Reducing the Cost of Demand Uncertainty through Accurate Response to Early Sales

Fisher and Raman 1996

Operations Research, vol. 44 (1), 87-99

Presentation by Hongmin Li

This summary presentation is based on: Fisher, Marshall, and A. Raman. "Reducing the Cost of Demand Uncertainty Through Accurate Response to Early Sales." *Operations Research* 44, no. 1 (1996): 87-99.

Fashion Industry

- Long lead time
- Unpredictable demand
- Complex Supply Chain
- Enormous inventory loss
  - 25% of retail sales
- Lost Sales

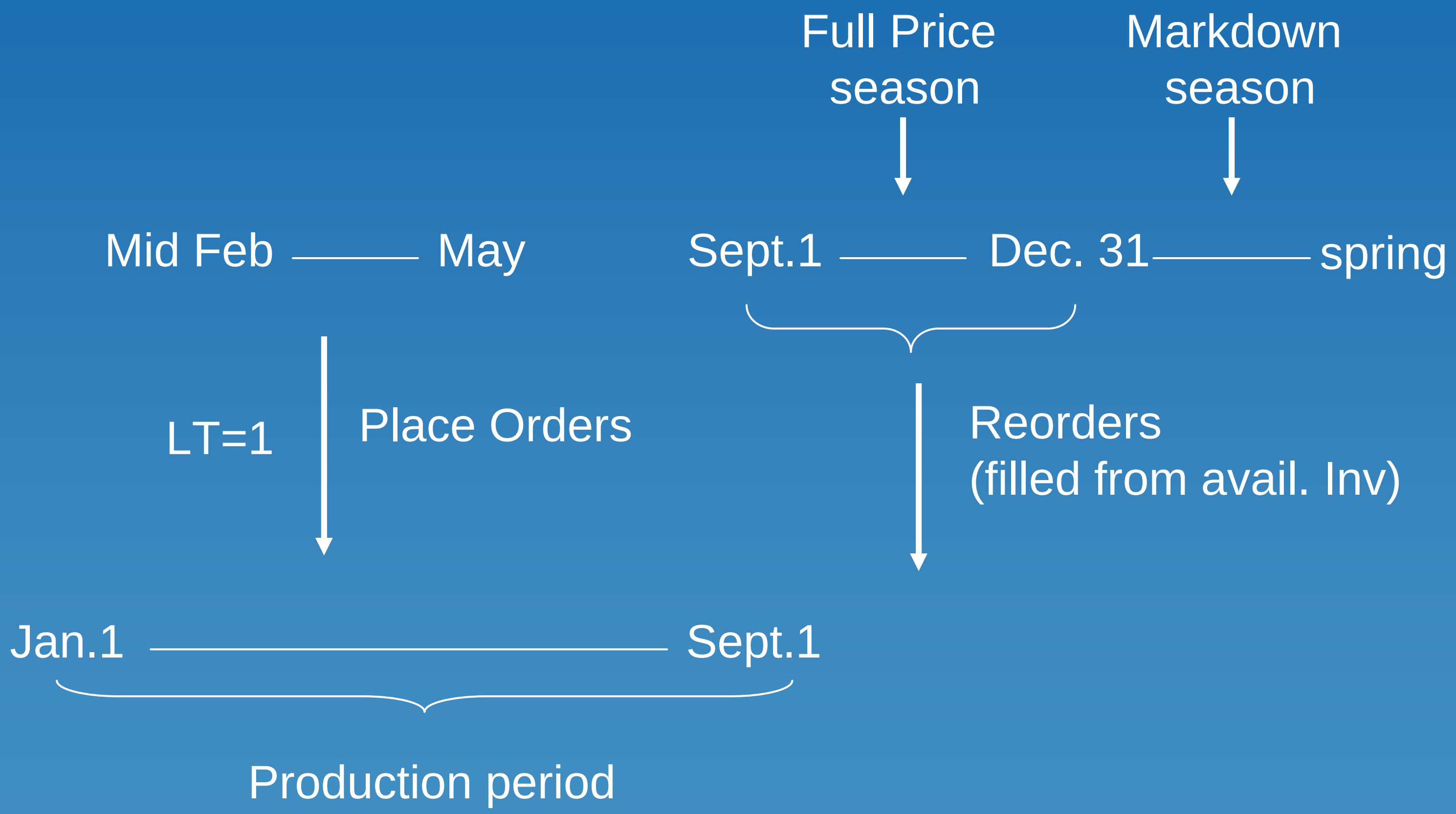
Quick Response

- Lead time reduction through
  - Efforts in IT,
  - Logistics improvement
  - Reorganization of production process
- Complications
  - Production planning (how much and when)
  - Need a method to incorporate observed demand information

This Paper

- Provides a model for response-based production planning
- A two-stage stochastic program
- Use relaxations to obtain feasible solutions and bounds
- Implementation at Sport Obermeyer

Time Line


Full Price season
Markdown season
Mid Feb
May
Sept.1
Dec. 31
spring
LT=1
Place Orders
Reorders
(filled from avail. Inv)
Jan.1
Sept.1
Production period

Notation

- n
- x_{i0}
- x_i
- D_{i0}
- D_i
- O_i
- U_i
- K
- $c_i(x_i,D_i) = O_i(x_i – D_i)^+ + O_i\ (D_i - x_i)^+$

(For notation descriptions, see page 90 of the Fisher and Raman paper)

Notation (2)

- $f_i(D_{i0}, D_i)$
- $g_i(D_{i0})$
- $h_i(D_i|D_{i0})$
- $f(D_0, D)$
- $g(D_0)$
- $h(D|D_0)$

(For notation descriptions, see page 90 of the Fisher and Raman paper)

Model

Expected cost of demand mismatch:

$$c_i(x_i, D_i) = O_i(x_i - D_i)^+ + U_i(D_i - x_i)^+$$

$$c(x, D) = \sum_{i=1}^{n} c_i(x_i, D_i)$$

$$E_{D|D_0} c(x, D) = \int_0^{\infty} c(x, D) h(D \mid D_0) dD$$

Choose x_0, observe D_0, then choose x to minimize total over and under production cost:

(P)

$$Z^* = \min_{x_0 \geq 0} Z(x_0) = E_{D_o} \min_{x_0 \geq 0} E_{D|D_0} c(x, D)$$

$$\sum_{i=1}^{n} x_i \leq K + \sum_{i=1}^{n} x_{i0}$$

$Z(x_0)$ is convex

(P²)

$$Z^1(x, D_0) = E_{D|D_0} c(x, D)$$

$$Z^2(x_0, D_0) = \min_{x \geq x_0} Z^1(x, D_0)$$

$$\sum_{i=1}^{n} x_i \leq K + \sum_{i=1}^{n} x_{i0}$$

$c_i(x_i, D_i)$ is convex in x_i
$\Rightarrow$ $c(x, D)$ convex in x
$\Rightarrow$ $Z^1(x, D_0)$ convex in x
$\Rightarrow$ Z^2 convex in x_0

However, no closed form expression for x in D_0, so convex opt. is not tractable

Approximation to P

"Replace the capacity constraint in 2nd period with a lower limit on total 1st period production"

x_0^* - optimal value of x_0

Define $$L^* = \sum_{i=1}^{n} x^*_{i0}$$

$$W(L) = \min_{x_0 \ge 0} E_{D_0} \min_{x \ge x_0} E_{D|D_0} c(x, D)$$

$$\sum_{i=1}^{n} x_{i0} = L \quad \sum_{i=1}^{n} x_i \le K + \sum_{i=1}^{n} x_{i0}$$

$$\overline{W}(L) = \min \overline{Z}(x_0) \qquad x_0 \ge 0$$

$$\sum_{i=1}^{n} x_{i0} = L \quad \textit{where} \quad \overline{Z}(x_0) = \sum_{i=1}^{n} E_{D_{i0}} \min_{x_i \ge x_{i0}} E_{Di|D_{i0}} c_i(x_i, D_i)$$

(P) **(<u>P</u>)**

Solve (P) for a Given L

1. Given $h_i(D_i|D_{i0})$,

   $x_i^*(D_{i0})$ = argmin $c_i(x_i,D_i)$

   (opt. newsboy solution)

   $x_i = \max(x_i^*(D_{i0}), x_{i0})$ solves $\min E_{Di|Di0}\, c_i(x_i,D_i)$

   Then $\underline{Z}(x_0)$ can be computed using numerical integration

$$\overline{W}(L) = \min \overline{Z}(x_0) \qquad x_0 \geq 0$$

$$\sum_{i=1}^{n} x_{i0} = L \quad \textit{where} \quad \overline{Z}(x_0) = \sum_{i=1}^{n} E_{D_{i0}} \min_{x_i \geq x_{i0}} E_{Di|D_{i0}} c_i(x_i, D_i)$$

(P)

2. If D_{i0} and D_i are positively correlated,

   There exists a D_{i0}^* s.t. for $D_{i0} \leq D_{i0}^*$, $x_i = x_{i0}$

   and for $D_{i0} > D_{i0}^*$, $x_i = x_i^*(D_{i0})$

   Partials of $\underline{Z}$ can be approximated with differences => x_{i0}^*

Choosing L

- x_0 (L) – value of x_0 that solves P with given L
- Use Monte Carlo generation of D_0 to evaluate $Z(x_0 (L))$
- Choose L by line search algorithm to the problem $min_{L>=0}\ Z(x_0\ (L))$

Lower bounds on W(L)

Relaxing the constraint $x >= x_0$, we get:

($\underline{P}_2$)

$$\overline{W}_2(L) = E_{D_0} \min_{x \geq 0} E_{D|D_0} c(x, D)$$

$$\sum_{i=1}^{n} x_i \leq K + L$$

- $\underline{W}$(L) is a lower bound
- $\underline{W}_2$(L) is a lower bound (a constrained Newsboy problem that can be solved with lagrangian methods.)
- $\underline{W}$(L) is smallest when L=0, $\underline{W}_2$(L) is largest when L=0

⇒ $\min_{L>=0}$ max ($\underline{W}$(L), $\underline{W}_2$(L)) is a nontrivial bound on Z*

Minimum Production Quantities

Let S_j defines the sets of products that satisfies a min. initial production level M_j^0, $j=1,\ldots, m$

(See equations on page 92, left hand column, of Fisher and Raman paper)

The RHS is evaluated by the following problem:

(See equations on page 92, left hand column, of Fisher and Raman paper)

can be solved similarly as P^2

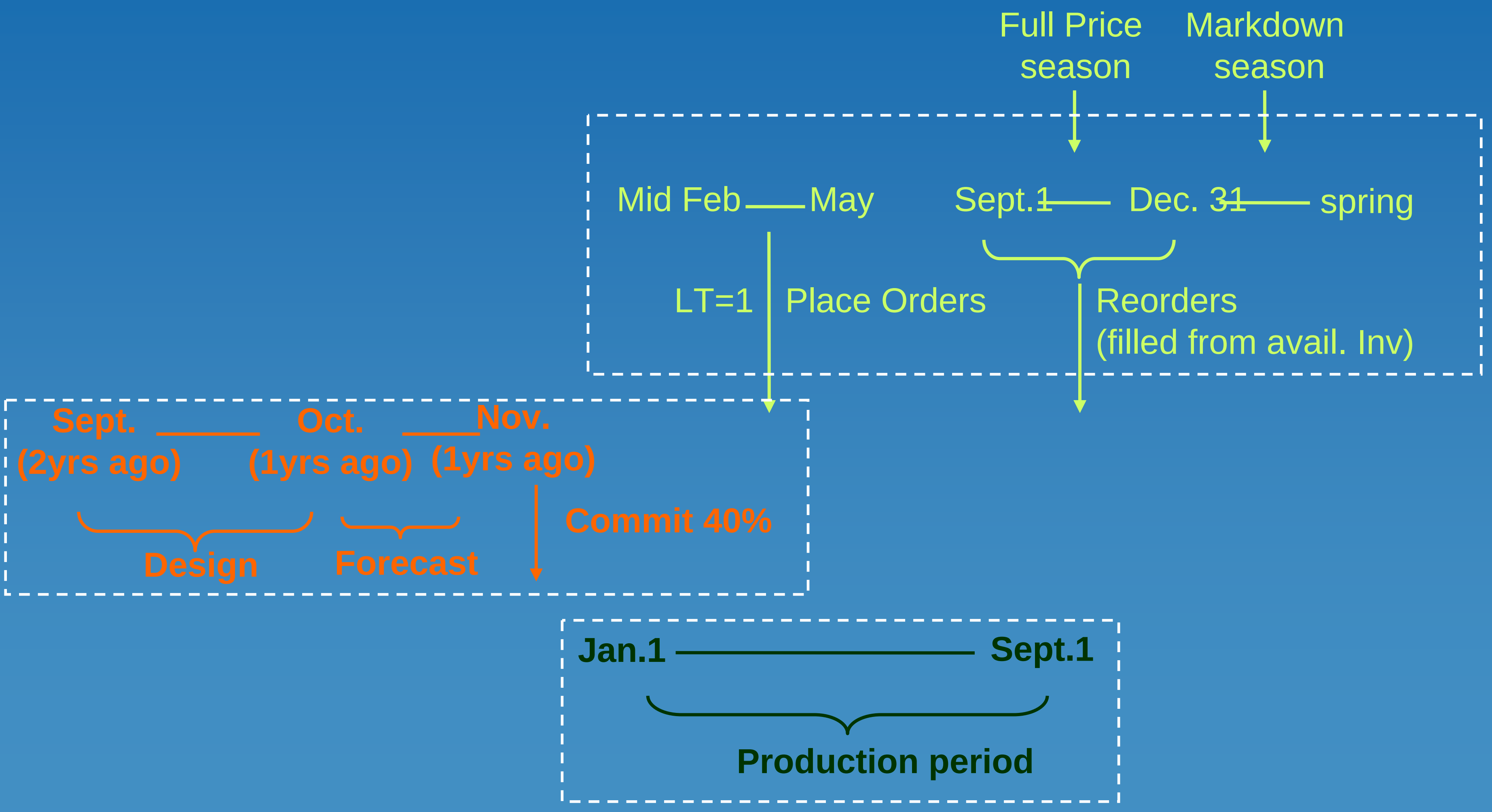
Minimum Production Quantities

- The modified problem can be solved as before with 2 changes:
  - Only allow movement away from $x_0^j = 0$ to a point that satisfies the min production constraints
  - Change the estimation of the derivative of $W(L)$ w.r.t. x_i^j at $x_i^j = 0$ to:

(See equations on page 92, left hand column, of Fisher and Raman paper)

  - Z_j^* has the same form as (P), thus can be solved similarly

Sport Opermeyer



Sport Obermeyer Application

- $L = 0.4D$
- U_i = wholesale price – variable cost
- O_i = variable cost – salvage value

  (a conservative measure)
- In general, $U_i = 3{\sim}4O_i$

μ

Demand Density Estimation

- D_{i0} and D_i follow a bivariate normal distrn
- Estimate μ_{i0}, μ_i, σ_{i0}, σ_i and ρ_i (correlation)

y_{ij} sales estimate of product i by member j

Actual deviation is well correlated with predicted standard deviation

Estimate

- Assume that ρ_i is the same for all products within 5 major product categories
- Estimate ρ_I as the correlation between total and initial demand in the previous season

- Let k be the fraction of season sales in period 1 => $\hat{\mu}_{i0} = \hat{k}\hat{u}_i$
- Let δ be the correlation coeff. between D_{i0} and $(D_i - D_{i0})$=> $\hat{\sigma}_{i0} = \hat{\sigma}_i \left[\rho_i - \delta_i \sqrt{\frac{(1-\hat{\rho}_i^2)}{(1-\hat{\delta}_i^2)}} \right]$ (Proposition 1)

Proposition 1

$$\sigma_{io} = \sigma_i \left[\rho_i - \delta_i \sqrt{\frac{(1-\rho_i^2)}{(1-\delta_i^2)}} \right]$$

σ_{i2} - std of (D_i - D_{i0})

Q-Q plot shows that the student's t distribution with t=2 may be better approximation.

(For equations and explanation, see Proposition 1 on page 95 of the Fisher and Raman paper)

Closed-form solution

Assume that all products have the same Ui and Oi, and have normally distributed demand with the same ρ

(See Theorem 2 on page 96 of the Fisher and Raman paper.)

Results

(See Table 1 on page 97 of the Fisher and Raman paper.)

Bounding Results

(See Figures 6 and 7 on page 98 of the Fisher and Raman paper.)

Summary

- Provides a model for response-based production planning
- A two-stage stochastic program
- Use relaxations to obtain feasible solutions and bounds
- Results at Sport Obermeyer

Critique

- Clear and practical motivation
- Take advantage of both expert opinion and early demand information
- Simple model
- Some details of the model is not explained very clearly
- Accuracy of the lower bounds is not discussed
- Application did not use the approximate method developed in the paper