# *Managing Demand and Sales Dynamics in New Product Diffusion Under Supply Constraint*

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# Motivation

**(Newspaper clipping about Apple Computer's worldwide launch of the iPod Mini to strong demand)**

- How will limited supply affect overall demand?
- How will the competition's reaction affect demand?
- How long should Apple delay the global launch?
- Should Apple have delayed original launch?

# Agenda

- Introduction
- Overview
- Related Literature
- Methodology
  - Model Formulation
  - Optimal Sales Plan
  - Supply-Constrained New Product Diffusion
  - Optimal Supply Decisions
- Discussion
- Critique
- Questions / Discussion

# Introduction

## Operations Literature

- Capacity sizing when launching a new product
- Specify an exogenous demand trajectory
- **Assumes:** ***Capacity does not affect Demand***

## Marketing Research

- Characterization of the demand process
- Social Diffusion Process including internal and external factors (price, advertising, population, etc.)
- **Assumes:** ***Unconstrained Supply***

***How does a new product diffuse in the presence of a supply constraint?***

# Overview

Joint Analysis of supply-related decisions and demand dynamics

- Improved characterization of the constrained demand and sales dynamics
  - Back-ordering vs. Lost Customers
  - Generalized diffusion model distinguishes between the demand process and sales process (min of demand and available supply)
- Improved Operational Planning
  - Capacity planning (Cost of backordering and lost customers vs. overcapacity)
  - Launch decision (MTS → delayed launch = *preproduction*)
  - *Should we sell less than is currently demanded* (given sufficient supply)?

(See Figure 1, page 189 in the Ho, Savin, and Terwiesch paper)

# Related Literature

- Analysis builds on the traditional Bass Model of new product diffusion (Bass, 1969)
  - Widely used in marketing to forecast demand
  - New product demand follows patterns of social diffusion processes similar to those in epidemiology and the natural sciences (e.g. SIR epidemic model)
  - Bass diffusion Model Basics:

    **Potential Adopters subject to two means of communication:**
    - External Influence (mass-media communication → advertising)
    - Internal Influence (word-of-mouth)
- Related Research
  - Jain, et al., 1991: Diffusion of telephone service in Israel
    - No competition = no customer losses
    - Capacity grows with backorders (assumed short lead time for capacity expansion)
    - Supply constraint is always binding → sales trajectory mirrors capacity
  - Kurawarwala and Matsuo, 1998
    - Model of procurement with Bass-like demand process with known parameters of internal and external influence, unknown market size
    - Extended newsvendor model
  - Fine and Li, 1988
    - Conditions for switching from one supply process to another during product life-cycle
    - Assume demand with symmetrical growth and decline stages
    - Assume that process switching will not influence the underlying demand dynamics (i.e. they assume that demand is *exogenous* to the model)

# Methodology

- Model Formulation
- Optimal Sales Plan
- Supply-Constrained New Product Diffusion
- Optimal Supply Decisions

# Model Formulation

The Firm:

- Introducing a new product (e.g. Mini-IPOD)
  - Short Lifecycle
  - Long lead times
- Key Decisions:
  - Capacity sizing (Assumes constant $c$ throughout the product life cycle)
  - Time to Market ($t_l >= 0$)
  - Sales Plan $s(t)$

# Notation

(For explanations of model notation, see Table 1
on page 191 of the Ho, Savin, and Terwiesch paper)

# Customer Diffusion

(See Figure 2 on page 191 of the Ho, Savin, and Terwiesch paper)

$$D(t) = S(t) + W(t) + L(t)$$

**Unconstrained Supply:**

$W(t)=0, L(t)=0 \rightarrow D(t) = S(t)$

**Else:** $$\frac{dL(t)}{dt} = lW(t)$$

# Demand Process

**Arrival of customers orders follows *Bass-like dynamics*:**

$$\frac{dD(t)}{dt} = p\left[m - D(t)\right] + \frac{q}{m} S(t)\left[m - D(t)\right]$$

**External Influence:**
- Innovation Dynamics
- Advertising

**Internal Influence:**
- Interaction Dynamics
- Word-of-Mouth

- **Assumes a uniqueness of the new product**
  - New brand, new product category (movies, video game console, Pentium III)
- **Allows for customer losses** (i.e. Does not require monopoly)
  - Cross-brand or cross-category substitution

# Production

**Connecting Demand to the Supply Process:**

**Total Production:** $$R(t) = I(t) + S(t)$$

**Production Rate:** $$r(t) = \begin{cases} c, t < t^* \\ \dfrac{dD(t)}{dt}, t \geq t^* \end{cases}$$

$$t^* = \min(t \,|\, dD(t)/dt < c, d^2D(t)/dt^2 < 0)$$

# Choosing a Sales Rate: *s(t)*

**Objective:** maximize life-cycle discounted profits, with $c$ and $t_l$ fixed

**Profit Function:**

$$P(c,t_l)=\max_{s(t)\geq 0}\left(\int_{t_l}^{+\infty}(a(t)s(t)-hI(t))e^{-\theta t}dt\,\middle|\,\{I(t_l)=ct_l\}\right), a(t)>0$$

Simplified by shifting the time origin to $t_l$:

$$\overline{P}(c,t_l)=\max_{s(t)\geq 0}\left(\int_{0}^{+\infty}(\overline{a}(t)\overline{s}(t)-h\overline{I}(t))e^{-\theta t}dt\,\middle|\,\{\overline{I}(0)=ct_l\}\right)$$

$$\overline{a}(t)=a(t+t_l),\overline{s}(t)=s(t+t_l),\overline{I}(t)=(t+t_l)$$

# Choosing a Launch Time: $t_l \geq 0$

**Given:** the optimal selling plan ***s*(t)***

**Discounted pre-launch inventory costs**

$$h\int_0^{t_l} cte^{-\theta t}dt = \frac{hc}{\theta}\left(\frac{1}{\theta}(1-e^{-\theta t}) - t_l e^{-\theta t}\right)$$

**Objective:** Maximize Profits

$$P^*(c) = \max_{t_l \geq 0}\left(P(c,t_l) - \frac{hc}{\theta}\left(\frac{1}{\theta}(1-e^{-\theta t}) - t_l e^{-\theta t}\right)\right)$$

# Choosing Production Capacity: $c$

**Given:** the optimal selling plan $s^*(t)$ and the launch time $t_l$

$$\max_{c} = \left(P^*(c) - Hc\right)$$

# Optimal Sales Plan

**Tactical Decision:** choosing the sales rate ***s(t)*** to maximize profits with ***c*** and $t_l$ fixed

**Profit Function:**

$$P(c,t_l) = \max_{s(t)\geq 0}\left(\int_0^{+\infty}(a(t)s(t)-hI(t))e^{-\theta t}dt\right)$$

**Proposition 1.** For any profit margin ***a(t)>0***, holding cost ***h>0***, and launch time $t_l$ >= ***0***, the optimal sales rate is given by:

$$s^*(t)=\begin{cases} r(t), & W^*(t)>0 \\ \min(r(t),d^*(t)), & I^*(t)=0, W^*(t)=0 \\ d^*(t), & I^*(t)>0 \end{cases}$$

**Where:**

***d*(t), I*(t)*** and ***W*(t)*** are optimal values

$I^*(t)W^*(t) = 0$

**Bottom Line: The firm should always favor the immediate sale.**

# Supply-Constrained New Product Diffusion

***Given the optimal sales plan:***

- Specify the demand $D(t)$ and sales $S(t)$ dynamics and compare to the unconstrained Bass demand dynamics
- Obtain an expression for discounted profits IOT determine the optimal capacity $c$ and time to market $t_l$

**Consider two cases:**

- Patient Customers ($l = 0$)
- Impatient Customers ($l > 0$)

# Patient Customers, *L(t)=0*

$$D(t) = S(t) + W(t)$$

$$R(t) + ct_l = S(t) + I(t)$$

$$\frac{dD(t)}{dt} = p[m - D(t)] + \frac{q}{m} S(t)[m - D(t)]$$

$$\frac{dR(t)}{dt} = \begin{cases} c, & t < t^* \\ \frac{dD(t)}{dt}, & t \geq t^* \end{cases}$$

$$\frac{dS(t)}{dt} = \begin{cases} c, & W(t) > 0 \\ \min\left(c, \frac{dD(t)}{dt}\right), & I(t) = 0, \; W(t) = 0 \\ \frac{dD(t)}{dt}, & I(t) > 0 \end{cases}$$

**Solve this set of equations given:**

$$t^* = \min(t \mid dD(t)/dt < c, d^2D(t)/dt^2 < 0)$$

$$D(0) = S(0) = R(0) = 0$$

**Analyzing the diffusion process for any chosen capacity *c* and launch time $t_l$, produces three different regimes:**

**Regime 1:** Unconstrained Diffusion (UD)
**Regime 2:** Initially Unconstrained Diffusion (IUD)
**Regime 3:** Initially Constrained Diffusion (ICD)

# Unconstrained Diffusion (UD)

$c$ and $t_l$ are high enough to ensure that $W(t)=0$ for all $t$.

Even with $t_l = 0$, capacity $c$ could be sufficient to ensure unconstrained Bass diffusion is preserved

- What is the ***smallest capacity*** level required?

or

- Given $c$, what is the ***earliest launch time***?

# Unconstrained Diffusion (UD)

**Determining the *smallest capacity* required to sustain UD.**

$$\tau_+ = \max(\tau | c = d_{Bass}(\tau))$$

$$c = \frac{pm(q+p)^2 \exp((p+q)\tau_+)}{(q + p\exp((p+q)\tau_+))^2}$$

$$\tau_+ = \frac{1}{p+q}\ln(\frac{q}{p}) + \frac{1}{p+q}\ln\left(\frac{1+\sqrt{1-\frac{c}{c_o^*}}}{1-\sqrt{1-\frac{c}{c_o^*}}}\right)$$

$$c_o^* = m(p+q)^2/4q$$

$$D_{Bass}(\tau_+) = \frac{m(q-p)}{2q} + \frac{m(p+q)}{2q}\sqrt{1-\frac{c}{c_o^*}}$$

For $t_l = 0$, the smallest capacity required

$$c_s^*(p,q,m)$$

is determined as the capacity $c$ where

$$c\tau_+ = D_{Bass}(\tau_+)$$

It follows:

$$c_s^*(p,q,m) < c_o^*(p,q,m)$$

# Unconstrained Diffusion (UD)

$$c_s^*(p,q,m) < c_o^*(p,q,m)$$

(See Figure 3 on page 195 of the Ho, Savin, and Terwiesch paper)

# Unconstrained Diffusion (UD)

Determining the ***earliest launch time*** that sustains UD.

**Lemma 1.** For a given $c$, unconstrained diffusion is sustained IFF:

$$t_l > t_l^*(c)$$

The critical launch time is a non-increasing function of $c$:

$$\partial t_l^*(c) / \partial c \le 0$$

**Provides the level of preproduction that avoids any supply shortages over the entire life cycle.**

$$t_l^*(c) = \begin{cases} 0, & c \ge c_s^* \\ \dfrac{m(q-p)}{2qc} + \dfrac{m(p+q)}{2qc}\sqrt{1-\dfrac{c}{c_o^*}} \\ -\dfrac{1}{p+q}\ln\left(\dfrac{q}{p}\right) & c < c_s^* \\ -\dfrac{1}{p+q}\ln\left(\dfrac{1+\sqrt{1-\dfrac{c}{c_o^*}}}{1-\sqrt{1-\dfrac{c}{c_o^*}}}\right), \end{cases}$$

# Initially Unconstrained Diffusion (IUD)

For a given $c$ and $0 < t_l < t_l^*$ pre-launch inventory is insufficient to support Bass diffusion over the entire life cycle of the product.

Given a finite amount of inventory at $t = 0$:

- It is possible to sustain an UD for a finite duration
- The diffusion process goes through ***3 phases***:
  - Initial Unconstrained Bass Diffusion (**UP1**)
  - Constrained Diffusion (**CP**)
  - Second Unconstrained Bass Diffusion (**UP2**)

# Initially Unconstrained Bass Diffusion (UP1)

Demand and Sales are identical and increasing: ***s(t)=d(t), ds(t)/dt>0***

$$D(t) = S(t) = pm\left[\frac{\exp((p+q)t)-1}{q+p\exp((p+q)t)}\right]$$

$$W(t) = 0$$

This phase lasts until ***production + inventory*** can no longer sustain the unconstrained diffusion. We can define the ending time of this phase:

$$\tau_1 = \min\left(\tau \middle| c(\tau + t_l) = m\left(1 - \frac{q+p}{q+p\exp((p+q)\tau)}\right)\right)$$

# Constrained Bass Diffusion (CP)

Customers are waiting and sales rate is equal to capacity:
***W(t) > 0, dS/dt = c***

$Given:$

$D(\tau_1) = S(\tau_1) = D_1$

$$D(t) = m - (m - D_1)\exp\left[-\left(\left(p + q\frac{D_1}{m}\right)(t-\tau_1) + \frac{qc}{m}\frac{(t-\tau_1)^2}{2}\right)\right],$$

$$S(t) = D_1 + c(t-\tau_1),$$

$$W(t) = -c(t-\tau_1) + (m - D_1)\left(1 - \exp\left(-\left(p + q\frac{D_1}{m}\right)(t-\tau_1) - \frac{qc(t-\tau_1)^2}{2m}\right)\right)$$

**Note:**

$$s(t) = c$$

$$s(t) \neq d(t)$$

This phase lasts until there are no customers waiting. Ending time of this phase:

$$\tau_2 = \min\left(t \middle| t > \tau_1, W(t) = 0\right)$$

# Second Unconstrained Bass Diffusion (UP2)

Demand and Sales are identical and ***decreasing***: ***s(t)=d(t), ds(t)/dt<0***

$$D(t) = S(t) = m - \frac{(m - D_2)(p+q)}{q - \frac{q}{m}D_2 + (p + \frac{q}{m}D_2)\exp((p+q)t - \tau_2))}$$

$$W(t) = 0$$

$$D_2 = D(\tau_2)$$

**Lemma 2.** Peak Demand and Sales Rates

$$\tau_{\max}^{D} \quad \Rightarrow \quad d(\tau_{\max}^{D})$$

$$\tau_{\max}^{S} \quad \Rightarrow \quad s(\tau_{\max}^{S})$$

$$\tau_{\max}^{S} \le \tau_{\max}^{D}$$

for all values of production capacity

# Initially Constrained Diffusion (ICD)

$t_l = 0$ and $c <$ initial inflow of potential adopters ($pm$). Initial constrained diffusion later replaced by unconstrained Bass process.

**Behaves much like the 2nd and 3rd phase of the IUD regime.**

$$W(t) > 0, \quad 0 < t < \tau_2$$
$$W(t) = 0, \quad t \geq \tau_2$$

**Lemma 3.** Demand and Sales Dynamics in ICD Regime

$$\tau_{\max}^{D} \quad \Rightarrow \quad d(\tau_{\max}^{D})$$

Maximum sales rate is equal to $c$.

$$D(t) = m\left(1 - \exp\left[-\left(pt + \frac{qc}{m}\frac{t^2}{2}\right)\right]\right),$$

$$S(t) = ct,$$

$$W(t) = -ct + m\left(1 - \exp\left(-\left(pt + \frac{qct^2}{2m}\right)\right)\right),$$

$$\tau_2 = \min(t \mid t > 0, W(t) = 0)$$

# Impatient Customers, *l > 0*

$$W(t,l) = -\frac{c}{l}(1-\exp(-l(t-\tau_1)))$$
$$+(m-D_1)\exp(-l(t-\tau_1))$$
$$\times\left(1-\exp\left(-\left(\tilde{p}(t-\tau_1)+\frac{qc(t-\tau_1)^2}{2m}\right)\right)\right)$$
$$+(m-D_1)\exp(-l(t-\tau_1))$$
$$\times\left(l\sqrt{\frac{2\pi m}{qc}}\exp\frac{m\tilde{p}^2}{2qc}\right.$$
$$\times\left(\Phi\left(\sqrt{\frac{qc}{m}}(t-\tau_1)+\sqrt{\frac{m}{qc}}\tilde{p}\right)\right.$$
$$\left.\left.-\Phi\left(\sqrt{\frac{m}{qc}}\tilde{p}\right)\right)\right)$$

$$D(t,l) = m-(m-D_1)$$
$$\times\exp\left[-\left(\left(p+q\frac{D_1}{m}\right)(t-\tau_1)+\frac{qc}{m}\frac{(t-\tau_1)^2}{2}\right)\right],$$
$$S(t,l) = D_1 + c(t-\tau_1),$$
$$L(t,l) = D(t,l) - S(t,l) - W(t,l)$$

**Proposition 2.** NP diffusion dynamics subject to customer loss behave as outlined by Lemmas 1-3. Unconstrained phases unchanged, constrained phases described:

**Enables firm to track the fraction of lost customers at any time.**

# Impatient Customers (cont)

**Proposition 3.** The length of the constrained phase is given by:

$$T_c(l) = \tau_2(l) - \tau_1$$

and is a decreasing function of ***l***:

$$\frac{\partial T_c(l)}{\partial l} < 0$$

**Suggests that the duration of the constrained phase decreases as customer impatience increases.**

**Impatience = fn (Competition)**

**Proposition 4.** The fraction of customers lost is given by:

$$f(l) = \frac{(m - D_1)\left(1 - \exp\left[-\left(pT_c(l) + \frac{qc}{m}\frac{T_c^2(l)}{2}\right)\right]\right) - cT_c(l)}{m}$$

and is an increasing function of ***l***:

$$\frac{\partial f(l)}{\partial l} > 0$$

# Optimal Supply Decisions

**Given:** characterizations of the demand and sales dynamics

**Tactical Decision:** choose the optimal capacity and time to launch

**Profit Function:** $$P(c,t_l) = \int_0^{+\infty} (as(t) - hI(t))e^{-\theta t}\,dt$$

**Proposition 5.** Performs the integration resulting in a “seemingly” complex equation for profit function which can be used for computing the optimal capacity and time to market.

pp 200-201

# Optimal Time to Market

(See Figure 5a and 5b on page 201 of the
Ho, Savin, and Terwiesch paper)

- For fixed capacity, optimal $t_l$ increases with both $p$ and $q$
- Optimal $t_l$ is more sensitive to $q$ than $p$
- Implies it is more important to estimate $q$ well

# Optimal Time to Market (2)

(See Figure 6 on page 202 of the Ho, Savin, and Terwiesch paper)

- For fixed $h$, optimal $t_l$ shortens as $c$ increases
- As the $h$ increases, the optimal $t_l$ decreases for the same level $c$
- Suggests that firms may want to substitute capacity with pre-production by delaying product launch

# Optimal Capacity Size

(See Figure 7 on page 202 of the Ho, Savin, and Terwiesch paper)

- Optimal capacity $c^{opt}$ increases with both $p$ and $q$
- $c^{opt}$ exhibits a saturation effect as the speed of diffusion inceases

# Optimal Capacity Size (2)

(See Figure 8 on page 203 of the Ho, Savin, and Terwiesch paper)

- $c^{opt}$ is a decreasing function of $H$ (capacity holding cost)
- Higher inventory costs $h$ push $c^{opt}$ down for the same level of $H$

# Value of Endogenizing Demand

(See Figure 9a on page 203 of the Ho, Savin, and Terwiesch paper)

- Value of endogenizing demand can be significant
- Value goes up and then down as both ***p*** and ***q*** increase
  - ❑ Slow diffusion → dynamics less likely to be constrained
  - ❑ Rapid diffusion → heavily constrained → lost customers

# Value of Endogenizing Demand (2)

(See Figure 9b on page 204 of the Ho, Savin, and Terwiesch paper)

# Value of Endogenizing Demand (3)

(See Figure 10 on page 204 of the Ho, Savin, and Terwiesch paper)

- Value of endogenizing demand can be significant when capacity costs ***H*** are relatively high
- As H decreases, the optimal capacity increases, demand diffusion becomes more Bass-like

# Discussion

- It is important to include supply constraints in the estimation of diffusion parameters
- Increase in pre-production (delaying product launch) can act as a substitute for capacity
- Shows how optimal time to market and capacity vary with diffusion parameters
  - Timing and capacity are more sensitive to imitation (q) than innovation (p)
  - Optimal capacity exhibits saturation effect as the speed of diffusion increases
- Value of endogenizing demand in supply-related decisions can be substantial
- Informs operational decision-making
  - Develop more accurate forecasts of demand
  - Challenges assumption that demand forecasts merely serve as inputs to operations planning processes and are not affected by supply decisions
  - Suggests it is optimal to pre-produce
- **Future Research**
  - Estimation of diffusion parameters
  - Using marketing mix variables to influence diffusion
  - Waiting time dynamics

# Major Contributions

(to the Operations and Marketing literature)

- Developed closed-form expressions of demand and sales dynamics in a Bass-like diffusion environment with a supply constraint
- Integrated capacity, time to market, and sales plan into a unified decision hierarchy
- Endogenized demand dynamics in determining the optimal capacity in a constrained diffusion environment

# Critique

- Aggressive agenda, but…
  - Needs a more robust application
- 52 equations in ~18 pages
  - Didn't provide the intuition for some of the math
- Proof of key "finding" $s^*(t)$ not included
- Missing graphs (3a-3b) referred to in paper
- Relies on many constant parameters that would not be constant in real life (e.g. loss rate)
- Many other factors "rolled up" in innovation parameter (e.g. price, competition, etc.)
- Offers some operationally useful ideas

# Discussion