ANDREW LO: First of all, any questions from last lecture? Yes?

AUDIENCE: [INAUDIBLE] he said he was [INAUDIBLE] possible [INAUDIBLE]?

ANDREW LO: OK. So let me repeat the question to make sure everybody heard. The question about net present value is that, is it possible, is it possible, that in one currency, the net present value of a project is positive, but in a different currency, it is negative? That's a very interesting question.

And it turns out that the answer is staring us in the face right here. Now remember, we're in a world of no uncertainty. So we know what future cash flows are going to be. And we know what future discount rates or discount factors are going to be. That's my assumption.

And in that world, when I give you the value of a sequence of cash flows, this v sub 0, if I wanted denominate it in dollars, then presumably all the cash flows have to be in dollars. If I want to denominate it in yen, then the cash flows have to be in yen. So strictly speaking, assuming that the exchange rates don't change over time-- and that's, again, a big assumption-- the question is, can I have a different result in terms of the sign of a net present value by changing the exchange rate? Any thoughts on that? What do you think? Yeah.

AUDIENCE: No.

ANDREW LO: No, why?

AUDIENCE: Because currency [INAUDIBLE].

ANDREW LO: OK. So the answer is no, because currency, the exchange rates always have to be positive. And presumably, you're multiplying the cache flows by the same number, either positive of one number or positive of another number. So when you multiply a sequence by a positive number, when you add that up, it is either still positive or still negative. In other words, you can factor it out. Right? You sure? Yeah.

AUDIENCE: I have a question. When we are doing this in the [INAUDIBLE], is it possible to have different [INAUDIBLE]?

ANDREW LO: Well. Right now, we're not talking about risk. So let's hold that off for seven or eight lectures. I want to ask this question. Have I got it right? We agreed that no matter what you multiply it by, as long as it's a positive number, it can't change the sign, so the currency doesn't matter. Yeah. Ernest?

AUDIENCE: But the exchange rate, so the actuals are at different times.

ANDREW LO: Yes.

AUDIENCE: So if your exchange rate is different at different times, then it's going to stay factored throughout the--

ANDREW LO: The assumption is that it's fixed. There's no uncertainty. But--

AUDIENCE: [INAUDIBLE].

ANDREW LO: I didn't say it was the same. So you said that it was the same. I didn't. You're right. So [? Shlomi, ?] you're right. If the exchange rate is the same over time, then when you multiply by one number, it's the same number for every cash flow. Then, it factors out. And then you're multiplying v sub-zero by a positive number.

So if v sub-zero is positive, it stays positive. If it's negative, it stays negative. But no uncertainty doesn't mean that it's fixed. So here's the subtlety. The subtlety is that if I assume that the exchange rate is fixed and known, but going up over time, whereas in US dollars, it stays fixed, that makes a difference.

Right? So it's possible. It's possible that if I change currencies and the currency is rapidly appreciating or rapidly depreciating, then you can actually change the net present value of the project. But it has to be the case that the particular path of the currency appreciation or depreciation is exactly opposite what's going on with the NPV.

So the bottom line is, you've got to do the calculation. And you have to use the currency that you care about. So if you're in US, you presumably care about getting paid in US dollars. You would use US dollars. If you're in Japan, you get paid in yen. You'll want to do it in yen. And you have to do the currency conversion. Now when we talk about uncertainty, that's going to make it much more complicated. It's going to introduce another component of risk in our calculations that has to be dealt with. So we're going to come back to that. But that's a good question. Anybody else? Yes?

AUDIENCE: I noticed that you used the term paper a couple of times. I just wanted [INAUDIBLE] definition of--

ANDREW LO: Of what?

AUDIENCE: Paper.

ANDREW LO: Paper. You mean, this is a piece of paper?

AUDIENCE: Well, I don't think [INAUDIBLE].

ANDREW LO: Right. Yeah, so typically by paper, people mean a security. And commercial paper is a security that is a debt instrument that is basically an IOU. It's like a bond. So we'll come back to that when we talk about fixed income securities. But that's what I mean. By the way, you raise a good point. When I mention terminology, feel free to ask me. But in turn, I'm going to feel free to tell you, you may want to look that up in [? Breeley, ?] [? Myers ?] and Allan, because I want you to read the book alongside of what we're doing in class, because you'll need to pick up this terminology, and we don't have enough time in this 20 lectures to cover all the terminology that you need to know.

So don't assume that just because I haven't covered it in class, or that I haven't defined it that you don't need to know it. The textbook is there to help you with the supplementary material that I would like you to cover. So that's why we assign those chapters. OK? Yeah, Justin.

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yes.

AUDIENCE: Then I read a news article, and they said the stock market jumps because they're getting bailed out.

ANDREW LO: Right.

AUDIENCE: So is there a simple reason as to why this is such a massive increase in stock--

ANDREW LO: In the stock market, while their stock has gone down.

AUDIENCE: Right. So that seems a little counter-intuitive. I'm going to give you a two minute answer now, but then I'm going to give you a much deeper answer in about three or four lectures, when we actually apply all of the framework we're developing to pricing common stock.

So as I said with Freddie and Fannie, there are two components. There are two sets of issues surrounding those companies. One is the value of the owner's equity, the folks who owned a piece of those companies. What are their investments worth? And the answer is very little.

The second piece is that Freddie and Fannie have issued all sorts of IOUs, all sorts of obligations to counter-parties. And the question is, what are those securities worth. The government bailing out Freddie and Fannie are basically saying, we will stand behind those IOUs.

The shareholders of the company-- sorry, you guys lost. The company has not done well. It suffered a lot of losses. So the fact that you own a piece of the company means that what you own is now worthless. But the pieces of paper that the company has issued, we will assume that obligation as the US government and make good on those obligations.

So the fact that those pieces of paper have much broader impact on the market as a whole, the fact that the US government is standing behind those pieces of paper will protect the stock market as a whole because there's confidence that business conditions will not be as bad as we thought. So that's what explains the fact that the stock market as a whole went up. It's because the market environment has been stabilized.

You can imagine what might have happened if Fannie and Freddie were to go under. Their pieces of paper, their IOUs, would be worthless. Which means the folks that own those pieces of paper, now they have a bunch of worthless paper. And when that happens, there are repercussion effects for those businesses, and those businesses will end up losing money, which will have repercussions for the entire market as a whole.

AUDIENCE: [INAUDIBLE] the amount that it went up shows how their paper was distributed to all these other companies.

ANDREW LO: It's a combination of how their paper was distributed. But more than that-- I mean, there are many companies in the S&P 500, for example, that don't own any of this paper. So why would their stock be void? It's because the business conditions have been stabilized, and there won't be any knock on effects. A good example of this is Lehman Brothers. As many of you know, Lehman Brothers is a big player in these kinds of securities, and they are currently under a lot of pressure.

Their stock prices dropped dramatically, even in the last few days, because they are a big mortgage lender, and CDO investor, so they're actually hit pretty hard by all of this. And while the rescue of Freddie and Fannie has had some positive effects on Lehman's stock price, it still is under fire and a lot of people want to get rid of it.

Imagine if Freddie and Fannie weren't rescued. It's almost a sure thing that Lehman would have gone under immediately as a knock-on effect. And if Lehman went under, well, I mean, there are other investment banks out there that might have gone under. And now all of a sudden, you have a series of very large companies that do business with all of Wall Street that it has gone under. That's going to have bad repercussions for the stock market as a whole. Yeah?

AUDIENCE: [INAUDIBLE] companies, like big companies?

ANDREW LO: Well, the short answer is I don't know. Nobody knows. I think that there is a concern that the Fed cannot be viewed as rescuing every possible financial institution that's out there. It's got to stop at some point. Many people said it should have stopped even before the Bear Stearns rescue. So the answer is we don't know. Wait and see, and we'll find out over the next few days.

As I said last time, these are very interesting times for financial markets. Very, very serious issues that are coming to the forefront literally every day. So we're going to be watching that, and we'll be talking about that. Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Where do I think that should stop? Well, well, there are a couple of issues that are at the heart of these discussions. The two issues are, how do you balance of the cost of bailing out these large organizations and the implicit moral hazard that it creates, the kind of potential promises that you're implicitly making to future equity holders of these organizations versus letting the market work against the potential disaster scenario of allowing these kinds of events to spread like wildfire.

I don't know how many of you actually know what happens during wildfires, during forest fires. But when forest fires get started, they're actually very difficult to stop. And every once in a while, they try to stop a forest fire by creating additional fires. Right? This may sound counter-intuitive.

But what they will do is around a raging forest fire, they will burn what's called a firewall. That term did not come out of IT. It actually came out of fighting forest fires. They will burn a ring around that forest fire, a controlled burn where they target very specific set of trees, and they would do it in a controlled fashion, so that when the forest fire gets to that ring, it burns itself out.

And one could argue that we need a firewall around these kinds of events. We need to have certain financial institutions fail and stop the spread of this kind of problem. The difficulty with that analogy is that with a forest fire, all you need is a helicopter to get up there and see what's going on. We don't have a helicopter. There's no helicopter that tells us where the fires are, and where the fires may be, and where the underground gasoline tanks are hidden for future explosions.

We don't know because a lot of this stuff is hidden. So my own opinion is that we are going to need to have at least one or two additional large failures, and people will have to lose money before they understand that this stuff really is risky, and that the price you pay for the benefits that you've gotten from these very handsome returns in the years before this kind of an event is the fact that every once in a while, in the parlance of Wall Street, you get your face ripped off. That's the nature of financial markets.

So I think that it's very dangerous to rescue these companies. But at the same time, you have to balance that against the risk of creating a mass panic. And if we do create that mass panic, there's virtually no way to stop it, and then we will run into a very deep recession and depression of the likes that we haven't seen since 1929. That's the balance and the danger. Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Well, you know, that might be. But let me suggest this. Let me put that off for a discussion point until we finish fixed income securities. Because at that point, I'm going to talk about the subprime problem specifically. And I'm going to use the tools that we develop== actually, you guys are going to use the tools that we develop to figure out exactly what's happened in these markets, why they're happening, and how maybe we can get around that.

So let me not give you my view now. I'd rather have you develop your own views based upon the tools we develop in this course. OK? Yeah?

AUDIENCE: Because of all this [INAUDIBLE] CEOs or executives were fired to get a big handsome buyout for all their hard work and efforts.

ANDREW LO: Yeah.

AUDIENCE: But now, should the market be able to self-regulate itself? Or does there need to be regulation in place? Or what will become of it?

ANDREW LO: Well, you know, that's again a very difficult question to answer because we're not done yet, so we don't know where this is going to end up. I think that there are some very important issues that we're going to have to come back to.

Let me put that off for even a bit longer because when we talk about corporate finance, we're going to talk about CEO compensation and ask the question, how do we relate compensation to performance, and does it make sense? It turns out that there's some incentive issues, such that if we don't do that, if we don't allow them to have these golden parachutes, then it may end up creating weird incentives when things are going well.

So every action has some kind of equal and opposite reaction in some other part of the system. And unless you know what that system is, it's hard to figure out the answer to the question. So by the end of the semester, I'm hoping that you'll be able to come up with answers to these questions. So let me put that off for a little while. OK. One more clarifying question maybe, and then we can move on.

AUDIENCE: During the Southeast Asian Crisis in '97, there was this discussion about the international financial institutions should risk [INAUDIBLE] countries and because of the bar [INAUDIBLE].

ANDREW LO: Right.

AUDIENCE: And they decided they should, so they rescued them and they survived. 10 years later, Latin America went into a crisis, and the same discussion started, and the international financial institutions, led by the United States decided not to rescue them. So we went into a crisis. And so I see now [INAUDIBLE]

ANDREW LO: That's right. Yeah, that's a very serious issue. But I would argue that issue actually goes even-- it goes to an even broader set of issues that have little to do with economics and finance, but political and social issues, which I won't comment on in this class, but which are important for determining those kinds of policy questions.

That's one of the things that I'd like to get across to you in terms of thinking about these issues, which is that there are multiple aspects to every issue. And rather than trying to come up with a single answer, what I would propose that you might do is when you think about a challenge like this, first of all, you try to identify the different issues and then come up with an answer for every single perspective of that issue.

So for example in the case of Latin America, there is certainly the economic issue and moral hazard. That's an important one. But there's also a political and social issue, which is that if you don't bail out countries that are in need, that's a recipe for creating social unrest. And if you don't do it, there is some dictator waiting with guns and other interesting possibilities for the people to try to take over.

That's right. And I mean, it's not rocket science. I mean, people are looking for solutions. And if you can't offer one, they'll go to the next person that has one. Whether or not it's true or false, they will try to come up with some kind of leadership.

So how do you balance off the economic considerations against the political and social? That's not something that an economist can answer, so I won't even try to begin. And by the way, my opinion is no better or worse than anybody else's. So I won't waste your time with that. But what I would suggest is from looking at these issues, first of all, try to think clearly about what the economic issues are, and then what the social and political issues are, and separate them out.

And then you can answer each of those questions in isolation and, at the end, decide on how you want to balance these kind of considerations. But don't use economics to try to answer a political question, and don't use politics to try to answer an economic question. You should use the tools that you have to answer the questions that those tools are designed for.

And in the case of Latin America, I would argue that's a very complex set of issues that economics alone cannot answer. The economic answer, never bail out countries that are failing, because you'll create moral hazard and increase the cost of borrowing for future generations in other countries. That sounds good until you see what happens when you don't, and you get these socialist dictatorships that end up creating all sorts of dislocation for the people in the country.

I mean, that there's a very big cost to that as well. And I'm going to have to beg the question about how you balance those costs against the benefits. Again, that's something for politicians and for voters to hopefully to decide. Yeah? Which?

AUDIENCE: [INAUDIBLE].

ANDREW LO: No. Sorry.

AUDIENCE: [INAUDIBLE] has renounced the United States treasury--

ANDREW LO: Yeah.

AUDIENCE: [INAUDIBLE].

ANDREW LO: That sounds good, but that wasn't my handout. So that might be my handout in about three weeks. But we have work to do now. So let me let me stick to that. And we'll come back to these interesting issues. But I want to give you the framework and the tools to be able to think about them.

OK. So let me continue on. This is Lecture Three. And we're going to continue looking at present value relationships and the time value of money. Last time, we were left with the expression for the value of an asset as simply being equal to the cash flows discounted with the appropriate discount factors, where I've assumed for simplicity that the discount rate between one year and the next is constant and given by the interest rate, or discount factor, or cost of capital, or user cost, or opportunity cost, r.

Fancy terms for the simple concept of the number that you use to construct these exchange rates between cash at different points in time. Now the solution of how you make management decisions given this simple framework becomes trivial. Take projects that have positive NPV. That's it. When you figure out what the value of a project is as a function of all of these exchange rates, you calculate what the present value is. And if the cost of the investment is included as a cash flow, possibly a negative cash flow, you've got the net present value.

And for things that are positive NPV, you want them, you want to take them. For things that are negative NPV, you don't want them, you don't take them, or if you can, you sell them. All right? Now, there are many different assumptions that got us to this point. We understand that. We're going to make those assumptions more and more realistic over time. That's, in fact, what the rest of the course is going to be doing.

We're going to be focusing on picking this expression and making it more realistic. And it's going to take us 12 more weeks to do that. So it's non-trivial, but that's exactly the objective. Yes?

AUDIENCE: Last week, you said [INAUDIBLE] summation of cash flow.

ANDREW LO: No. I said the asset was a sequence. What is an asset? An asset is a sequence of cash flows. That's the definition of an asset, not the value. The value of the asset, remember, is that function that you stick in a cash flow sequence, and out pops a number.

So the value of an asset is not the same thing as the asset itself, right? You can have a rocket ship that can go to the moon. That is an asset. The value of a rocket ship that goes to the moon, that's a different thing, right? You need to have this v function in order to figure out the value of an asset. But I can't really talk about the value of an asset unless I've defined the asset to begin with. So v sub-zero is the value of the asset. It's not the asset itself. It's the value of the asset. The asset itself is the sequence of cash flows.

Now, here's a simple example about how these discount factors work. This is just an interest rate example. If you let little r equal 5%, then you can figure out what the value of a dollar is in the future, or you can figure out what the value today of a future dollar is. It's just using simple arithmetic to be able to do that.

So this is just a simple concrete illustration. And if you graph the present value of a dollar, over time, you'll notice that as time goes out farther, the present value of a dollar declines. Not surprisingly, $1 today is worth more than the dollar tomorrow. But $1 tomorrow is worth more than $1 two years from now. And $1 two years from now is worth much more than $1 an infinite number of years from now. Right?

Now here's an example of how you use this valuation approach. And the problems that we handed out last time will give you practice in how to think about present value. So I urge you to do those problems to make sure you really understand these concepts.

Here's an example where a firm spends $800,000 every single year for electricity at its headquarters. And by installing some kind of specialized computer lighting system, it turns out that you can reduce your electricity bills by $90,000 in each of the next three years. Now, of course, it costs money to install that system. It costs $230,000 to install that system. So the question is, is this a good deal? Should you do it? That's a management decision.

And the management decision relies on valuation first. Once you value it, then you can make a decision. So you've got 90,000, 90,000, 90,000 in the three years as your cost savings, but it's going to cost you $230,000 upfront.

Now if it turns out that the interest rate is 4%, you can figure out what the answer is. At 4%, it turns out that the NPV of this project is about $20,000.

So it's a good deal. On the other hand, if you change the assumptions, and you make the interest rate something else, well, it might not be a good deal. How would you have to change the interest rate to make this a terrible deal? Increase or decrease it?

Increase it. Why? Why does that make sense? Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Exactly. With a higher interest rate, money now is more valuable than the cost savings to your electricity. How do you know it's more valuable?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Exactly. The opportunity cost is 10% as opposed to 4%. It's a lot more valuable. If you stick it in the bank, you get 10%. So the cost savings depends on the interest rate at hand. Once you have the interest rate, you can make a decision. Where does interest rate come from?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Exactly. The market. You don't pick the interest rate out of the air. You don't say, I sort of feel like it's a 2% kind of day. The interest rate is what you can get on the open market. See, that's why the market matters. It's because if that's a market interest rate, by saying it's a market interest rate, it means you can actually get that rate from the market.

And therefore, it's a real number that can be actionable. It's not a fictitious theoretical construct that may or may not have any practical bearing. It's a number that actually you can achieve.

And as a manager, if you're trying to increase the value of shareholder wealth, if that's the objective, is to make more money for the shareholders, this is the way to do it. So this is what I meant when I told you at the very beginning of this course that finance is the most important subject you'll ever study.

It's because with proper valuation, management decisions are easy. Now, it's not always easy to get to the point where the numbers tell you so much. And so, management is trying to understand all of the various different factors and balancing them out. Like, the kind of questions you were asking me at the very beginning of class, I can't answer many of them in the abstract. It depends on the situation. And I'm hoping that by the end of this course, you will know enough about the basic framework to make those trade-offs yourself.

And then, the art of management works together with the science of management to come up with good decisions. OK. So this is simple. And in the next few slides, I'm going to ask you to take a look at examples on your own. Here's an example, a real live example, where CNOOC, the Chinese oil company, made an offer to acquire Unocal about a year, year and a half ago. And I would suggest you take a look at this example and just do the back of the envelope calculation to see whether or not they provided a good deal or a bad deal.

But I want to turn now to the main subject of today's lecture, which is one of the most beautiful formulas in this entire course. Now it might seem strange for me to talk about a formula as being beautiful.

You know, a while ago, Paul Samuelson, the great economist here at MIT, once said that, you know, either you think that probability theory is beautiful or not. And if you don't think it's beautiful, then I feel sorry for you. And I suppose the same can be said for this formula. It's hard to believe that a formula can be beautiful, but trust me, it is. And if you don't think so, I feel sorry for you.

By the end of the semester, hopefully you will think it's beautiful. Let me explain what we're about to do. I want to come up with the value of a very specific asset, an asset with a very, very simple and interesting cash flow. So this is one of the two special cash flows that we're going to analyze in this class.

And this cash flow is known as a perpetuity. A perpetuity is exactly what it sounds like. It pays cash forever. Now we can debate whether or not forever really exists. I won't try to argue with you that we will live forever. But it's a hypothetical construct. OK? So this is a figment of our imaginations. There exists in my imagination a piece of paper that has a claim, such that whoever holds the piece of paper will be entitled to a cash payment of C dollars every year forever, out to infinity. OK?

And the question is, how much is this worth? How much is this piece of paper worth? It's an asset, because it's a sequence of cash flows. It just turns out that this cash flow is an infinite sequence. It never ends. It's the gift that keeps on giving.

So you would think that it should be worth an infinite amount, because it pays an infinite amount of cash, right? No, that's not right. And the reason it's not right is because $1 today is worth more than $1 tomorrow, which is worth more than $1 a year from now, which is worth more than $1 two years from now. And so the value of a dollar paid out into the far future declines. And it turns out that it declines at a rate for which you can actually figure out what the value is today.

So here's we're going to do. Using the same principle of discounting that we did for the previous set of cash flows, we're going to take a sequence and discount it. I'm assuming with the perpetuity, that it starts paying next year. So that's the very first cash flow. We're sitting here at date zero, and it pays C dollars next year, and then another C dollars the year after, and then another C dollars the year after that, and so on.

So we're going to dis count them by 1 plus r, 1 plus r squared, dot, dot, dot, forever. And so this is an infinite sequence. And those of you who were on your high school math team, you'll know that a quick and dirty way of some summing that infinite sequence is basically to multiply both sides by 1 plus r. And you'll notice that when you do that, you get the series back again, but with an extra C.

And when you do the subtraction and division, you end up with this incredibly simple formula that says that the present value of this claim that pays C dollars forever is not infinite. In fact, it's quite finite. It's C divided by r. What a simple formula.

If I have a piece of paper that pays $100 a year forever, and the interest rate is 10%, what is this piece of paper worth? Yes?

AUDIENCE: $1,000.

ANDREW LO: Exactly. $1,000. Isn't that amazing, that we could actually value something like that? If the interest rate is 5%, what is it worth then? Yeah. $2,000. Right. Simple. We have complete analytical solution for a cash flow that, on the surface of it, seems like it should be worth a huge amount of money. It's not that huge. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, yes. We're assuming-- assume that interest rates are constant. Absolutely. So if interest rates vary. This formula is not right. We're going to come to the case where interest rates vary over time. So, absolutely. This is still under the simplistic assumption that interest rates are the same. But under that case, I think it's still pretty cool that we're able to come up with the formula for value, right? Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, that's a good question. I was afraid you were going to ask that. But I am prepared. I am prepared to answer that. In the United Kingdom, there is a bond issued by the government called a console. And this bond is a perpetuity. That is, it pays to the holder a fixed amount every year forever.

Now in that case, forever means as long as the British government is still in existence. You know, it's still around. But that's an example.

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yes, absolutely. It trades. You can buy it, sell it, observe the price. Absolutely. Yeah. Yes?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right. Good question. Where do we get the interest rate? The market. Exactly. So you can either get it from the marketplace-- so I have a piece of paper. It pays $1 a year forever. Who will pay me $5 for this piece of paper. $6? $7? I'll auction it off to the highest bidder, and that price will translate into an interest rate determined by the marketplace.

So the short answer is the market. Now you're asking me probably a deeper question, which is where does that come from? Because there are all sorts of factors, like future, famine, and plagues, and wars, and all these other issues. And the answer is, it's an approximation that market participants make, and they're willing to live with. Right?

I'll give you an example. A few years ago, Walt Disney, the entertainment company, issued bonds, corporate bonds. They were 100 year bonds. They were going to mature in 100 years. Now, I don't know how many of you are high school math team jocks, but if you are, one test is to ask the question, with this infinite series, if you take it out to 100 terms, instead of all the way out to infinity, what percentage of the total market value will you capture?

It turns out that 100 terms is pretty darn close to infinite in this grand scheme of things with interest rates that we use. So that's an example, where when they issued that bond, and they auctioned if off to the market participants, whoever bought those bonds, whoever the highest bidders were, they set the price. Once you have the price, you can back out and calculate the r. In fact, let me ask you this. If I tell you what C is, C is $100, and I tell you the market price, say it's $500, what's the interest rate? Can you figure that out? Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Right. Exactly it's basically determined. So the market price for an instrument like this will give you the market's assessment of what that interest rate is.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Let me repeat the question in case people didn't hear. The question is, am I telling you that with all the PhDs out there, there is nothing more sophisticated in terms of pricing these instruments than simply auctioning them off, as we did to a bunch of MBAs?

Well, first of all, I wouldn't denigrate MBAs that way. I would argue that the PhDs who are doing research are ultimately advising the MBAs as to what to bid, and then the MBAs take into account the business considerations, as well as the analytics. And so it's actually a highly complex and sophisticated process by which the bidding occurs.

In other words, you're not getting amateurs doing it. You're getting professionals who know how to price these things. That said, are they going to make mistakes? Absolutely. So market pricing is an imperfect mechanism. But the imperfect mechanism actually works pretty well. And so far, nobody else has figured out anything that works any better. So, yeah?

AUDIENCE: [INAUDIBLE] price [INAUDIBLE] $1 [INAUDIBLE]. Obviously, they're not just issuing one to the highest bidder.

ANDREW LO: So the question is, isn't there a problem in terms of the auction if what we're doing is determining the price based upon the highest bidder. Because the highest bidder is typically the individual that's the most confident. Or it's possible that that particular bidder knows something that the rest of the market doesn't.

So I don't know which of those two possibilities might be the case. It depends on the market circumstances. One of the things about auctions, though, is that the design of the auction can actually have a big impact on how informative the price is. So the standard auction is actually very, very complicated in terms of the various incentive effects. But there are more intelligent auctions that are designed to elicit true responses based upon not just kind of anxiousness to win, but on what the economic valuation is.

In fact, there's an example of an auction that works something like this. You have bidders bidding for a particular commodity. And it turns out that the highest bidder wins. But the highest bidder will pay a price that is the second highest bidder's price.

So that actually has a very interesting incentive in the sense that it ends up forcing you to actually reveal your true preferences. And we'll come back to that as we talk later on about market mechanisms and pricing. Yeah?

AUDIENCE: [INAUDIBLE] mechanisms in auction, for example, for public contracts--

ANDREW LO: Yeah.

AUDIENCE: In which they do the average and they rule out people who have more than 15% deviation from that. So it could really go for a very low price.

ANDREW LO: Yeah.

AUDIENCE: It's interpretative that you're like, [INAUDIBLE]. So you're kicked off the deal.

ANDREW LO: That's right. So there are mechanisms to try to make the auctions smarter. And that's one example. Another example of that. But we're going to assume for now that the auction mechanism produces a good price. Later on, after we figure out how markets work, we're going to come back and question that. And the very end of this course, I'm going to question all of this and confront you with empirical evidence that describes psychological biases that all of us have that are hardwired into us that would make you think that markets don't work well at all. And we'll give you a framework for thinking about those two kinds of phenomenon. Yeah?

AUDIENCE: I'm just curious to see-- [INAUDIBLE] would you have bought this [INAUDIBLE] at market price. [INAUDIBLE]

ANDREW LO: OK. The question is, do people's bids actually reflect interest rates over time? Well, remember that market conditions are changing. So the question is, do they reflect people's information as of when. I mean, you never step into the same river twice. So what you bought last year at last year's price may have no bearing on what you're willing to buy at this year's price, right? Things change.

So I'm not sure that that question is well-posed. At every point in time, if an individual will pay this C divided by r for a security that pays C forever, that's the fair market price. Now in the future, if interest rates change, the price will change. But what this does say is a very interesting point that I think you're getting to, which is that suppose that C never changes by contract. If interest rates never change, then this security will never change in price.

It will have absolutely no price growth. So here's an example where you buy a piece of paper-- let's say the interest rate is 10% and C is $100. You pay $1,000 today. If next year the interest rate is 10%, this piece of paper's still worth $1,000. And then five years from now, if he interest rate is 10%, the piece of paper's still worth $1,000.

Does that makes sense? Does that seem to suggest that you got stiffed because you bought a security and it didn't grow in price? In fact, the rate of return on that security is 0. Right?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Or I mean, a $100 payment.

AUDIENCE: You have one coupon payment plus--

ANDREW LO: Every year. Right, exactly . So it's wrong that the return is zero. The price return is zero. There's no price growth. But meanwhile every year, you've been getting checks for $100. And if the piece of paper was $1,000 and you've been getting checks for $100 every year, what's your annual return? 10%.

What's the interest rate? Oh, funny how that works, huh? That's great. You get a 10% return. Why? Because you're holding this piece of paper that generates coupons, and the coupons end up giving you a 10% rate of return, because the price of the security is those coupons discounted at 10%. Nothing magic about it. It all adds up. It all works together. OK? Yes?

AUDIENCE: [INAUDIBLE] for example--

ANDREW LO: We're going to get to that. Yes, we're going to get to that. That's my next example. Let me hold off on that. I want to make sure everybody understands the perpetuity though. And then we're going to get to the example where C changes.

Now to your example, what happens if C changes. In fact, let's be optimistic and let's say that C grows. So not only am I going to pay you something forever, but that something, I'm going to let that grow by a rate of growth of g. So next year, I pay you C. But the year after, I'm going to pay you C, multiplied by 1 plus g.

So let's say g is 5%. Then next year, I pay you $100. The year after, I pay you $105. And the year after that, I'll pay you whatever 1.05 squared is times 100, and so on. Now, what is this piece of paper worth?

And if you do the same kind of high school math team trick and solve for the present value, you get an answer, PV is equal to C divided by r minus g. r minus g. So you subtract this growth rate. Now when you subtract the growth rate, that makes the denominator smaller, which makes the whole thing bigger, which is the right direction because you're getting a cash flow that is not steady over time, but it's growing over time. So it should be worth more.

And it's worth r minus g more. All right? Now you notice, I have a little condition at the end of that. r has to be greater than g. Why do I have that condition? Yeah?

AUDIENCE: [INAUDIBLE] infinite [INAUDIBLE] the infinite [INAUDIBLE].

ANDREW LO: That's right. So let's suppose that r equals g. Let's see what happens. If r equals g, then the infinite series on top, c divided by 1 plus r plus C times 1 plus g over 1 plus r squared, that's just C over 1 plus r, because I'm assuming g and r are the same. Plus C over 1 plus r, plus C over 1 plus r, plus C over 1 plus r.

I have an infinite number of C over 1 plus r. And C over 1 plus r is a finite constant. The sum is infinite. So at some point, that's going to exceed total world GDP, and then beyond it, and then the other planets of the solar system, and so on. What's going on here? Why is it happening? Anybody give me the intuition for what's happening?

AUDIENCE: Because the numbers are going smaller and smaller [INAUDIBLE]

ANDREW LO: Right.

AUDIENCE: But compared just to zero, the amount of [INAUDIBLE].

ANDREW LO: Right. Yes.

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yeah. That's right. It's growing. But what's the intuition for why that can't persist?

AUDIENCE: Sounds like you're [INAUDIBLE] the 10,000 [? quantity. ?]

ANDREW LO: Right. That's right. It's basically working against the time value of money because the numerator is growing as fast as the denominator is growing. So what it says is that the cash that you're presumably going to be paying to somebody is actually increasing at the exact same rate that the discount rate is growing.

So there's no way to sustain that forever. You can't do that forever. So it has to be the case that the amount that the cash is growing can never exceed the discount rate.

Now remember, these are all theoretical concepts where I'm assuming that growth rate stays the same forever, and the interest rate stays the same forever. This doesn't rule out for short periods of time growth rates exceeding interest rates. You just can't do it forever. For the last 15 years, China has been growing at a rate of approximately 10%. Their entire economy has been growing at 10% a year for every single year over the past 15 years.

That can't persist. If it did, not only would we all be speaking Chinese, but all of the planets in this entire galaxy would end up speaking Chinese. I mean, growth rates cannot persist forever. But here, we're assuming, we're assuming, that this growth rate is an infinite growth rate. It applies forever. So in that sense, it has to be smaller than the discount rate. Question? OK.

AUDIENCE: [INAUDIBLE] rest of the world.

ANDREW LO: Well, there are a couple of problems with that. So the question is, what happens when r is actually less than g? Right? You would think that that gives you a negative number. In fact, it doesn't, because there's a discontinuity at zero. And so this formula is-- that doesn't even apply.

What happens, if you do the infinite sum, when g approaches r, this infinite sum already goes to infinity. When g gets above r, it gets to be even more infinite, whatever that means. Right? Because the numerator is then not growing at the same rate, but it's growing at a faster rate than the denominator.

So the formula, you wouldn't even get the formula, because now you're dealing with infinities. OK?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Right. Right.

AUDIENCE: [INAUDIBLE].

ANDREW LO: It would just be an infinite value, but an even bigger infinity, whatever that means. And so, this formula really only holds under this condition. If it were equal to or negative, this formula just would not be appropriate. You'd have to go back to that formula. And what that formula would show you is that you're getting an infinity.

OK? So that's a perpetuity. And we're going to use this, by the way. This may seem kind of theoretical. But trust me, it's going to come in very handy when we start pricing bonds and stocks. So we're going to use this quite a bit. Now I want to tell you about a formula that is my second favorite formula in this entire course.

And in a way, this is much more practical, and it's very closely related to the perpetuity. This formula is a formula for an annuity. An annuity is a security that pays a fixed amount every year for a finite number of years, and then it stops paying.

So an example of an annuity is a bond. Another example is an auto loan. Another example is a mortgage. And I think I told you that this mortgage valuation formula is one that you're going to use when you start thinking about making a home purchase decision. And it will actually be this formula exactly that you're going to need to use. Question?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yes.

AUDIENCE: Just one question. The principle is returned within these payments, or at the end?

ANDREW LO: Let me talk about that later. Right now, we don't know what principle is. So when I talk about bonds, I'm going to come back to that. OK? So let's not get ahead of ourselves. I want to make sure we understand the formula and then I'll come to that. That's an important point that we're going to get to in about a lecture and a half. OK? OK. So let me explain what a perpetuity and an annuity are in relationship to each other. A perpetuity pays a fixed amount forever. An annuity pays a fixed amount for a finite period of time.

So there's a relationship between the two. And in particular, you can think about the value of an annuity as the value of a perpetuity where you only get to have it for a finite period of time. Right? Let me explain.

An annuity, the value of that, is given by the expression on the top line. Right? C, C, C, C for T periods, discounted at the appropriate discount rate. Now, it turns out that you can come up with an expression for what that present value is, again, using the high school math team kind of an approach. You simply multiply both sides by 1 plus r, and then you solve for the present value, and you get an expression that looks like this.

Well, this looks an awful lot like it's related to the perpetuity formula. You've got to C over r here, but then there's some annoying other terms over here. So let me give you a thought experiment that will show you how to derive this formula in less than one minute without any kind of high school math team tricks.

And the experiment goes like this. Suppose that you want to create an annuity, but you don't have an annuity at hand. Well, one way you can do it is to buy a perpetuity, hold it for T periods, and then get rid of it and sell it.

Now look at the cash flows that you get. If you were to take a perpetuity, which is the top cash flow, and you would subtract from it a perpetuity as of date T plus 1-- so you've gotten rid of the perpetuity at this point. When you take the top cash flow sequence and you subtract from it the next cash flow sequence, you get the bottom cash flow sequence, which is just an annuity. Right?

So an annuity is a perpetuity on borrowed time. So what is it worth? Well, it's worth whatever it is to buy a perpetuity, hold it for T periods, and as soon as it pays off in that Tth date, you sell it. OK?

So what's it going to cost? What's the value of that? The value of that is this is what it costs to purchase the annuity today-- the perpetuity, sorry. Right? C over r. That's what it costs to purchase the perpetuity today. And you're going to hold on to that perpetuity for T days or T periods. And at date T plus 1, you're going to sell the perpetuity.

What are you going to get when you sell it? What would you get as the payment? C over r. That's right, because that's the price of a perpetuity. The price never changes. It's always C over r. When do you get paid that price? At T or T plus 1?

AUDIENCE: T plus 1.

ANDREW LO: Because I want to have T periods a cash flow. So I've got to hold onto that perpetuity at least until T periods. After the Tth date, I sell it, which means I sell at the next date, which is T plus 1. And so I get paid a cash flow of c over r at day T plus 1. What is that cash flow worth today?

Remember, it's at two different points in time. I need to use the exchange rate to convert it to the same currency. What's the exchange rate between date 0 and date t plus 1? Yeah? [? Scholmi? ?]

AUDIENCE: [INAUDIBLE]

ANDREW LO: By t, or by t plus 1?

AUDIENCE: By t.

ANDREW LO: No. Close, but no cigar.

AUDIENCE: t plus 1.

ANDREW LO: Why t plus 1?

AUDIENCE: That's the period where you're getting paid.

ANDREW LO: That's the period where you're getting paid. So let's go back. And remember, the first thing you do? Draw a time line. Right? So here's the timeline. And you see why it's confusing. You know, I don't blame you for thinking it's t, because I said two periods and you're going to sell it after two periods but when I say sell it after two periods if it's after two periods it's plus 1.

So take a look at this diagram, and you've got to draw the diagram. You've got to draw the diagram to really get this. OK? The top part is a perpetuity. The middle part is that same perpetuity at day T plus 1. So if you own the top piece, and at the same time you sell the middle piece, that means at time T plus 1, you're going to give up all of your future cash flows because you're going to sell the perpetuity. Then you're left with the actual annuity cash flow that we want.

So the question is, what does this transaction cost? I buy that it's going to cost me c over r. I sell this. This is a sequence of cash flows. So if I'm selling a sequence of cash flows, I'm selling that value. I'm going to receive that value as payment. So it's going to reduce my cost, and so like any other sequence of cash flows, when I sell this, I have to value it, and it turns out that this is equal to the value at this date, the value of the perpetuity at this date. And what is that value?

C over r. And if it's C over r at this date, what is the value at this date? I've got to discount it by 1 plus r to the T plus 1, because it's T plus 1 periods going back. OK?

Well, actually, sorry. T periods, the convention is a little confusing. It's T periods because you're at t plus 1, and you want to figure out what the value is. And the value of the perpetuity at T is a perpetuity that starts paying off at T plus 1.

So you're right. It's T, but you're discounting it as of the payment as of T plus 1. OK? How many people are confused? OK. Yes.

AUDIENCE: [INAUDIBLE].

ANDREW LO: Let me-- let me do this on the board. Right. Exactly. Let me do this on the board, because the notation is confusing. Let me just switch on the light. Whoops. OK.

So we're going to start by assuming that we've got a perpetuity at date 0. So this is date 0. And remember, the definition of perpetuity is that it starts paying the next period. And so it pays C,C until this date. And sorry. T Plus 1 pays T plus 2, and so on. The annuity that we want to value is an annuity that is just consisting of the first T cash flows. Right?

So what I claim is that if you engage in the following transaction, at date 0, you buy a perpetuity, and you also agree to sell that perpetuity at date after date T. So you sell after T. What that means is that you will hold onto the perpetuity until it pays you C dollars.

And as soon as it does that, after it does that, you sell it. Now when you sell it, what do you get for it? You get C over r. But the question is, when do you get that C over r? If you have a sequence of cash flows that starts in year T plus 1, then the value of it at day T is C over r right? Because a perpetuity by assumption is a piece of paper that starts paying off not today, but next year.

So if it starts paying off next year, for every single year thereafter, the value at that point is going to be equal to C over r. Any questions about that? OK . So we've now established that when you sell these cash flows going out into the infinite future, the value at date T is C over r.

And therefore, if the value at date T is C over r, what is the value of date 0? You have to bring it back to date 0. You're discounting it by T periods. So it's C over r times 1 over 1 plus r to the t. That's what you get when you sell this perpetuity. And what you paid for it is C over r. So the value of this particular set of actions that you've engaged in is C over r minus C over r times 1 over 1 plus r to the T.

That's the annuity discount formula in a nutshell. And this formula is the basis of how you figure out your mortgage payments. Because a mortgage is where you have an obligation every month to pay something to the bank in exchange for a pile of money, the money that you used to buy your house.

And the first time I was buying my, house I actually went through this transaction. I decided that I was going to just calculate this myself, because the interest rate was not exactly given by what was in the particular banker's table. So I went to the mortgage company. It was a bank. And I think the interest rate that day was something like, I don't know, 8%, 8 and 1/2%, or 8 and 3/4%.

And it turns out that the table, this book, that had all of these calculations, all of these numbers, didn't have that interest rate. It didn't have 8 and 3/4. It had 8 and 1/2 or and 9, but it didn't have 8 and 3/4. And so I just used this formula, punched in a few numbers, and I got my monthly payment.

And you know, I told the banker, well, you know, this is what I'll pay every month. And he said, well, you can't just do that. I said, well, what do you mean? And he says, well, you know, I don't know that that's the right number. We have to wait for the senior vice president to tell me what the right number is. Because we don't have the book. And he contacted the senior vice president. It turned out he did have the book either.

So they had to call the main branch, and somebody had to look it up in this book. And sure enough, when they came back with the number, it was my number down to the fourth decimal place. And so he was amazed like, wow, how did you do that? You know, this is amazing. You're incredible.

It's incredible if you don't know this very basic secret. So you're going to do this. You're going to do this in the problems. You're going to calculate mortgage payments, auto loan payments, consumer finance payments. All of it is based upon this simple formula. And you can construct tables, as people have done, of what are called annuity discount factors.

So the annuity discount factor is simply separating the interest rate from the cash flow. And so when you're going out for a mortgage, the amount that you're borrowing, you borrow $200,000 for your house, that's the left-hand side. Your monthly payment, C, that's the right-hand side. And the prevailing interest rate, that's the r.

So if you know the annuity discount factor, which is based just on the interest rate, and you know the amount of the loan that you want, PV, you can divide and figure out what your monthly payments are, or vice versa. If you have a particular set of cash flows every month, and you have an interest rate, you can figure out what your total value of that loan is going to be in terms of market terms.

And so once you have this expression, you can use a simple table of numbers to calculate these annuity discount factors. And then you can compute mortgage payment yourself. So this is the kind of number I was talking about. Given various different interest rates, you can come up with these particular annuity discount factors.

And once you do, you can calculate monthly payments very easily. So you only need one set of tables. And for any kind of mortgage, for any kind of consumer loan, you can compute the monthly payments. Right? Whether it's an auto loan, or a mortgage, it doesn't matter. What you need is this table right here.

Nowadays, we can do it in Excel. It's not a big deal. But still, you should know what the underlying basis is for those calculations. OK. Now before you finish this, I would like you to take a look at a few examples. I've given you some here, numerical examples. And I want to close this class with a discussion about compounding, and then next time, finish up with a discussion of inflation. Because I don't think we'll have time to do both.

Compounding is a matter of convention. And I want to explain what that convention is and try to give you a little bit of motivation for the logic of it, so that at least it doesn't look like I'm just making it up out of the blue. The idea behind convention is to take into account calendar effects, and in particular, the possibility of early withdrawal.

Let me explain. When I tell you that an interest rate is 10%, typically, that quote is in terms of an annual interest rate. Almost all interest rates in the world are quoted on an annualized basis, meaning if you were to keep an investment for a 12-month period, that's what the rate of return for that investment would be.

The problem with that quote, 10%, is that what do you do if you want to withdraw your money after six months? What should you get paid then? Well, it would seem fair, if you agree to a 10% interest rate per year, to say, all right, if I take it out in six months instead of a year, maybe you should only pay me half the interest rate.

Right? That seems like a fair deal. Right? Instead of 10%, pay me 5%. And maybe if I keep it in for only a month, it would be fair not to pay me 10% for that month, but to pay me 10% divided by 12 for the month. The reason that that discussion matters is that if you agree that that's the fair thing to do, well then 10% is not what you're going to get.

Because if you get paid 5% interest over the first six months, you take your money out of the bank, and they give you that 5%, and then you put the money back in the bank literally the next minute and keep it in for the next six months, you're going to earn another 5% on your original amount, plus you're going to earn 5% on the first six month's 5%. You're going to earn interest on the interest. And the banks know that.

And after a while, they were OK with that. That's a convention there's no reason it has to be that way. The bank could say, I'm going to give you 10% interest. But if you want to withdraw your money in six months, I'm going to give you the amount of interest such that if you were to take the money out and put it right back in and hold it, you would get 10%.

Does anybody know what that interest rate would be? How you figure that out? Yeah, [INAUDIBLE]?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Roots. That sounds painful . Is that like a root canal? What root do you mean? You're right. You're right. What is it?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yes.

AUDIENCE: [INAUDIBLE].

ANDREW LO: Yes The square root. Right.

AUDIENCE: [INAUDIBLE] you get [INAUDIBLE].

ANDREW LO: That's right.

AUDIENCE: [INAUDIBLE]. Exactly. So what you would do in order to figure out what the six month interest rate would be so that when you held the interest on the interest over through the whole year, it would add up to exactly 10%. The way you would do it is 1.10, take the square root of that. Minus 1, that's the interest rate that you would have for the first six months and the second six months. A little less than 5%, such that that number, when you add 1 and multiply it by itself, you'll get 1.10.

They don't do that, mainly because nobody likes dealing with roots, except dentists. OK? So what they do is they say, OK, as a matter of convention, here's what we're going to do for you. This is the deal we're going to give you. When we say 10% on an annualized basis, what we mean is that it's going to be compounded, typically on a monthly basis, and nowadays on a daily basis.

What that means is that the interest rate that you're actually going to get is the stated equivalent. It's the stated annual rate divided by the compounding interval. Now that's a good deal when you're a depositor. That's not a good deal if you're a borrower. Because when they tell you, you want to borrow money from me, I'll give it to you at a great rate, it's going to be at 10%. But when you actually look at how much interest you're paying, you're going to find out that, actually, it's more than 10%.

So that's where the term APR and ERA came from. What does APR stand for? Anybody know? When you see this ad on TV for auto loans, you know, [? buyer ?] loans, buy a car, no money down, we'll give you a loan, the APR is x%. Annual percentage rate. That is the stated rate. That's not the rate including the effects of compounding.

So as a depositor, when you're lending money to the bank-- that's what it means to deposit money in the bank-- that's a good thing. Because the annual percentage rate of 10% is actually not what you're getting. You're getting more than that, because it's going to be compounded on a monthly, or in some cases, on a daily basis. OK?

In other words, the compounding means you get interest on your interest on your interest's interest going forward. Right? So you've got to keep in mind that when you see these discount rates being quoted, ask whether or not they are APR, annual percentage rate-- that's like the 10% stated rate-- or EAR. EAR is the equivalent annual rate.

That's what you're really going to get. That's what you would actually get in terms of literal dollars at the end of the year if you did nothing but left the money in there for that entire year. It would include the interest on the interest on interest on the interest and so on. Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Annual percentage yield. Yeah, that's right. Now, this is a very clear example of it. OK? If you've got $1,000, and there's no compounding effects and the interest rate is 10%, you're going to get $1,100. If you compound twice a year, which is what the old banks used to do because they didn't have calculators in those days-- it was kind of hard to compute these numbers-- they would compound it twice a year. And so you would get credit for the interest, and then you would get interest on that interest as well as on the original deposit or principle.

Then that turns into $1,103. So being able to compound more frequently gives you an additional bonus, right? Not much. $3. But if you think about this as billions of dollars, this starts adding up to be real money. Now, if you compound on a quarterly basis, it's $4. If you compound on a monthly basis, it's $5. That's actually starting to add up to something important. Right? Yeah?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Well, I mean, I think it's six of one, or half a dozen of the other, as they say. Banks will compete with each other to offer ultimately what the market rate is. So they won't play any tricks with this kind of stuff some. Banks did play tricks with this early on in the early days of banking. That's why banking is such a highly regulated industry, to make sure that no funny business goes on. And frankly, that's why banks are forced now to tell you what whether it's an APR or an EAR.

It's a truth in lending kind of a commitment that they are now being forced to make. So nowadays, when you get an auto loan or a mortgage, they have to tell you, yeah, this is NPR. This is the annual percentage rate. But your actual rate earned may vary, and it may vary because of compounding effects. And if you ask them what the effective annual rate is, they are obligated to tell you.

AUDIENCE: [INAUDIBLE] all the information. Because with APR, you also need to know the compound--

ANDREW LO: Compounding rate. But it's now taken for granted that compounding happens on a daily basis. So that's a given. OK? Any questions about that?

AUDIENCE: [INAUDIBLE].

ANDREW LO: Compounding does, yes.

AUDIENCE: For checking accounts?

ANDREW LO: For checking accounts, for savings accounts. Yes, it's daily. And you know why? It's because they allow you to take out money on a daily basis. So if they didn't do it on a daily basis, they'd have to figure out on a one-off, if you were to take your money out in the middle of the month, and I was to take my money out after the first three days, and you were to take your money out after five days, they'd have to do all these custom calculations for each of those circumstances.

So now they do it simply. They say, fine, we're going to give you your interest rate every day. Every day, we're going to compute your interest. So whether you come or go, you will figure out when you get the interest. For certain market applications, people compute interest intraday. Like the number of hours you borrow money, they will calculate interest.

There are cases where you need to borrow money, only for four hours or three hours. I know this sounds like drug money. But that's not-- that's not what I'm talking about. There are cases where you need very, very short-term financing, and you need to borrow the money. And in those cases, they compute it on a minute to minute. And in some cases, on a continuous basis.

So I'm going to leave you with a little puzzler, which is if this tells you what the effective annual rate is, where you're compounding at intervals of n-- so if r is an APR, an annual percentage rate, and n is denominated in months-- so monthly would be 12-- what would happen, what would your effective annual rate be, if you compounded not every day, not every hour, not every minute, not every femtosecond, but literally every possible time slice, the narrowest time slice you can think of. If you did it continuously, if n were to go to infinity, what would you get?

Think about that. That's a little puzzle. It turns out that's called continuous compounding. So you're compounding continuously. It turns out that you actually get a number. And what that number is is really bizarre. So I want you to think about that, and we'll take that up next time. Thank you.