



System Dynamics Group  
Sloan School of Management  
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System Dynamics II, 15.872  
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## Assignment 1 Structure and Behavior of Delays

Assigned: Monday 28 October 2013; Due: Wednesday 6 November 2013

*This assignment should be done in a team totaling three people and submitted digitally*

Delays are a critical source of dynamics in nearly all systems. Thus far in our modeling, however, we have represented delays in causal diagrams qualitatively. In this assignment you explore the structure and behavior of delays and test their responses to a range of inputs. You will also apply your knowledge of delays to an important real-world issue: the current glut of natural gas in the US. The assignment helps you understand the dynamics of delays so that you can use them appropriately in more complex models. The assignment also develops your skills in model formulation and analysis. **To do this assignment it is essential that you first read chapter 11 in the text.**

### A. Material Delays

- A1. Do the challenge on page 425 of the text (Response of Material Delays to Steps, Ramps, and Cycles). Note that you are asked to:
  - Sketch your intuitive estimate of the response of a first order material delay to the four test inputs shown in Figure 11-9.
    - To assist you, Figure 11-9 is reproduced on the following pages so you can sketch your estimates directly on the graphs by hand or digitally; you will need to submit your response digitally so you can take a picture of your hand-drawn graphs and paste it into your write up.
    - Your grade will not be affected by your answer to this part of the question. Please ***be honest*** and sketch your responses ***before*** simulating the model; this is important for building your intuition about delays.

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Originally prepared by John Sterman; last updated by JS and HR October 2013.

- denotes a question for which you must hand in an answer, a model, or a plot.
  - \* denotes a tip to help you build the model or answer the question.
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- A2. After sketching your intuitive estimates, build and simulate the model for the first-order material delay.
  - The structure and equations for a first-order delay are described in the text (section 11.2.4). Build the delay explicitly as a stock and flow structure shown in Figure 11-4 (that is, do not use Vensim's built-in function DELAY1).
  - Set the initial value of the stock of letters in transit so that the delay is always initialized in equilibrium, independent of the initial value of the input. In equilibrium, the stock in transit is unchanging, so the inflow and outflow to the delay must be equal. Solve this equation for the equilibrium value for the stock in transit, and enter this parametric expression as the initial condition for the stock in your model. Run your model with a few different constant input values to confirm that it begins and remains in equilibrium. You do not need to hand in the equilibrium run.
  - We have created a “test input generator” that will make it easy for you to generate the step, ramp, sine wave, and other test inputs you need to run the tests. Download the model, TestGen.mdl, from the course website and use it to generate the input patterns you need. The test input generator is fully documented and dimensionally consistent (see appendix 1).
  - To learn more about the parameters for the built-in functions used in the Test Input Generator (specified in the Appendix) and in the following exercises, use the pull-down “Help” menu in Vensim. Click on “Keyword Search...” and type in the function name (e.g., STEP), and hit Enter.
  - Note that the test generator has an initial value of one, so the exponential growth rate you need to replicate the test input in the graphs is 0.05/day. The ramp slope needed is 0.05/day.
  - Set the initial time to –5 days, the final time to 25 days, and the Time Step (dt) to 0.125 days. Be sure to save output every time step (in the **Time Bounds** tab under **Settings...**, in the **Model** menu).
  - To show the relationship between the input and output, define a custom graph that shows the input and output rate on the same graph, and with the same scale. Also use the strip graph tools to examine the behavior of the stock in transit.
  - It may be helpful to envision the delay as the post office, with the input representing the mailing rate, the output representing the delivery rate, and the stock in transit representing the letters in the post office system.
- After you run the model for each of the test inputs, compare the results to your intuitive estimates and comment briefly. Were your mental simulations correct? Why? / Why not? In particular,
  1. Consider the linear ramp. The response of any system to a shock consists of a *transient*, during which the relationship of input and output is changing, and a *steady state* in which the relationship between input and output is no longer

changing (even though the input and output might both be growing, for example). Does the output of the delay (e.g., the delivery rate of letters) equal the input (e.g., the mailing rate) in the steady state for the linear ramp? Explain.

2. How does the response to the sine wave depend on the delay time relative to the period of the cycle? In particular, as the delay time increases relative to the period of the cycle, what happens to the amplitude and timing of the output?

A3.  Next, repeat the steps above (*both* mental and formal simulation) for a third order material delay with the same average delay time of 5 days.

\* Build the delay explicitly as shown in section 11.2.5. Do not use the Vensim built-in function DELAY3. Figure 11-6 shows a second order material delay; the third order delay is analogous but contains three intermediate stocks of material in transit (see equation 11-4).

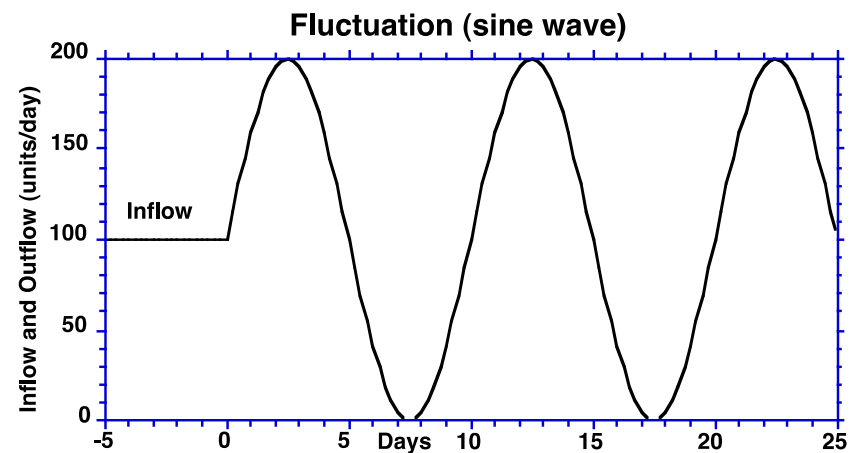
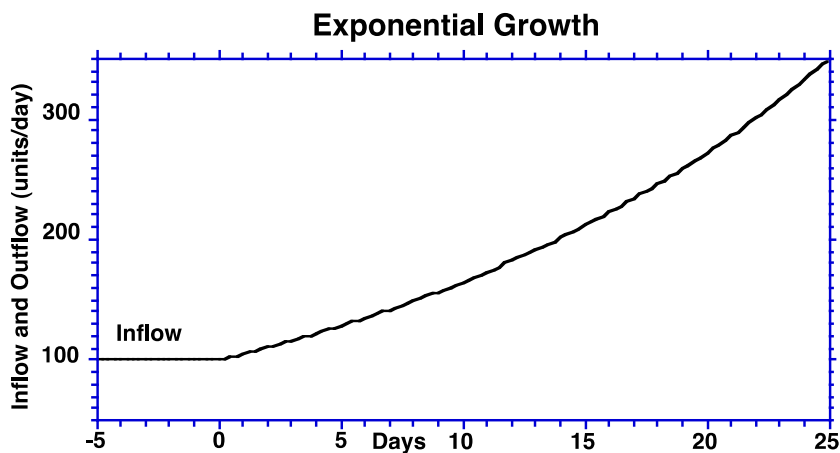
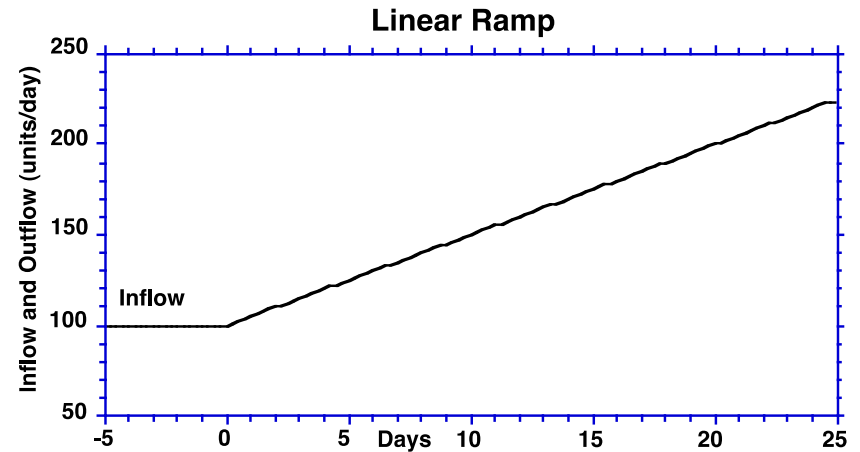
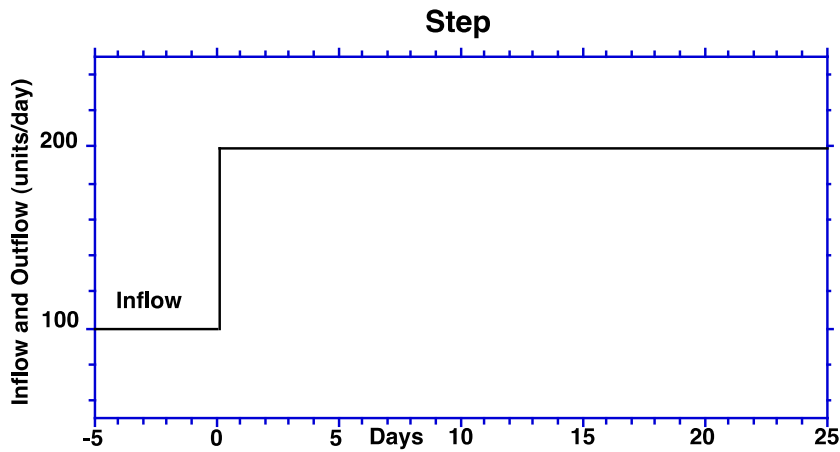
\* Build the third order delay in the same model as your first order delay and use the same input test generator. This allows you to compare the behavior of the third order delay to the first order delay.

A4.  Next, repeat the steps above (again *both* mental and formal simulation) for a pipeline delay with the same average delay time of 5 days.

\* Build the pipeline delay using the DELAY FIXED function in Vensim. The pipeline delay is described in section 11.2.3. To access the DELAY FIXED function from the variable definition dialog in VensimPLE, click on the tab labeled Functions; you will then see an alphabetical list of all the built in functions available in PLE. The syntax for the DELAY FIXED function is:

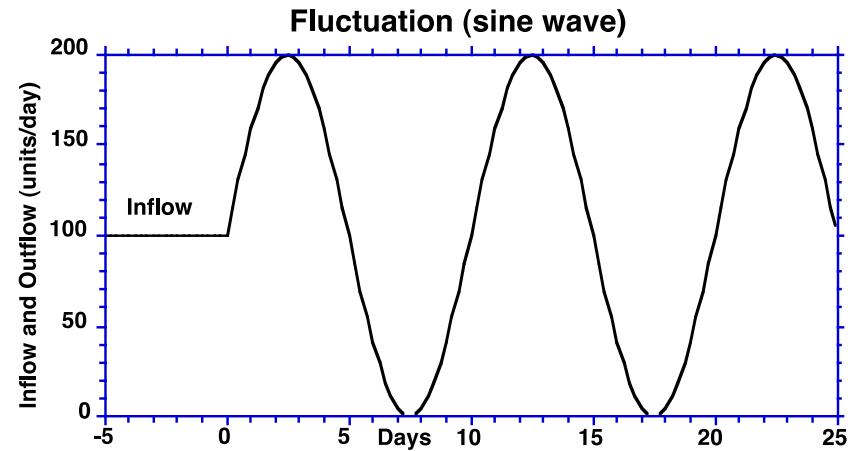
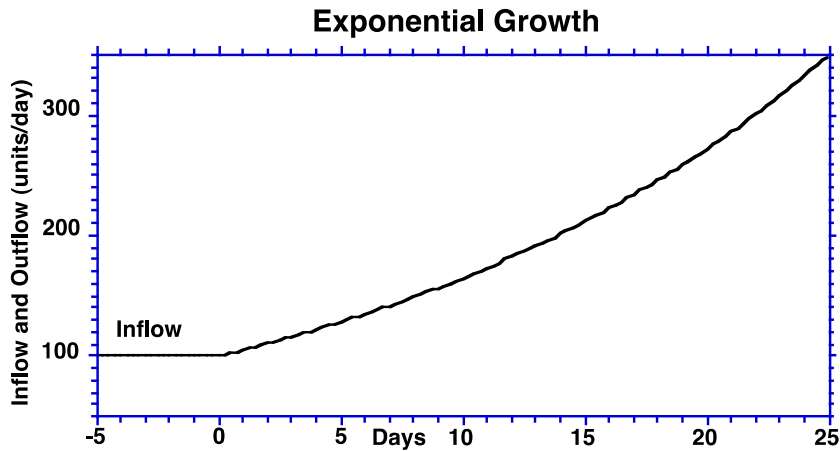
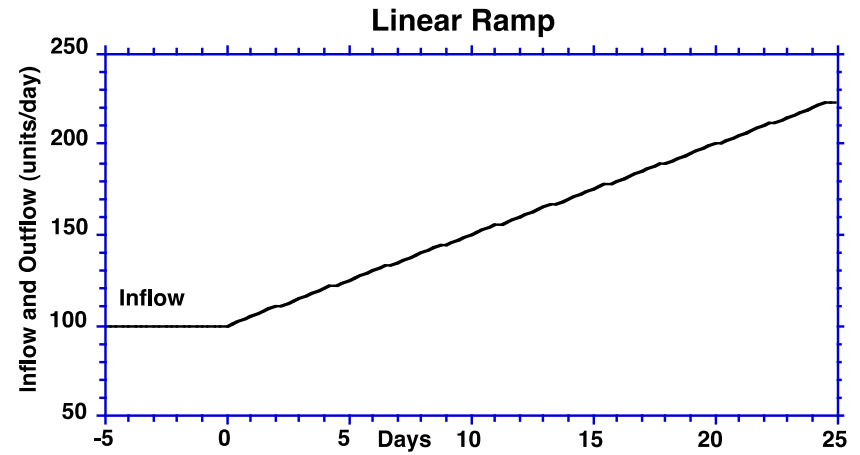
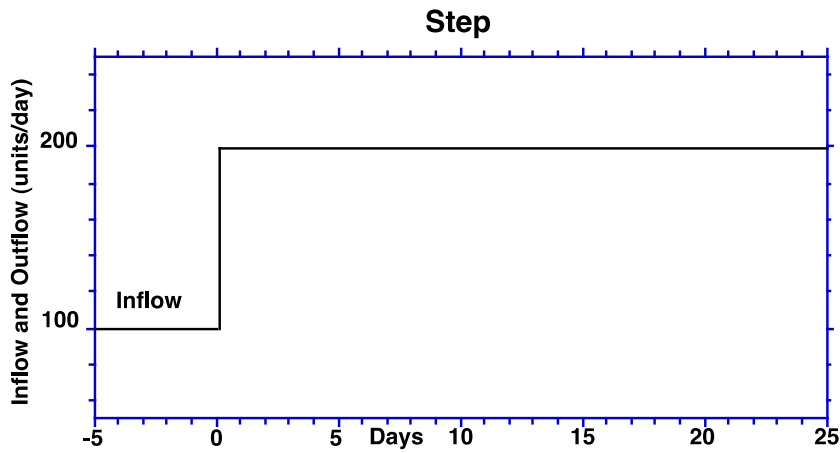
OUTPUT = DELAY FIXED(INPUT, DELAY TIME, INITIAL OUTPUT)

\* Build the pipeline delay in the same model as your other delays so you can easily compare the output of the three delay types.



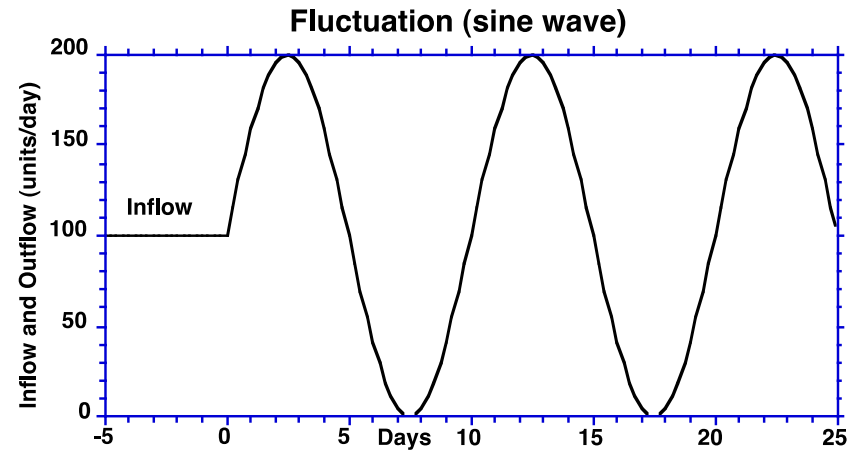
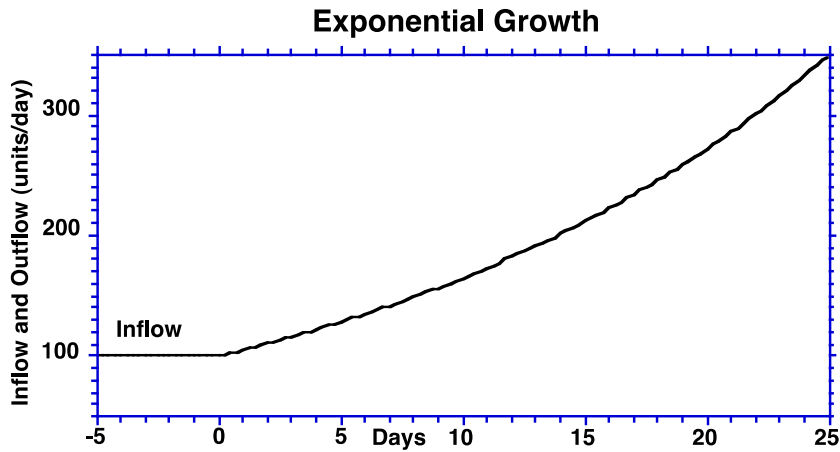
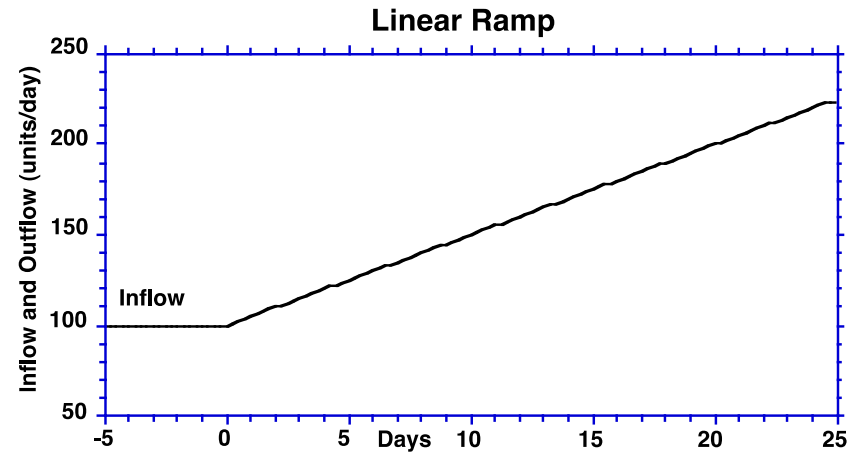
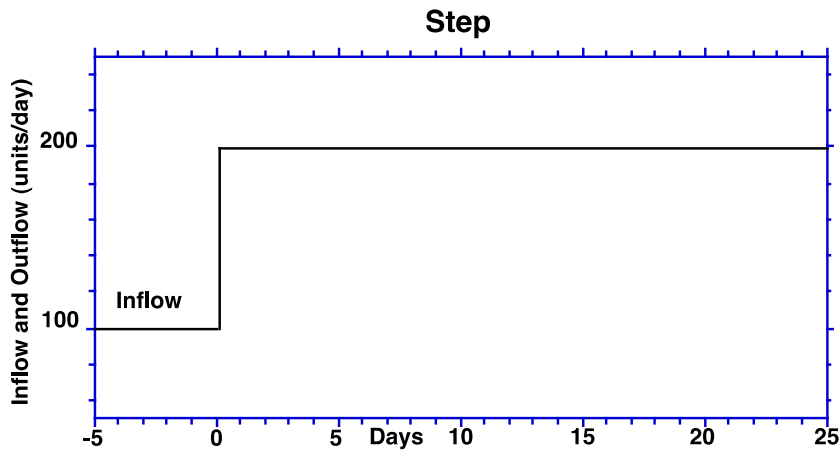
#### Four test inputs to a delay

*Before simulating the model*, sketch the response of a *first order material delay* to each of these inputs, assuming a 5 day average delay time.



#### Four test inputs to a delay

*Before simulating the model*, sketch the response of a **third order material delay** to each of these inputs, assuming a 5 day average delay time.

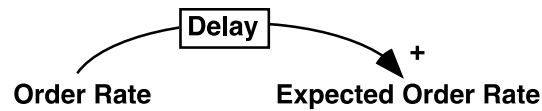


#### Four test inputs to a delay

*Before simulating the model*, sketch the response of a *Pipeline delay* to each of these inputs, assuming a 5 day average delay time.

## B. Information Delays

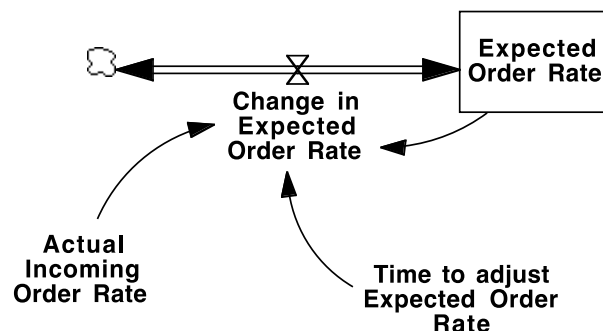
In the previous sections we considered material delays, where the input to the delay is a physical inflow of items to a stock of units in transit, and the outflow is the physical flow of items exiting the stock. Many delays, however, exist in channels of information feedback—for example, in the measurement or perception of a variable, such as the order rate for a product.



In this example, the expected order rate is management’s belief about the value of the order rate. The expected order rate lags behind the actual order rate due to delays in measuring and reporting orders, and due to the time it takes managers to update their beliefs about the order rate. The physical order rate does not flow into the delay; rather information about the order rate enters the delay. For reasons discussed in *Business Dynamics*, Ch. 11, a different structure is needed to represent these information delays.

As an example of an information delay, consider the way a firm forecasts demand for its products. Why does forecasting inevitably involve a delay? Firms must forecast demand because it takes time to adjust production to changes in demand, and because it is costly to make large changes in production. They don’t want to respond to temporary changes in demand but only to sustained new trends. A good forecasting procedure should filter out random changes in incoming orders to avoid costly and unnecessary changes in output while still responding quickly to changes in trends to avoid costly stockouts and lost business.

One of the most widely used forecasting techniques is called “exponential smoothing” or “adaptive expectations.” As discussed in section 11.3.1, “Adaptive expectations” means the forecast adjusts (adapts) gradually to the actual stream of orders for the product. If the forecast is persistently wrong, it will gradually adjust until the error is eliminated. The structure of an exponential smoothing delay is shown in Figure 11-10. Specifying this structure for the case of a demand forecast leads to the following:



The equation for the rate at which expectations adapt—the rate at which the forecast is revised—is:

$$d(\text{EOR})/dt = (\text{IOR} - \text{EOR})/\text{TEOR}$$

where

EOR = Expected Order Rate (widgets/month)

IOR = Actual Incoming Order Rate (widgets/month)

TEOR = Time to adjust the Expected Order Rate (months)

The “time constant” TEOR represents the time required, on average, for expectations to respond to a change in actual conditions. The forecast, EOR, is a *stock*, since the forecast is a state of the system, in this case representing a state of mind of the managers regarding what the future order rate will be. This stock remains at its current value until there is some reason to change it. In adaptive expectations, the stock (the managers’ belief about the future order rate) changes when there is a difference between the current order rate and their belief about the expected order rate. This structure is known as a first-order information delay, or ‘first-order exponential smoothing’.

Build a model to represent this forecasting procedure. Your model should include the equations above exactly. Do not add any additional structure.

- Assume TEOR is 6 months.
- Assume the order rate IOR is exogenous. Use the Test Input Generator in the Appendix to determine the incoming order rate. You will have to change the units for time in the test generator from days to months.
- Set the initial value of the forecast to 100 widgets/month.
- Use a simulation time step of 0.25 months and simulate the model for 60 months.
- Create a custom graph that traces the behavior of both IOR and EOR.

- B1. Run your model with a random input to demand. Set the Noise Standard Deviation to 0.50, and the Noise Start Time to 5 months. That is, the noise should have a standard deviation of 50% of the initial input.

To filter out random variation in incoming orders, should the adjustment time be long or short?

- B2. Consider the response to a step input. Set the Step Height to 0.50 and the Step Time to 5 months.

To respond quickly to a permanent change in incoming orders should the adjustment time be long or short?

- C. *0 points--unless you don't document your model, in which case the grader may deduct an arbitrary number of points.* Hand in the dimensionally consistent and documented Vensim model files (.mdl) along with your submission (including the first order, third order, and pipeline material delays, and for the information smoothing delay).



## D. Delays in Action: the Natural Gas Glut

Now let's apply your knowledge of delays to model an important issue: the dynamics of natural gas supply and prices.

Hydraulic fracturing (fracking) has led to a boom in exploration for, and production of, natural gas. Modern fracking was introduced around 1998. Fracking suddenly made it possible to find and produce large quantities of gas that were previously too expensive or technically unrecoverable. As shown in the graphs below, drilling activity in the US boomed, reaching a peak around 2006-2008 about three times higher than the average for the 1980s-1990s. Proved reserves soared. However, the demand for gas is quite inelastic in the short run. Consumption has been growing modestly and did not increase nearly as much as production capacity.

As a result, the price of natural gas plunged. As seen below, real prices (measured in constant 2010 dollars) fluctuated from about \$6 to over \$10 per thousand cubic feet (tcf) from about 2005 through 2008. Since then, however, prices have tumbled, reaching a low of less than \$2/tcf, an enormous decline (the price has rebounded to about \$3/tcf today). Some of the decline can be attributed to the recession and weak economy since 2008. But as you can see in the graph of consumption, demand did not fall much from 2008-2010. The main source of the price crash lies on the supply side: The US is awash in gas—At \$2-3/tcf, gas is now so cheap that many exploration and development firms are losing money. As the New York Times reported (20 October 2012; the full article is appended to this assignment):

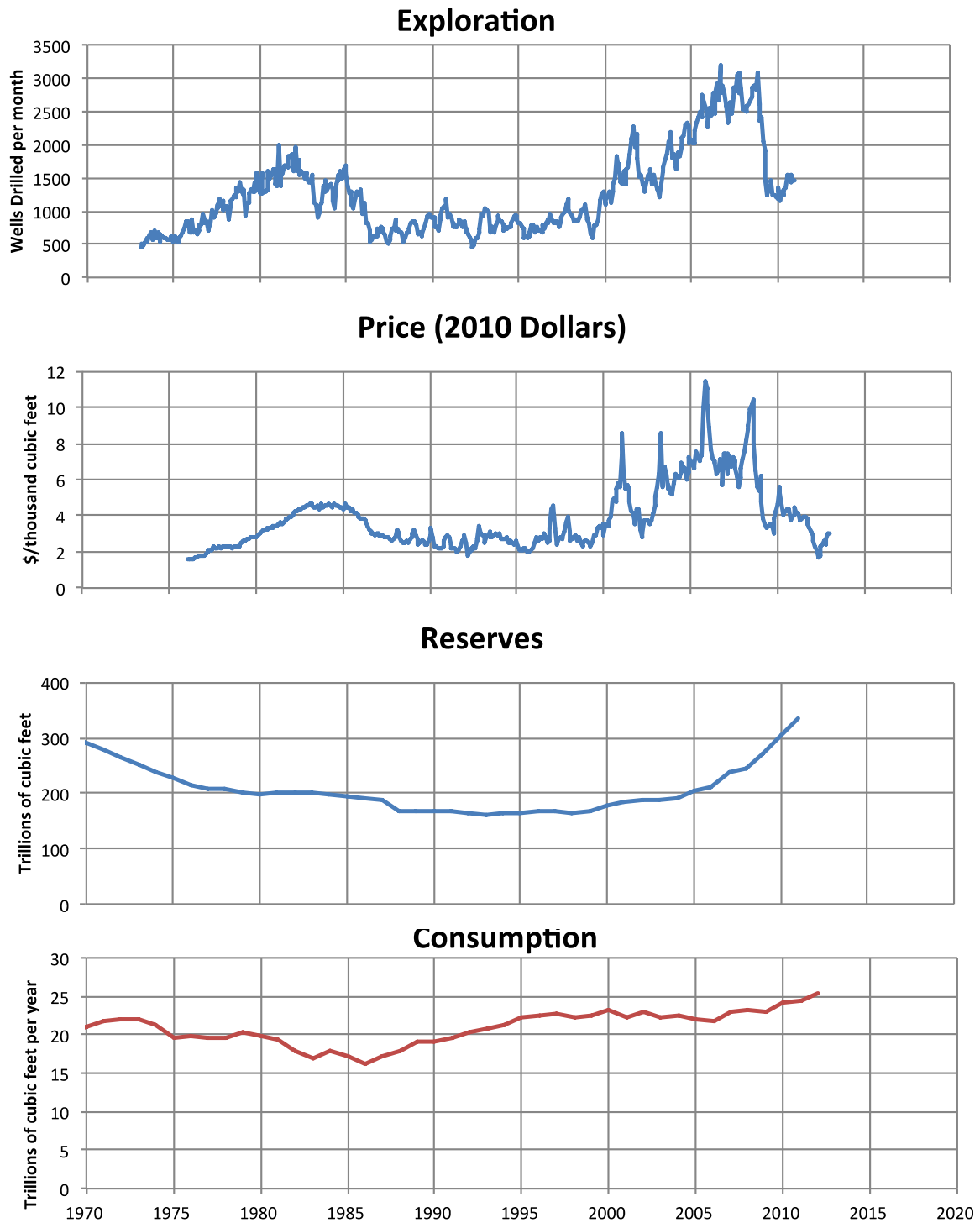
“...while the gas rush has benefited most Americans, it's been a money loser so far for many of the gas exploration companies and their tens of thousands of investors.

The drillers punched so many holes and extracted so much gas through hydraulic fracturing that they have driven the price of natural gas to near-record lows. And because of the intricate financial deals and leasing arrangements that many of them struck during the boom, they were unable to pull their foot off the accelerator fast enough to avoid a crash in the [price of natural gas](#), which is down more than 60 percent since the summer of 2008.

...most of the industry has been bloodied — forced to sell assets, take huge write-offs and shift as many drill rigs as possible from gas exploration to oil, whose price has held up much better.

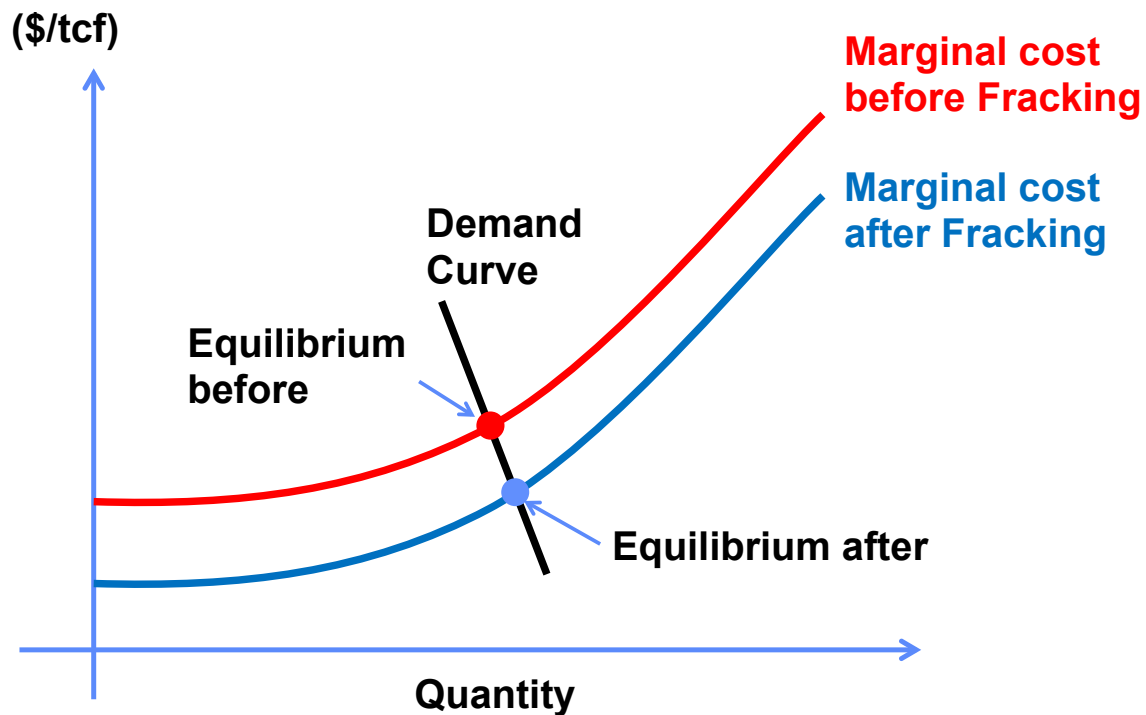
Rex W. Tillerson, the chief executive of Exxon Mobil, which spent \$41 billion to buy XTO Energy, a giant natural gas company, in 2010, when gas prices were almost double what they are today, minced no words about the industry's plight during an appearance in New York this summer.

“We are all losing our shirts today,” Mr. Tillerson said. “We're making no money. It's all in the red.”



The US natural gas industry. From top: exploration activity (wells drilled/month); spot price (2010 \$/tcf, Henry Hub. Henry Hub is an important gas pipeline distribution center in Louisiana; Henry Hub spot and futures contracts are traded in commodity markets including the NY Mercantile Exchange [NYMEX]. Prices deflated using the Producer Price Index [PPI]); proved reserves (trillion cubic feet); total US consumption, in trillion cubic feet/year. Source: Energy Information Administration, US Department of Energy.

We can conceptualize the impact of fracking technology as a shift in the long run supply curve that increased the quantity of gas that can be produced at a given price (a hypothetical supply curve is shown in the graph below). In the short run, fracking lowered the marginal cost of developing a new well below the market price, creating excess profits and a huge incentive to drill. Standard economic theory suggests (1) price should fall to a new equilibrium at the new, lower marginal cost, and (2) exploration should expand capacity until the marginal cost of gas once again equals the market price. Profits would rise until this happens and then return to normal levels, but not drop below normal. However, as shown in the data and described above, drilling expanded too much and too long, resulting in a glut and prices significantly below marginal cost for many producers. The industry dramatically overshot equilibrium. How did this happen? In this section, you will use your knowledge of delays to build a simple model of the exploration process.



Note on model boundary: To keep this assignment simple, the model is not intended to be a complete representation of the market. In particular, we will focus on the supply side, assuming (for simplicity) that demand is constant. We will also use a simple model for price determination and market clearing; a more sophisticated approach is described in *Business Dynamics*, Ch. 13.2.12 and Ch. 20.2.6. Further, although gas is a nonrenewable resource, the model omits the depletion of the resource base. Finally, natural gas production and combustion generate methane and CO<sub>2</sub> emissions, the two most important greenhouse gases contributing to global climate change. These impacts are also omitted. We also omit environmental issues around fracking including seismicity, water pollution, air quality, and other local environmental impacts.

**Causal Structure of the Market:** The diagram below shows a causal diagram of the supply side of the natural gas market. There are two main stocks, Production Capacity and Capacity Under Development. Production capacity (measured in million cubic feet/year of potential production from existing wells), increases when new exploration and development projects are completed, adding to the stock of producing wells, and decreases when the wells and associated production and distribution equipment reach the end of their useful life and are decommissioned. There are important time delays in the reaction of exploration and production to changes in market and technological conditions. When entrepreneurs believe that it is profitable to develop new resources, they initiate new development projects. The flow of new projects increases the stock of projects under development. After some time, these projects are completed and the number of producing wells increases. Capacity under development captures all exploration and development projects that have been initiated but not yet completed, including capital budgeting and financing, planning, site location, lease acquisition, acquisition of drill rigs, and the time required to carry out the actual drilling. For simplicity, these stages are not explicitly separated.<sup>1</sup>

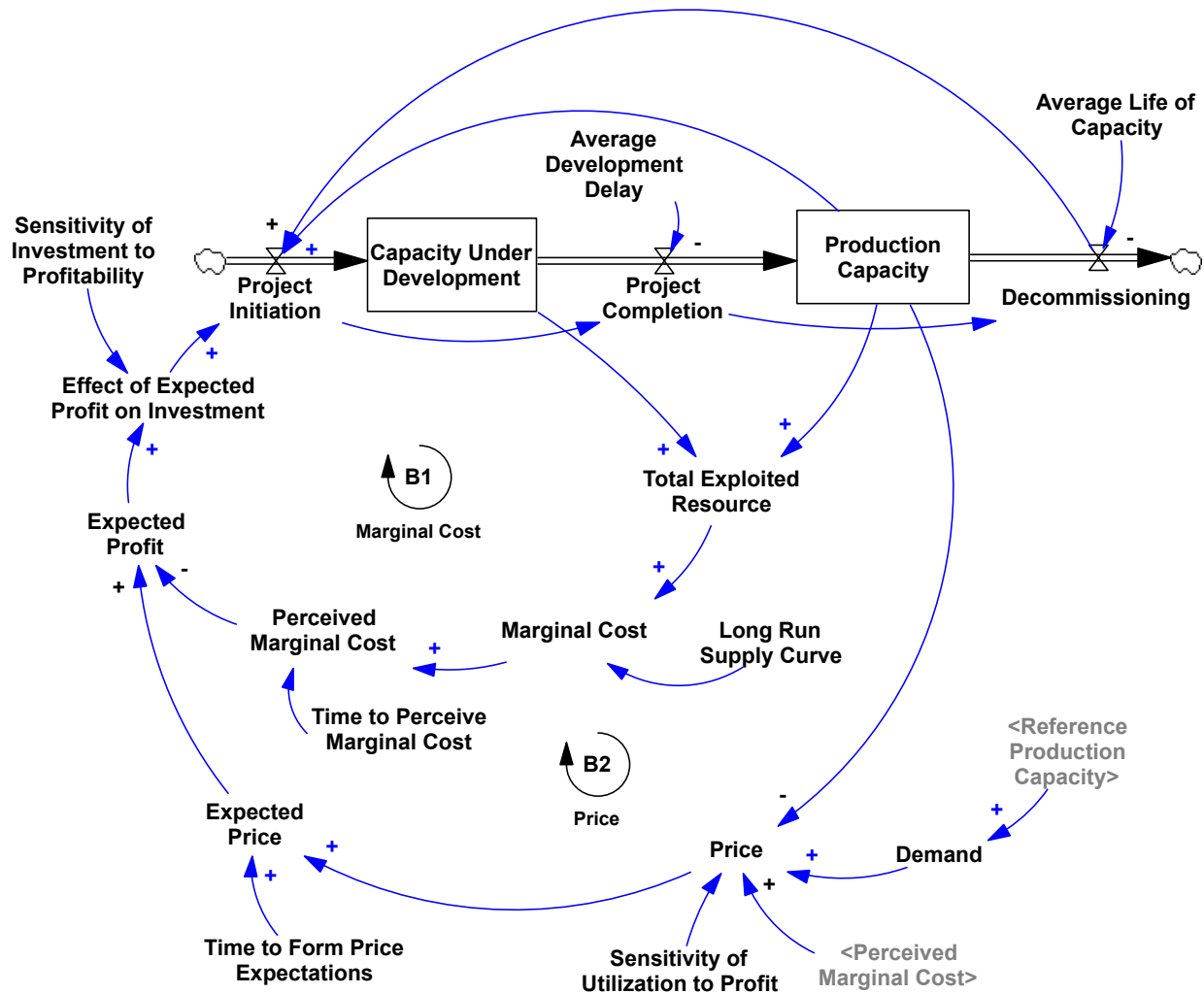
The decision to expand or contract drilling activity is based on entrepreneurs' and energy company managers' beliefs about whether new exploration will be profitable. If they believe that the price of gas (in the future, when the wells they begin to develop now will begin to produce) exceeds the marginal cost of development, they will expand drilling activity. If they believe future prices will be below marginal costs they reduce the number of projects they start. Marginal costs depend on the long run supply curve: the larger the total exploited resource, the higher the marginal cost will be. These relationships create the balancing Marginal Cost feedback: an increase in profitability leads to more exploration and development, raising the marginal costs of the next development prospects until profitability drops enough to stop further development. Note however that it takes time to assess the marginal cost of new wells. Each region, geologic zone, and region differs, and the costs of new technologies are uncertain. Although fracking lowered the marginal cost of development in many places, its benefits were not immediately obvious: Developers had to gain enough experience with the technology to update their prior beliefs about costs. Hence there is a lag in the response of developers' perceptions of marginal cost to changes in actual costs.

The second feedback in the diagram shows the determination of spot prices. Prices respond to the balance of demand and production capacity. When there is excess capacity, prices fall, causing the most expensive wells to be shut in. Price, and utilization, continue to fall until price reaches a level that clears the market. The result is the balancing Price feedback, shown as B2 in the diagram. If profitability is high, exploration increases, eventually increasing production capacity. Prices then fall, eroding profitability.

Expected profitability depends not on the current spot price, but on the expected future price. Energy prices are notoriously volatile—a rise in price today, or this month, may not persist long enough to justify an expansion in drilling. Entrepreneurs tend to wait and see whether prices will stay high before adjusting their price expectations upward. Research shows that people's price expectations are strongly conditioned on past prices (see *Business Dynamics*, Chapter 16).

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<sup>1</sup> A more detailed model would recognize that some wells lead to large discoveries while others are dry holes. For the purpose of this model we aggregate over all development prospects. That is, we assume developers account for the expected value of well productivity and the probability of dry holes when assessing expected profitability.



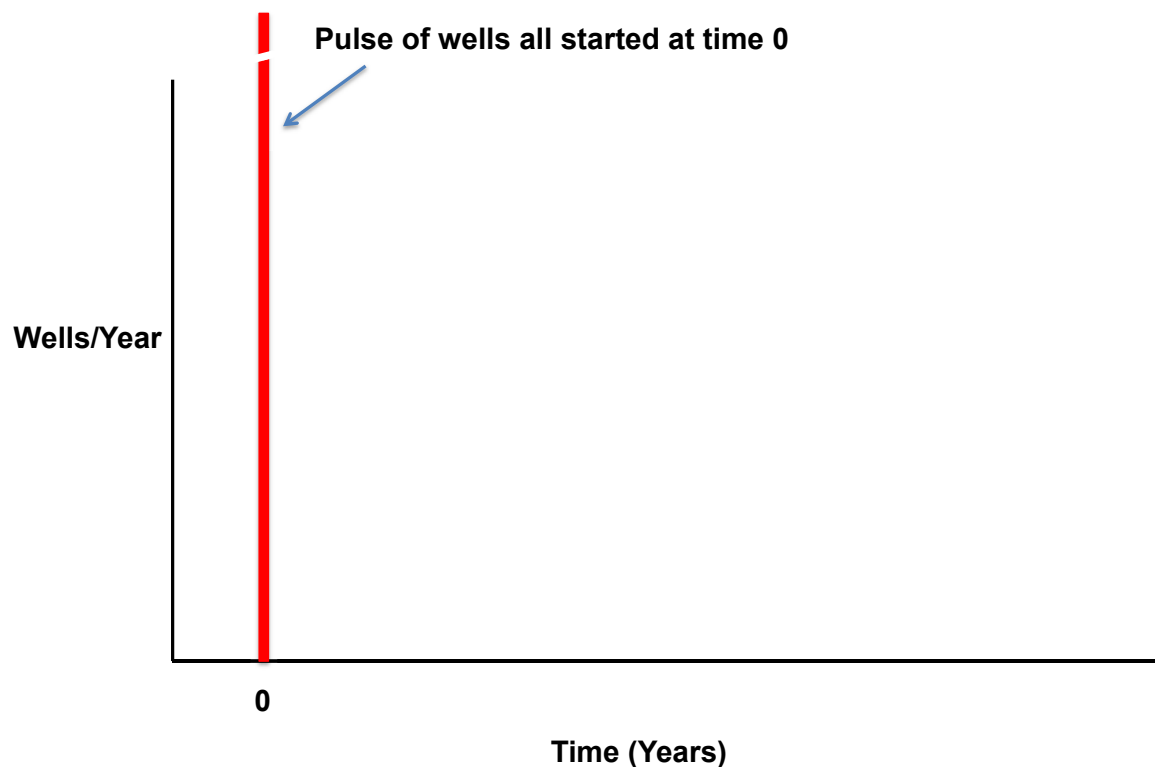
### Tasks:

□ D1. The causal diagram above shows several delays. These are:

- Time to form price expectations** (affects the **Expected Price**)  
(the average time required for developers to adjust their expectations regarding the future price as prices vary);
- Time to perceive marginal costs** (affects **Perceived Marginal Cost**)  
(the average time required for developers to adjust their beliefs about the marginal costs of exploration and development to new information);
- Average development delay** (affects **Project Completion**)  
(the average lag between the decision to develop new wells and project completion);
- Average life of capacity** (affects **Decommissioning**)  
(the average useful life of a well and associated production equipment).

Using your best judgment, estimate the average duration of each delay, in years (or fractions of years). **Do not do any research or gather any data to respond:** we are interested in your intuitive estimate of the average delay time for each of these processes. Provide a brief (1-2 sentence) justification for your estimate. Hint: Consider the relationship among these delays, particularly the first three. Your grade for this part does not depend on the estimate you provide, but on the clarity of your explanation.

□ D2. Consider the shape of the response of the development delay. Using the graph below, sketch the shape of the project completion flow to a pulse in project initiation. That is, if a thousand new wells were started all at once, what would the pattern of completions look like as time unfolds? Provide a time axis for your sketch. Briefly explain the shape you choose. See *Business Dynamics*, Ch. 11.2 and Figure 11-2 for an example and discussion of the relevant considerations.



□ D3. Download and open the model, Natural Gas Market.mdl from the course website. The model corresponds to the causal diagram above. The equations and units of measure are all specified for you, with the exception of the formulations for the delays. Given your responses to the questions above, specify the equations for perceived costs, expected price, and the development delay. **Hint: Pay particular attention to the type of delay (material or information), the average duration of the delay, and the order of the delay.** Do not agonize over the order of the delays: the main choices, as described in *BD* Ch. 10, are first order, intermediate order, and a pipeline delay. You can use the built-in third order delay functions in Vensim if you want to specify an intermediate order delay.

D4. Provide documentation for your formulations (enter the explanation in the comment field in the equations for the delays you have specified). Check to be sure your model is dimensionally consistent.

D5. Simulate the model with your delay formulations and values. The model is set up to begin in equilibrium. The model is set up to simulate the introduction of fracking. To do so, the long run supply curve shifts down by 20% in year 0 (see the Marginal Cost view in the model). Note: the simulation begins in year -2 and runs through year 40, with a time step of 0.015625 years (1/64 of a year, or a little less than 1 week).

In your write-up, provide a screenshot of the Dashboard view in the model showing the response of the system to the change in marginal costs with your estimated parameters. What happens, and why?

D6. Explore how the behavior of the market and exploration changes as you vary the duration of the time delays (use Synthesim and the sliders for each delay time). Briefly explain the impact of changes in the delays on the response.

D8. Briefly explain the policy implications of your results. Why has the introduction of fracking led to a glut of natural gas and steep losses for exploration and development firms? What do you think is likely over the next decade or more in terms of exploration, demand, and prices?

E. *0 points--unless you don't document your model, in which case the grader may deduct an arbitrary number of points.* Hand in the dimensionally consistent and documented Vensim model file (.mdl) along with your submission.

**F. What to hand in and how to submit your work**

Write up your responses to the questions above in a word (.docx) document. In addition, you need to submit the various Vensim model (.mdl) files called for in parts C and E.

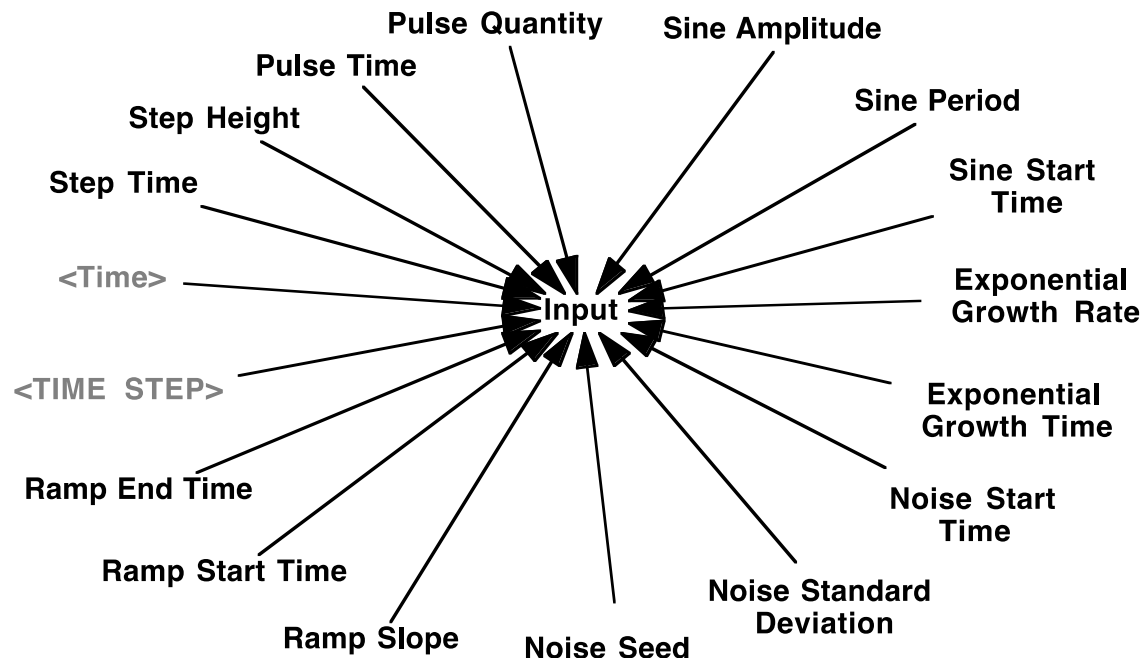
Upload your team's assignment by 5 PM on November 6<sup>th</sup>. Submit your assignment as a single .zip file including your response document and models.

Name files with your team's name, and for multiple files of the same type, the assignment section, e.g. "*Team21.docx*", "*Team21-C.mdl*", all submitted as part of "*Team21.zip*".

## Appendix 1: Test Inputs

System dynamics modeling software provides functions for the most commonly used test inputs: step, pulse, ramp, sine wave, exponential, and uncorrelated (or “white”) noise, etc. These inputs are not necessarily intended to correspond to anything that really happened in the past or that will happen in the future. Rather, the inputs are designed to help you easily explore the dynamics of a model. With a very simple input, it is easy to see the dynamics generated by the model. More complicated input patterns, such as actual historical data, make it difficult to isolate the behavior generated by the model’s structure from the input pattern. Once the dynamics of the structure are understood, it is usually possible to grasp how the structure will behave with more complicated inputs such as the actual historical input data.

A useful “Test Input Generator” is provided by the following structure. Note: When using this in other models, be sure to check the units for time and modify if necessary, along with the default parameters.



### Equations:

```

Input=
1+STEP(Step Height,Step Time)+
(Pulse Quantity/TIME STEP)*PULSE(Pulse Time,TIME STEP)+
RAMP(Ramp Slope,Ramp Start Time,Ramp End Time)+
STEP(1,Exponential Growth Time)*(EXP(Exponential Growth Rate*Time)-1)+
STEP(1,Sine Start Time)*Sine Amplitude*SIN(2*3.14159*Time/Sine Period)+
STEP(1,Noise Start Time)*RANDOM NORMAL(-4, 4, 0, Noise Standard
    Deviation, Noise Seed)
~ Dimensionless
~ The test input can be configured to generate a step, pulse, linear
ramp, exponential growth, sine wave, and random variation. The initial value

```



of the input is 1 and each test input begins at a particular start time. The magnitudes are expressed as fractions of the initial value.

Step Height=0

- ~ Dimensionless
- ~ The height of the step increase in the input.

Step Time=0

- ~ Day
- ~ The time at which the step increase in the input occurs.

Pulse Quantity=0

- ~ Dimensionless\*Day
- ~ The quantity added to the input at the pulse time.

Pulse Time=0

- ~ Day
- ~ The time at which the pulse increase in the input occurs.

Ramp Slope=0

- ~ 1/Day
- ~ The slope of the linear ramp in the input.

Ramp Start Time=0

- ~ Day
- ~ The time at which the ramp in the input begins.

Ramp End Time=1e+009

- ~ Day
- ~ The end time for the ramp input.

Exponential Growth Rate=0

- ~ 1/Day
- ~ The exponential growth rate in the input.

Exponential Growth Time=0

- ~ Day
- ~ The time at which the exponential growth in the input begins.

Sine Start Time=0

- ~ Day
- ~ The time at which the sine wave fluctuation in the input begins.

Sine Amplitude=0

- ~ Dimensionless
- ~ The amplitude of the sine wave in the input.

Sine Period=10

- ~ Day
- ~ The period of the sine wave in the input.

Noise Seed=1000

- ~ Dimensionless
- ~ Varying the random number seed changes the sequence of realizations for the random variable.

Noise Standard Deviation=0

- ~ Dimensionless
- ~ The standard deviation in the random noise. The random fluctuation is drawn from a normal distribution with min and max values of +/- 4. The user can also specify the random number seed to replicate

simulations. To generate a different random number sequence, change the random number seed.

```
Noise Start Time=0
~ Day
~ The time at which the random noise in the input begins.
```

```
*****
. Control
*****~
Simulation Control Parameters
|
```

```
FINAL TIME = 25
~ Day
~ The final time for the simulation.
```

```
INITIAL TIME = -5
~ Day
~ The initial time for the simulation.
```

```
SAVEPER =
TIME STEP
~ Day
~ The frequency with which output is stored.
```

```
TIME STEP = 0.125
~ Day
~ The time step for the simulation.
```

### **A Note on Random Noise**

The random input is useful to simulate unpredictable shocks. The `RANDOM_NORMAL` function in Vensim samples from a normal distribution with parameters the user specifies. The function has the following syntax:

`RANDOM_NORMAL (min, max, mean, std dev, seed)`

Vensim uses a default random number “seed.” You can specify a different seed by defining a constant called “Noise Seed” in your model and setting it equal to some value (e.g. Noise Seed = 1000). Vensim generates a single random sequence for any given seed. Let’s say the sequence is: 0.500, 0.213, 0.678, 0.932, 0.340, 0.015. If there is a single random number function in the model it will simply yield the random sequence. If there are two or more random functions, the functions will take turns accessing the sequence. For example, if you have two functions, the first will yield 0.5, 0.678, 0.34; and the second will yield 0.213, 0.932, 0.015. If you run two simulations with the same seed, you will get exactly the same sequence of random numbers. This is important so that you can compare two runs with different policies and be sure the differences in behavior are due only to the policies and not to different realizations of the random number generator. When you do want to examine runs with different realizations of the random process, you need to change the value of the random number seed.

Note also that the use of a function such as `RANDOM_NORMAL` means a new random number is selected every time step. Cutting the time step in half would then double the number of random shocks to which the model is subjected, and increase the highest frequency represented in the random signal. This is generally not good modeling practice. In realistic models, one must not only select the standard deviation of any random processes, but also specify its frequency spectrum (or, equivalently, the autocorrelation function). Failure to do so can lead to spurious results and make your model overly sensitive to the time step. These issues are discussed in Appendix B.

### **A Note on the Pulse Function**

The pulse function is used to simulate the effect of instantaneously adding a fixed quantity  $Q$  to a variable. To ensure the entire quantity is added all at once (within a single time step, or  $DT$  [delta time]), the duration of the pulse is set to the smallest interval of time in the model, that is, to the time step  $DT$ . The height of the pulse is then the quantity to be added divided by the time step in the model,  $Q/DT$ . The inflow increases by the height of the pulse and remains at the higher level for one time step, so that the total quantity added to the accumulation is  $(Q/DT)*DT = Q$  units.

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