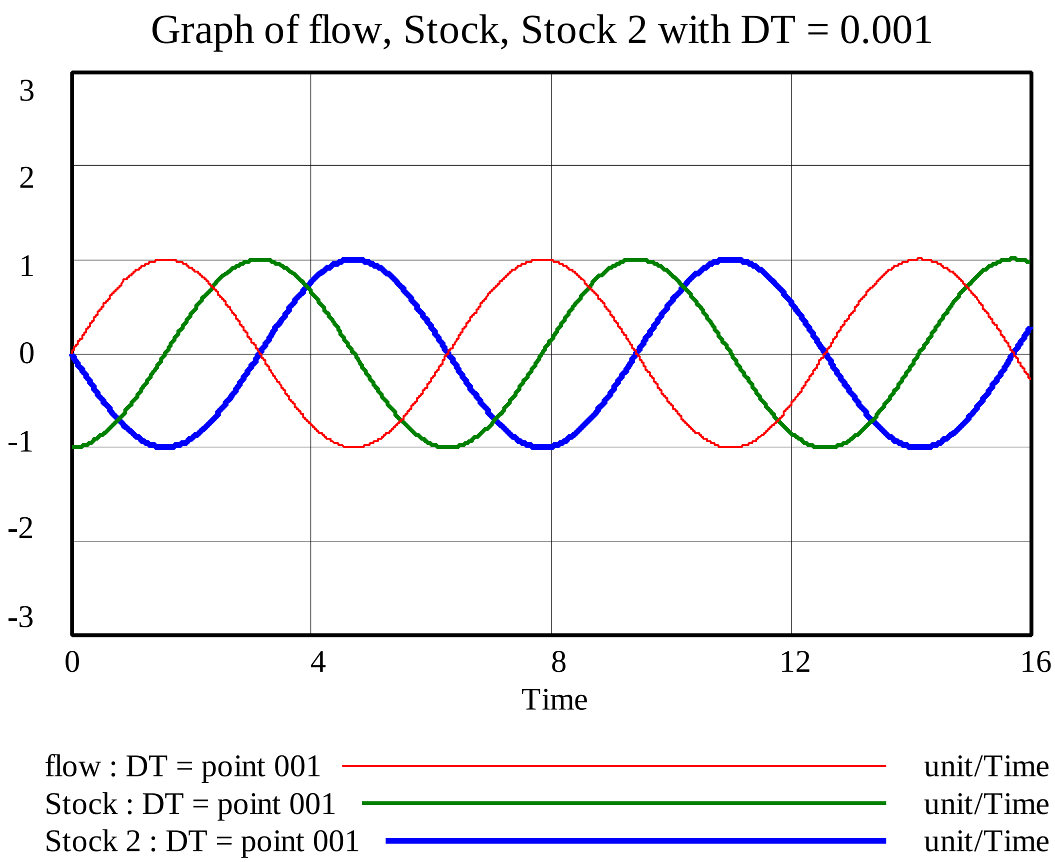
Appendix

While simulating a model, the time step acts as an accumulation and becomes a delay in the system, much like a stock. Therefore, a second-order system with two integrations and therefore a delay in two places may exhibit some characteristic behaviors of a fourth-order system because of the delay introduced by the time step. One such type of behavior as seen in this special case of two levels in a single loop is expanding oscillations. More general second-order systems that have a cross loop inside the outer loop of these exercises can exhibit either growing or decaying oscillation. In the real world, almost no systems exhibit perfectly sustained oscillations unless, like a clock, they have an energy source to compensate for frictions, shocks, and other exogenous effects.

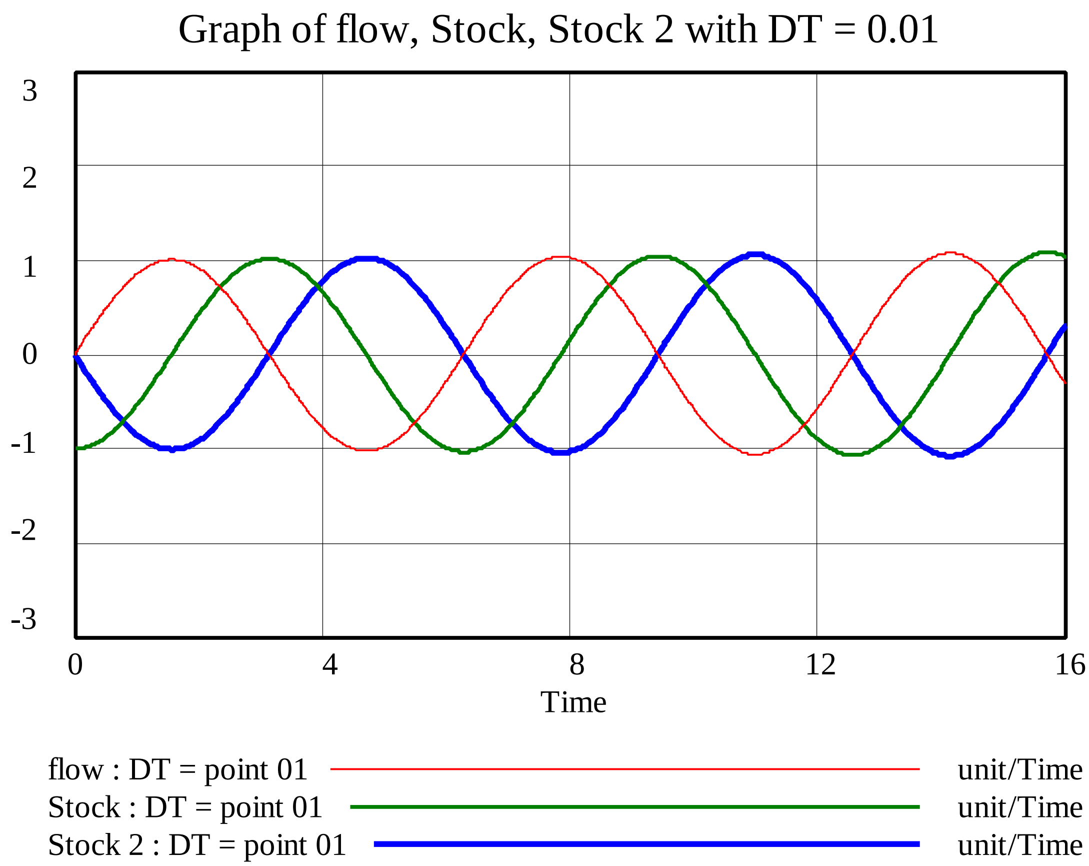
Decreasing the value of the time step can reduce the influence of the time step. Test by repeatedly dividing the time step by 2 until there is no longer an important effect. Do not make the time step too small because it will require more computer time, and, with extremely small steps can lead to roundoff errors (which occur when rates of flow are too small to be properly represented by the number of digits available in the computer.

The graphs below show the model used in exercise 1 when simulated with time steps of various values.

Example 1: DT = 0.001

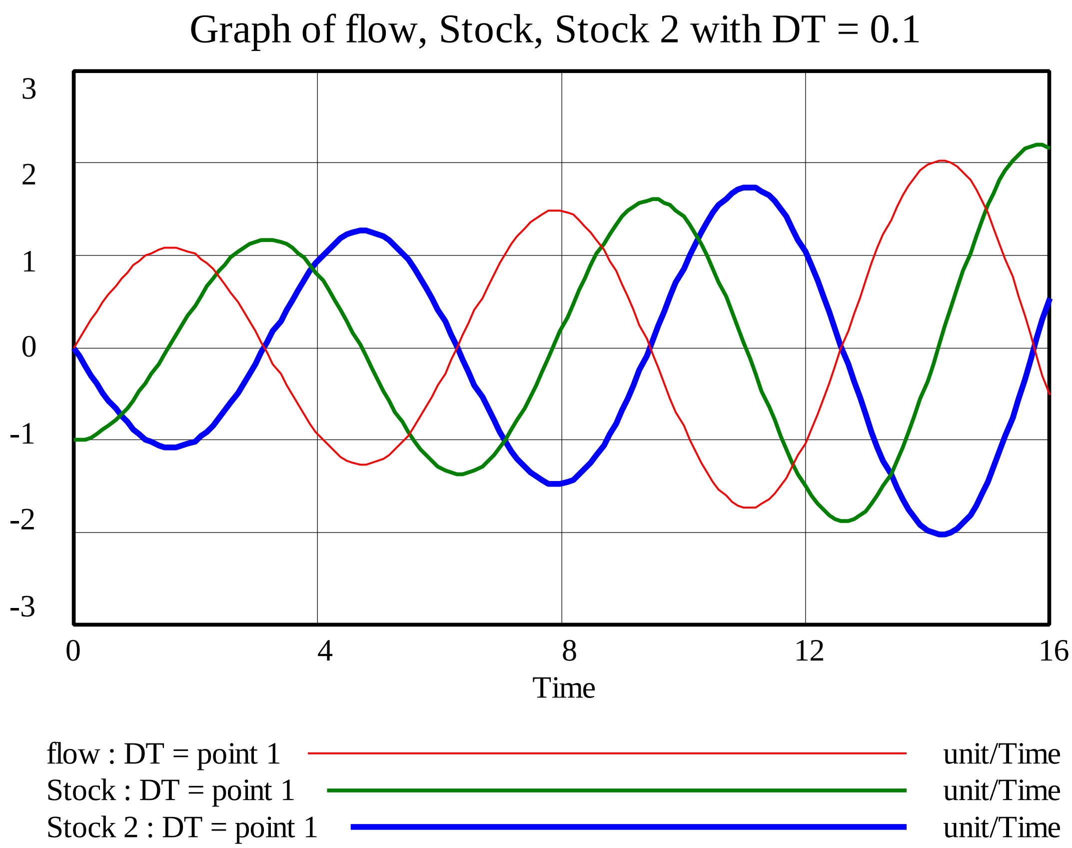


Example 2: DT = 0.01



Notice the slightly expanding oscillations.

<u>Example 3: DT = 0.1</u>



Notice the significantly expanding oscillations when DT = 0.1. Clearly, such a small DT is inappropriate.

The expanding oscillations are produced due to the computation process of the First-Order Euler Integration Method, which is normally used in system dynamics simulation. Higher-order integration methods exist to perform these calculation, but those methods lead to other subtle problems under certain circumstances. Hence, do not use any method other than the Euler First-Order Method.