ANDREW LO: Well, let me pick up where we left off last time and give you just a very quick overview of where we're at now, because we're on the brink of a very important set of results that I think will change your perspective permanently on risk and expected return. Last time, remember, we looked at this trade-off between expected return and volatility. And we made the argument that when you combined a bunch of different securities that are not all perfectly correlated, what you get is this bullet-shaped curve in terms of the possible trade-offs between that expected return and riskiness of various different portfolios.

So every single dot on this bullet-shaped curve corresponds to a specific portfolio, or weighting, or vector of portfolio weights, omega. So now what I want to ask you to do for the next lecture or two is to exhibit a little bit of a split personality kind of a perspective. I'm going to ask you to look at the geometry of risk and expected return, but at the same time, in the back of your brain, I want you to keep in mind the analytics of that set of geometries. In other words, I want you to keep in mind how we got this bullet-shaped curve. The way we got it was from taking different weighted averages of the securities that we have access to as investments. So every one of these points on the bullet corresponds to a specific weighting. As you change those weightings, you change the risk and return characteristics of your portfolio.

So the example that I gave after showing you this curve where I argued that the upper branch of this bullet is where any rational person would want to be. And by rational, I've defined that as somebody who prefers more expected return to less, and somebody who prefers less risk to more, other things equal. So if you've got those kind of preferences, then you want to be in the Northeast. You want to be as north, sorry, Northwest as possible. And you would never want to be down in this lower branch when you could be in the upper branch because you'd have a higher expected return for the same level of risk.

So after we developed this basic idea, I gave you this numerical example where you've got three stocks in your universe. General Motors, IBM, and Motorola. And these are the parameters that we've estimated using historical data. Now there's going to be a question, and we've already raised that question, of how stable are these parameters. Are they really parameters, or do they change over time. And I told you, in reality of course, they change over time. But for now, let's play the game and assume that they are constant over time, and see what we can do with those parameters.

So with the means, the standard deviations, and most importantly, the covariance matrix-- So this is the matrix of variances and covariances-- With these data as inputs, we can now construct that bullet-shaped curve. The way we do it is of course, to recognize that the expected return of the portfolio is just a weighted average of the expected returns of the component securities, where the weights are our choice variables. That's what we are getting to pick, is how we allocate the 100% of our wealth to these three different securities.

And the variance, of course, is going to be given by a somewhat more complicated expression where you have the individual security variances entering here from the diagonals. But you also have the off diagonal terms entering in that same equation for that variance of the portfolio. And when we put these two equations together, the mean and the variance, and we take the square root the variance to get the standard deviation, and we plot it on a graph, we get this. This is the curve, the bullet-shaped curve, that we generate just from three securities, and from their covariances.

And where we left off last time is that I pointed out a couple of things that was interesting about this curve. One is that unlike the two asset example, where when you start with two assets, the curve, the bullet goes through the two assets. In this case, with three or more assets, it's going to turn out that the bullet is actually going to include these assets as special cases, but they won't be on the curve. In other words, what this curve suggests is that any rational person is going to want to be on this upper branch. What that means is that it never makes sense to put all your money in one single security.

You see that? In other words, if we agree that any rational investor is going to want to be on that efficient frontier, that upper branch, why would you ever want to be off of that branch? You'd like to be Northwest of that, but you can't. You'd never want to be below that branch, or to the right of that branch because you could do better by being on that branch.

So what this suggests is that we never are going to want to hold 100% of IBM, or 100% of General Motors, or 100% of Motorola. If we did, we'd be on those dots, and those dots would lie on that efficient frontier. But in fact, they don't. So right away, we have now departed from Warren Buffett's world of, I want to pick a few stocks and watch them very, very carefully. Yeah, Brian?

AUDIENCE: Would you expand that to say that you'd never want to invest in less than three stocks at a given time?

ANDREW LO: That's not necessarily true. There are points on this line where-- and they may be pathological, so in other words, they may be very rare-- but there may be points on the line where you are holding two stocks, but not the third. So you've got to be careful about that. But those are exceptions. As a generic statement, you're absolutely right. The typical portfolio is going to have some of all three of them. And if you had four stocks, the typical portfolio would have some of all four.

Yeah, [INAUDIBLE].

AUDIENCE: You answered my question, which is if you take one more stock, you'll always have your package [INAUDIBLE] n stocks include all the [INAUDIBLE], all the n stocks so, at the limit you should have an infinite number of stocks [INAUDIBLE]

ANDREW LO: Well, let me put it another way that may be a little bit more intuitive. What this diagram suggests-- you guys are already groping towards-- is the insight that the more, the merrier. As you add more stocks, you cannot make this investor worse off.

So in other words, I've now shown you an example with three stocks, we used to do two. Is it possible that by giving you an extra stock to invest in, I've made you worse off?

Yeah?

AUDIENCE: No

ANDREW LO: Why

AUDIENCE: Because you can just not invest in that stock.

ANDREW LO: Exactly. I can never make you worse off in a world where you're free to choose, that is. Because you always have the option of getting rid of the stock that you don't like. You can always put 0 on it. So to your point, [INAUDIBLE], as I add more stocks, first of all my risk-reward trade-off curve will get better. What does it mean to get better? What does it mean for the risk-reward trade-off to be better? Yes?

AUDIENCE: It means you get a higher return for the same level of risk.

ANDREW LO: That's right. A higher return for the same level of risk, or a lower risk for the same level of return. In other words, your upper branch actually moves to the Northwest. That's what it means to get better. As I add more stocks, this will move to the Northwest. And therefore, you have available all of the opportunities to the south and to the east, but you would never take those because you're rational in the sense that you always prefer less risk to more, and more return to less. Yeah?

AUDIENCE: If we put all of the stocks on the index, on [INAUDIBLE]. And if we looked at all the possible combinations that-- we can look at them all at the same time, but then all the subsets that you can think of, then you must come up with some most efficient frontier in that market.

ANDREW LO: Hold onto that thought for 10 minutes. We're going to come back to that. Let's do three first, and then we could do all of them. Yeah, Chris?

AUDIENCE: Trying to better understand Buffett's strategy relative to this one. Is the correlation of these stocks due primarily to just psychological factors of the market, or is it due to intrinsic correlation? And then the follow on is when Buffet says invest in one stock and just watch it carefully, isn't that sort of assuming that the market will determine at some point that the stock is undervalued, and what was he--

ANDREW LO: So those are two good questions. Let me take each of them separately. Let's first talk about the correlation. Why is there correlation? We haven't really talked much about it, but it turns out that there are many different arguments for why there is correlation. Probably the most compelling is that a rising tide lifts all boats, and vice versa. In other words, when business conditions are good, then that helps all companies. Just like when business conditions are bad, it hurts all companies. So there's some macroeconomic type of commonality among businesses that create correlation. That's one reason.

But the second reason is something you pointed out, which is quite apt, particularly over the last few weeks, which is the psychological factor. When the entire economy is under stress, and people are scared to death about what's going to happen to the market, what they will do is withdraw money in mass from equities and put them into safer assets like cash, or treasury bills, or money market funds, or whatever they can do to get to safety.

So I would say that the answer is both. There are good economic reasons where correlations should exist among different companies, but there are also psychological or behavior reasons that exacerbate those kinds of commonalities.

Now your second question about Buffett versus this approach. There's one fundamental difference between what Buffett would say about a company that he decides to buy versus how we're approaching it. The fundamental difference is that Buffett would say that he's been able to identify a severe mispricing. In other words, he would argue that markets are not in equilibrium. He would argue that Goldman Sachs is dramatically undervalued where it is today. And seven years from now, he may be right. And that's the kind of time frame he has in mind, if not longer.

So far, I've made no such argument at all about deriving these analyses. I've not made any argument about whether prices are good or bad. In fact, I'm arguing, in a way, that these prices I'm taking as given. And the question is, what can I do to construct a good portfolio irrespective of whether markets are crazy or markets are rational. In a few minutes, I'm going to argue that when markets are rational and in equilibrium, then there is something that we can say about the relationship between risk and reward that's extraordinarily sharp and meaningful from the perspective of financial decision making.

And then at the end of the course, I'm going to try to explain to you what the limitations of that set of assumptions are. Dennis?

AUDIENCE: Just as we wouldn't put anything in bond's half of this frontier, does this graph imply that we strictly prefer IBM over GM? That we pretty much never weigh anything for GM?

ANDREW LO: Well, from a risk-reward perspective, let's take a look. IBM has a higher expected rate of return, and it's got a higher level of risk. So you really can't say that you would never prefer GM over IBM, because GM has lower risk and lower expected return. If on the other hand, GM were over here, then you would be right. Because any point to the direct Northwest of a particular point on this curve is strictly preferred. And GM and IBM don't have that relationship.

In other words, the way you can identify securities that are dominated in both dimensions is-- So this is your risk dimension, this is your expected return dimension. Pick a point in this space, and ask the question, what are the other portfolios that are strictly preferred to that point. Well the answer is pretty simple. Any portfolio that has higher expected rate of return for the same level of risk, so the vertical line. Any portfolio that has less risk for the same level of expected returns, So the western direction. And anything in this segment, that orthant, or quadrant, is strictly preferred.

So in the case of IBM, if you draw the vertical and the horizontal and ask the question, does

GM lie in that area? No. If you do GM, and you draw the vertical and then the horizontal and asked does IBM lie in that strictly preferred quadrant? The answer is no.

So the answer to your question about IBM versus GM, no, there isn't any strict relationship that would say one would always dominate the other. But if GM were here, then IBM is clearly contained in that preferred quadrant. So then the answer to your question would be yes. Yeah, Justin?

AUDIENCE: Theoretically then, wouldn't everyone just buy IBM, sell GM, then wouldn't there be some sort of equilibrium where then GM--

ANDREW LO: So the answer is, it depends on other things going on. Everyone would not do that. Everyone would do something else, and I'm about to tell you. So I'm about to give you the tools to make that exact conclusion, and the reason is that when I show you what people will do, that's going to far dominate what you think people want to do. Just with pairs. So instead of doing it with pairs, let's do it with all the securities, as Zeke wanted to do. We're going to do that in just a minute.

But I want to make sure everybody understands this basic framework first, because we're going to now start making this a little bit more complex.

Where we left off at the very last moment of Wednesday's lecture was I showed you this diagram with the tangency portfolio, but we hadn't really gotten to talking about it. Remember the case where we had only one risky asset and one riskless asset, treasury bills? And in that case, when you are combining a portfolio with one risky asset and one riskless, you've got a straight line. It turns out that that is much more general. You get a straight line anytime you combine a riskless asset with any number of risky assets.

So let me give you an example. Suppose we picked an arbitrary portfolio which is this red dot, p. And I wanted you to tell me what is the risk-reward possibilities that you could achieve by mixing p with treasury bills. Well, you get that straight line, right? We derived that last time. So any point along the straight line is what you could achieve, right? Anybody tell me where the portfolio would be that invests 100% of your assets in T-Bills? Where is that on this graph?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right this dot right here. How about 100% in portfolio p? Right, the red dot over there. How about 25% in T-Bills, 75% in p? Where would that lie?

AUDIENCE: [INAUDIBLE] along the line.

ANDREW LO: It would be along the line, but where? Here? 25% T-Bills, 75%--

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right, exactly. It would be 3/4 of the way up towards this dot, because it's 75% of the risky, 25% of the riskless, so you're going to get closer to the risky asset. OK, great.

So we've now demonstrated that what I can achieve as an investor, just mixing portfolio p with the risk-free rate, is anywhere along that line. Now this analysis applies to any portfolio p. So for example, suppose I wanted to ask you, what risk-reward trade-offs could I generate by mixing the risk-free rate with General Motors? What would that look like?

Yeah, Ken?

AUDIENCE: The line from T-Bills through GM.

ANDREW LO: Exactly, that's right. If I wanted to mix T-Bills with General Motors, I get that straight line right through that dot. If I wanted to mix T-Bills with IBM, I'd go through that dot, with IBM. If I wanted to mix T-Bills with Motorola, I'd go through Motorola. And if I wanted to mix T-Bills with any portfolio on that frontier, on that upper branch, it would just be a line between T-Bills and that point on the upper branch. Right?

So question. If I were to give you the choice of mixing T-Bills with only one portfolio, just one, which would it be? Which would you prefer? [INAUDIBLE]?

AUDIENCE: The one where the line is tangent to the curve.

ANDREW LO: The one where the line is tangent-- so you're talking about right around here, right? Somewhere here. That's where the line is just tangent to that curve. Now why is that? How'd you come up with that? Yeah?

AUDIENCE: If you took anything below that then it'd be, I would say, preferable to stay back. [INAUDIBLE]

ANDREW LO: Exactly. If you picked any other portfolio besides the tangency portfolio, let's pick one and see. If you picked, let's say this one right here. If you drew a line between this point and that portfolio, it's going to turn out that there are other points over here that are strictly in the

Northwest of that line, that you could do better.

There exists only one portfolio that you can mix with T-Bills, such that you can never, ever do better in terms of generating risk-reward trade-offs for everybody that likes expected return, and doesn't like risk. And it turns out that that portfolio happens to be the tangency portfolio. That's the portfolio that all of you in this room would love to have. I don't know anything about you, I don't know your backgrounds, I don't know your risk aversions, but I don't have to know. As long as I know that you like expected return and you don't like risk, those are the only assumptions that I need.

Then I know, all of you in this room, are going to want that portfolio. You may not be at that portfolio. For example, some of you who don't like risk, you're going to be down here. Those of you who are budding hedge fund managers, you're going to be up here. But the point is, you're going to be on this line. You're not going to be on this line down here. Why? Because why be on that line when you could get higher return for a given level of risk, or a lower risk for a given level of return. You're giving up something for no good reason.

So this is a remarkable insight of modern portfolio theory. This basically tells us that regardless of our differences in preferences, as long as we satisfy the hypothesis that we like expected return and we don't like risk, that means that everybody in this room will agree that the only line that they would ever want to be on is that tangency line.

Questions? Ingrid?

AUDIENCE: Is there a particular level of risk that makes you accepting to the tangency fund?

ANDREW LO: Yes. In fact, this tangency portfolio is one very particular and special portfolio. So in other words, it's a particular weighting of IBM, General Motors, and Motorola, that gives you this particular portfolio.

AUDIENCE: Which one? [INAUDIBLE] something intuitive?

ANDREW LO: It's something you can solve analytically. It has a solution, and if we were using matrix algebra, I can actually solve it for you. But it's a little bit complicated, so I'm not requiring that people know how to do that. Only that you know that it exists.

Yeah?

AUDIENCE: You go above the red dot, and into leverage [INAUDIBLE]?

ANDREW LO: Exactly

AUDIENCE: So it means that I have costs for my debt.

ANDREW LO: Yes

AUDIENCE: So maybe some debt, it would be more efficient to buy a portfolio with other weights--

ANDREW LO: Yes. If you assume that there are borrowing and lending differences, then obviously these analyses don't apply. So in particular, if you're here, you're actually lending. If you're here, you're fully invested in the stock market. If you're here, you're borrowing. If you're borrowing and lending rates are different, then it turns out that the curve that you want to be on actually has a kink in it. And that means that there is a potential for being on this curve, and then there's another tangency line that goes out at a different slope. That's possible. But that's more complicated than what we want to talk about at this point.

So here I am assuming borrowing and lending rates are the same. Zeke, and then Rami.

AUDIENCE: If I had a choice of, if I had control over the volatility of the market, then if the yield goes down, of the deals, then I would want to have a more volatile market so that I can intersect the curve at the higher return point. Tangent to the curve--

ANDREW LO: OK, hold on. You're changing the assumptions here. Why are you controlling the volatility of the market? The volatility of the market is a data point that you're basically using as an input. OK?

AUDIENCE: I know, I know. I'm just trying to figure out what I am-- because you're basically connecting, I see this as a connection between the yield curve and the market. Because it's not only retrospective in the sense that if the yield goes down, there's cash flowing from the market--

ANDREW LO: Let's not worry about the dynamics. This is not meant to be a dynamic story. I didn't say anything about this happening over time, and there's lots of different changes going on. This is a static snapshot, today versus next period. These returns and covariances and all that apply to the returns from this period to the next, whether it's monthly or annual, that's a static snapshot as of today. So we're not talking about any term structure effects yet.

Yeah, Rami?

AUDIENCE: This is assuming three stocks. So if you had four or five, are you going to actually move the bullet left? And then you're gonna change--

ANDREW LO: Yes, absolutely. If you start adding more stocks to this cocktail, what's going to happen is the bullet is going to shift to the left, and it's going to shift up. And so the tangency point will change. But the curve, that straight line, the tangent line, what you're going to see is that tangent line is going to go like that. The slope, that's right. You're going to get more expected return per unit risk. And that is something we're going to take as a measure of how good this particular trade-off is. We're going to look at that slope of this line. And the slope of this line will give us a measure of the expected rate of return per unit risk. That's exactly what it's going to do for us.

AUDIENCE: So if you're in the upper right beyond the tangency [INAUDIBLE], you know that line? Then you're borrowing [INAUDIBLE] portfolio?

ANDREW LO: That's correct.

AUDIENCE: If you were to extend the line leftwards, down to the left, would you be then shorting the market to invest in T-Bills?

ANDREW LO: Yes. And if that happens, you know what you would do? It would not go this way, because of course standard deviation can't be negative. It would go like this. It would go this way. Because standard deviation is always non-negative. It's the square root of the variance, which is always positive. So if you decided to short the tangency portfolio and put it in T-Bills, well, you'd be a knucklehead. But you would be on this line right here. You would have higher and higher risk, because you're taking a short position on equities, and you'd have a lower and lower return because you're shorting the high yield asset and buying the low yield asset.

Any other questions about the geometry of this point. It's very important, this is a major insight. Yeah?

AUDIENCE: Earlier we discussed about how we bring market [INAUDIBLE] portfolios and expected return changes as a function of n and standard deviation from certain changes [INAUDIBLE]

ANDREW LO: Yes.

AUDIENCE: Is that the reason why the shift is more towards left? Because as we add more and more portfolios, the n dominates the [INAUDIBLE]?

ANDREW LO: That's right, that's right. As we add more securities, you get more and more impact of diversification. So that increases your expected rate of return per unit risk because you can't make somebody worse off by giving them choices. They can always put a 0 for the new stocks that you give them if they don't like it. So the only thing you can do is to make you better off, meaning the only thing you can do is to give you a higher level of expected return per unit risk, or a lower level of risk per unit of expected return. So by adding more securities, you're basically increasing the slope of this line.

So let's talk about the slope of the line. The slope of that line is equal to the expected return of that tangency portfolio, minus the T-Bill rate, divided by the volatility of that tangency portfolio. If you just calculate rise over run, that's what you get as the slope. There's a name for this. The name for this is called the Sharpe ratio. You may have heard of this, particularly those of you who have interest in hedge fund investments. Hedge fund managers will often quote their Sharpe ratio very proudly. The Sharpe ratio is simply a measure of that risk-reward trade-off. The higher the Sharpe ratio, the better you're doing. If you're a mean variance optimizer, meaning you prefer more expected return and less risk.

So the idea behind the tangency portfolio is that it is the one that will give you the highest Sharpe ratio. Let's look at it again. If you pick a portfolio like here, take a look. Look at the slope. The slope is going to be lower. Take a point over here, in the inefficient branch of the bullet. Then the slope is going to be even lower than the upper branch. The biggest slope occurs when you invest between T-Bills and that tangency portfolio. That's what you're optimizing. Yeah?

AUDIENCE: I have a hard time understanding why the bullet would go left when I add additional stocks. I understand it analytically, standard deviation goes down. But then on the other hand, wouldn't it happen that the likelihood of correlation between these additional stocks would decrease and therefore as we--

ANDREW LO: How would the correlation increase by adding another stock?

AUDIENCE: I mean the likelihood that I have 20 stocks, the overall correlation is higher rather than [INAUDIBLE]

ANDREW LO: How could that be? You've got 20 stocks, and they've got a correlation among those 20 stocks. Now, I want you to think about adding a 21st stock. When you add that 21st stock, you don't affect the existing correlations, right? I mean, it is whatever it is. Those are parameters. At least for now, we're going to call them parameters. When I add my 21st stock, I'm giving the investor an extra degree of freedom. Now instead of investing among 20 securities, I'm going to let you invest among 21. You don't have to invest in the 21st. Or another way of thinking about it is that when you only had 20 stocks, you really had 21 portfolio weights. But the 21st weight, I've arbitrarily constrained to be 0. Now, I'm going to loosen the constraint and I'm going to say, OK, now you can invest in the 21st stock. You won't affect the existing correlations, but the new stock that you add in can benefit in providing additional diversification benefits.

AUDIENCE: But as a general rule, we've always been trying to have negative correlation so that the bullet was left.

ANDREW LO: Oh, well actually, you don't need a negative correlation to make this go to the left, you just need to have something less than 1. Remember? From the last lecture? This is a case where you had perfect correlation. Anything less than perfect correlation, brings you to the left. So as long as my 21st stock is not perfectly correlated with the existing stocks in that portfolio of 20, I'm going to move things to the left.

AUDIENCE: In general, [INAUDIBLE] any stocks?

ANDREW LO: In general, that is true.

AUDIENCE: I mean I would try to have the negative correlation.

ANDREW LO: You would, but what this suggests is that negative correlation is a very rare thing.

AUDIENCE: It's difficult.

ANDREW LO: It's very difficult, it's extremely difficult. Now, from the analytical perspective, we can conclude it's very difficult. Let me ask you from an economic perspective, why is it difficult to find a stock that's negatively correlated with all other stocks? Anybody give me a business rationale for that? Yeah, Ingrid?

AUDIENCE: There's what you said before that when the economy goes down, everything goes down and we get vice versa. Just that we mention [INAUDIBLE] different countries, in different economic regions, in different industry, and they should not be--

ANDREW LO: Let's actually spend a little bit more time thinking about this. I want you guys to tell me right now, give me a stock that you would put your money in right now, today. S&P has gone down by 45% since the high several months ago. The stock market's doing terribly, and it doesn't look like it's getting any better. So you tell me, what stock would you put your money in right now, today?

Yeah, Terry.

AUDIENCE: Campbell's Soup.

ANDREW LO: Campbell's Soup. Why is that?

AUDIENCE: It's a food stock. It's a foodstuff people need, will purchase, inexpensive, pretty much just for--

ANDREW LO: OK. But on the other hand, if people are poorer all around, might not they start consuming even less of canned soup and try to make their own soup from little packages of ketchup and hot water? I saw that on an I Love Lucy episode years ago. It's pretty cool. So are you sure? Are you sure that Campbell's Soup is going to go up over the next few months, in response to the current crisis?

AUDIENCE: It'll stay pretty stable.

ANDREW LO: It'll stay stable. Ah, but that's not negative correlation. That's 0 correlation. I want something that's going to go the opposite direction of where the economy is heading. Tell me where that is? Yeah?

AUDIENCE: [INAUDIBLE] short financials.

ANDREW LO: OK, fine. So you're going to short the market. That's a cheap answer. Sorry, you don't get any credit for that. I want to answer the question that was raised by David which is, show me a stock that can get me even more to that left. I want a negatively correlated stock. Yeah, [INAUDIBLE].

AUDIENCE: Wal-Mart.

ANDREW LO: Wal-Mart?

AUDIENCE: It's been going up.

ANDREW LO: Well, that's not the same thing as saying that it is going to go up over the next several months in response to this economic crisis. You don't think that there's going to be a decline in consumer spending that will affect retail as well?

AUDIENCE: So far, everybody's going to Wal-Mart. If they don't go to Wal-Mart, where would they go?

ANDREW LO: Well, that's what I'm asking you. Where are you going to go? So you're telling me now that you believe that Wal-Mart is the answer? You think it will be negatively correlated? Historically, just to let you know, retail has not been negatively correlated with the business cycle. Yeah, Zeke?

AUDIENCE: What about Freddie Mac?

ANDREW LO: Freddie Mac?

AUDIENCE: Yeah.

[LAUGHTER]

ANDREW LO: If you like that investment, I have something else for you--

[INTERPOSING VOICES]

--afterwards.

AUDIENCE: We could do it this time.

ANDREW LO: I don't know if you want to argue that Freddie Mac is negatively correlated with market downturns. I mean, the reason that Freddie Mac got into the trouble that it did was because of the economic downturn. All right, one more. [INAUDIBLE]?

AUDIENCE: Philip Morris.

ANDREW LO: Philip Morris. That's an interesting one. Obviously, people are very nervous now. When you're nervous, you're going to be smoking. On the other hand, again, one could argue that it's not negatively correlated. It might be either slightly positively correlated, but even there, people have argued that cigarettes are a consumption good that can get hit with a downturn in markets. The bottom line is that it's really hard to come up with negative correlated stocks. Let me tell you, if you found one that was negatively correlated, if you found one that was really, really, negatively correlated, what would all of you do?

AUDIENCE: Buy it.

ANDREW LO: Exactly, you'd buy it. The effect of that would be to increase the price and depress the expected return. Remember what the expected return is. It's the expected future price, divided by the current price. If now all of you go out and buy Wal-Mart, or whatever stock you think is negatively correlated, that would have the impact of increasing the current price and therefore decreasing the expected return.

Now if you have a stock that's got a negative covariance and a negative return, that doesn't help. Because in fact, that was a suggestion that was put forward here. Let's just take the S&P and short it, and then you get a negatively correlated stock. The problem is that it's also got a negative expected return and then you're not helping things.

The key is to find negative correlation with a positive return. If you can find that, then you've really found something worthwhile. But my guess is it won't last, for exactly this reason.

Other questions?

OK, so now, let's go back and ask the question, what does this mean if we agree that all of us want to be on that tangency portfolio. What does that tell us? Well, that allows us to then make an argument that managers that are trying to provide value added services for us, they need to be doing something above and beyond what we can do ourselves.

Now, here's where Warren Buffett meets modern finance theory, in a way. If I want to see whether or not Warren Buffett or any other managers are adding value, one simple criterion that I can put forward is this. This Is what I can do on my own. I can get that line pretty much by just using my basic finance skills that I've learned here at MIT. If you're going to manage my money and charge me 2 and 20, show me what you can do above and beyond this. I want you to tell me where you can get me on this graph. Can you get me up here? Can you get me over here? Can you get me anywhere either to the left or above that curve?

We can use that as a measure of performance, and there's a name for that. It's called Alpha. Typically, when people talk about Alpha, they're talking about deviations from a line like this. We're going to get to that more formally, you don't have to write it down or make note of it just yet. It's on the next slide. But we're going to show you how to measure that explicitly so now, not only is this a good idea for you as a baseline to manage your own portfolio, but you can then use it as a metric to gauge whether other people are adding value to you.

So Warren Buffett would say, no problem. I think I've got Alpha. So I'm not going to bother with this, I think I can get you up here. That is, if you want to invest with me. And in fact, if you looked at Warren Buffett's performance over the last 25 years that he's been doing it, or 30 years, his Sharpe ratio is a lot better than the tangency portfolios. So he actually has added value if you use this as a criterion. But the problem is, you have to identify the Warren Buffetts before they become Warren Buffetts. Because after they become Warren Buffetts, it's not clear that they're adding the same amount of value. It's already, the cat's out of the bag. Yeah, [INAUDIBLE]?

AUDIENCE: Question about the Sharpe ration. So is this a stag ratio, or is it dynamic? Because in my mind, as you gain more stock options, it's going to become sharper.

ANDREW LO: Yes

AUDIENCE: Opportunity cost for switching to T-Bills is going to be greater, so you're going to shift preferences away from T-Bills. Then isn't that point going to increase and flatten out?

ANDREW LO: Yes, so the dynamics of this are very complex. This is, right now, a static theory. Static meaning today versus next period. We're not looking at the dynamics over time. In order to do that, there's lots of different effects that are much, much more complicated. For that, you've really got to take 15 433 and even 433 won't cover those kinds of questions in complete detail because they rely on some very complex kinds of analysis.

But I'm going to get to that at the end. So if I don't, please bring it up again. I want to make a comment about that, and how you can take this relatively simple static theory and make it dynamic in an informal way even though the analytics become very hard when you try to do it formally.

So the key points of this lecture are, oh, sorry, question?

AUDIENCE: [INAUDIBLE] every point on the line, it's indifferent? Or--

ANDREW LO: No, no no. Not at all, not at all indifferent. Any point on this line is a different risk-reward combination. So in other words, it depends on your preferences.

AUDIENCE: You take a function for the industry?

ANDREW LO: Yes, yes. Now, we haven't talked about utility functions yet, but we're going to in a little while.

Let me preview that, since you asked. You all remember what indifference curves are? From basic economics? An indifference curve, when I first came across that, I was rather offended because I don't view myself as an indifferent individual. I have lots of passions. And so, why should we be indifferent about two choices? In fact, that's an economic term. It simply means that you are just as well off between two combinations and therefore, these two combinations you're indifferent to, you're indifferent between.

So if I had to ask you to draw on this graph, an indifference curve of risk-reward trade-offs for you, the typical individual, what would it look like? Can anybody give me a sense of what different kinds of risk-reward trade-offs you would be indifferent among? And to make the question a little bit simpler, let's start off with a particular point.

So let's suppose that this point right here is the point that I want you to draw the indifferent curve from. Which is a standard deviation on a monthly basis of about 6%, and an expected return of about, say 1.4% or something. So you've got a monthly return of 1.4% and a risk of about 6%. Give me another point that you would be indifferent between, versus that one? Anybody? Any volunteers? Yeah?

AUDIENCE: There could be a point above the tangent line, but to your right. Somewhere where you're pointing.

ANDREW LO: OK, so do you have a particular number in mind? In other words, let me ask you this. If I cranked up your volatility from 6% to 8%, how much extra return would I have to give you in order for you to be just as well off as you were at 6% and 1.4%?

AUDIENCE: Something slightly higher than the 1.8% [INAUDIBLE].

ANDREW LO: OK, higher than 1.8% OK.

AUDIENCE: Or what about the corresponding value. 1.41%.

1.41%

ANDREW LO: OK, this is 1.41%. And if I said, now I want to be at 8%, how much risk do I have to give you, how much expected return do I have to give you to make you just as well off as this point?

AUDIENCE: Higher then 1.2%, right? Or 1.4%, higher than 1.4%.

ANDREW LO: Right, but how much higher? That's the question. It's a personal question. Rami?

AUDIENCE: 1.3 times your initial expected return. So 33% on top.

ANDREW LO: You would have to have an increase in 33% of your expected return, even though I'm only giving you a 25% increase in the risk.

AUDIENCE: Well, no. You'd have to at least do 25% of the-- sorry. Put 6%-8% is--

ANDREW LO: That's a third, you're right. So I'm increasing the risk by 1/3, you want me to increase the expected return by 1/3. So your trade-off is linear, is that right? You're looking at it linearly? Anybody else? You know, you may want to translate this into annual numbers. Because I'm sensing that you may not have a good feel for what your own preferences are. And by the way, this is a challenge. Not everybody understands what their own personal preferences are for these numbers. This is not a natural act of human nature, that we automatically have preferences on these numbers.

But the bottom line is that if I make you take more risk, I'm going to have to compensate you and give you more expected return. There's got to be a reason why you want to take that risk. For some people, it's linear. For other people, it's much more than linear. They don't want to take any more risk. In fact, right now most investors don't even want to answer that question. Because they don't want to take more risk. And you say, well what if you did? Well I don't want to. Well, but just what if? I don't want to what if. I just don't want to take the risk.

So they can't even answer that question. But if they could, my guess is that it would be way up here. So you'd have to give them a lot of expected return to make people take more risk today. Alternatively, if you want to give people less risk, my guess is that you can actually subtract a lot of return in order to take away a little bit of risk. How do I know that? Take a look at the yield on the three month treasury.

So an indifference curve. It's going to look like this. It's going to look like it'll be increasing, but it'll actually be bowed this way. And the theory behind why it's going to be convex, holds water as opposed to spills water, the reason it's going to be convex is because there is a decreasing, or diminishing, marginal utility between risk and expected rate of return. Like anything else, economists have this notion of diminishing marginal utility between any two commodities.

If you've got ice cream sundaes and basketballs, there's only so many basketballs that you can enjoy before the next incremental basketball provides relatively little pleasure for you. The same thing with ice cream sundaes. You can only consume so many ice cream sundaes before the next incremental sundae provides somewhat less benefit to you. That kind of diminishing marginal utility gives you this kind of a bowed curve.

So where you are on this straight line depends upon how bowed your curve is. Somebody that's really risk-averse has a curve that looks-- let me draw this because it's a little bit easier to see rather than trying to follow my laser pointer. So here's the trade-off, this is the line. Somebody that's extremely risk-averse is going to have curves like this. Those are indifference curves. And as you go to the Northwest, you're happier and happier.

So the optimal point is where this indifference curve hits this particular line. On the other hand, if you're very risk-seeking, if you don't need a lot of compensation of expected return per unit risk, then your difference curve's not going to look like that. It's going to look like this. In which case, you're tangency point will be farther to the Northeast. You'll be taking more risk and getting more expected rates of return.

But the bottom line for this graph and this lecture is that everybody, no matter what your risk preferences are, everybody's going to want to be on that line, that tangency line. And it turns out that that insight is going to translate into a remarkable, remarkable conclusion about risk-reward trade-offs.

So the key points for this lecture are diversification reduces risk. In diversified portfolios, covariances are the most important characteristics of that portfolio. It's not the variances, but the covariances. Investors should try to hold portfolios on the efficient frontier, that upper branch. And with the riskless asset, everybody is going to want to be on the tangency line. Those are the major conclusions from this analysis. And you can work all of this out analytically using the mathematics of optimization theory, but in fact, all of this can be done graphically as we have geometrically. Question, [INAUDIBLE]?

AUDIENCE: Sorry, this is sort of a simple-minded question, but I'm having trouble thinking of the expected return. I know it's absolute, but so much of the portfolio metrics, I guess, are relative to the benchmark of the market rather than [INAUDIBLE]. So I don't know.

ANDREW LO: OK, so we're going to get to that. We're going to talk about benchmarks because you're right, that most investments today are all benchmarked against something, right? And you're probably wondering how that got to be. That whole direction of analysis and performance attribution, that came out of this. In other words, it was because of this particular academic framework that was developed by Harry Markowitz, and Bill Sharpe, and others, that indexation and benchmarking came to be.

So I'm going to get to that. Let me put that off for another lecture or so. After we derive the implications of everybody wanting to hold the tangency portfolio, it's going to turn out that that tangency portfolio happens to be the benchmark. So we'll get to that. Yeah?

AUDIENCE: Is there any assumption behind all this analysis that would say, if you have these preferences, then you should choose this portfolio. If everyone did that, does that change [INAUDIBLE]

ANDREW LO: So you would think that it would, but in fact, I'm going to show you that there is exactly one case where it doesn't. And that's the case of the equilibrium that I'm about describe. So let me turn to that right now. There any other questions?

AUDIENCE: We've been using risk and standard deviation kind of interchangeably, whereas I think of risk as the risk of not making anything. Is there a way to mathematically translate from a standard deviation in your portfolio to the risk of not making [INAUDIBLE]?

ANDREW LO: Well, there is. Although, I would have to say that if you have a preference about the downside, so not making anything as you point out. Then that changes this analysis. So this analysis really requires that you use standard deviation as the sum total of your perception of the risk of a portfolio. If you have other kinds of sensitivities, then you need to bring them into the analysis, and that will change these outcomes.

AUDIENCE: The way we do this, if we measure the risk of a company, if they're historically-- Nevermind. I guess if they were historically, they varied higher, if that wasn't strictly normal, and they end up being higher in market [INAUDIBLE] lower, they would still have a larger deviation so you're correlating that with companies that are also [INAUDIBLE]. Does that make sens?

ANDREW LO: That's true, but again, you've made an assumption there that I'm not making. Which is you're assuming companies are outperforming or underperforming. I'm assuming that the data are given, and I'm not making a bet on whether any companies are likely to succeed or fail. I'm merely looking at companies as investment opportunities that provide certain expected returns, volatilities, and covariances. You want to go down the path of Warren Buffett, and I'm resisting that because I don't have the skills of a Warren Buffett. So I don't know what's a good value and what's not a good value.

And the case in point is a discussion we just had today. You tell me what is a good value today? Do you really believe that Campbell's Soup or Wal-Mart should be the companies you invest in today? I don't know. I mean, another argument is entertainment. Why don't you invest in movie theaters? Lots of people now are going to see the James Bond movie, and they want to escape from reality. Wouldn't that be a growth industry, given market conditions? Well, that's true, but how many people have $12 to spend on a movie? Plus, you've got to get the popcorn and the bonbons, and all those. And by the time you're done, it's like a $60 evening. I mean, I don't know.

So the point is that unless you are willing to make predictions, this is the only alternative that provides a disciplined approach to investing in so-called good portfolios. So it's a different approach.

So now, let me turn to the next lectures. Lectures 15 through 17, where we're now going to talk about equilibrium. We've already identified that all of us in this room, assuming we have mean variance preferences, that's an important assumption, I grant you, but it's not an unreasonable one. It's just, it is an important assumption.

We've all agreed that we're going to take on portfolios that lie on that line, and therefore, the portfolio that is the tangency portfolio, I'm going to give it a special name. I'm going to call it M, portfolio M. What we now know is that, given a choice between holding n securities and T-Bills, versus holding T-Bills and a single portfolio, all of you would be indifferent between those two choices, if that single portfolio were M. The tangency portfolio. Do we agree on that?

So therefore, I could, in principle, construct a mutual fund called M. This mutual fund holds stocks in exact proportion to the weights given by that tangency portfolio. In other words, it is the tangency portfolio. So what that suggests is that all of you in this room would be absolutely indifferent between investing among the n stocks and T-Bills on the one hand, versus investing in two securities on the other. One security is T-Bills, and the other security is shares of mutual fund M. Do we agree on that? Any controversy there? I know I've made a number of assumptions to get us here, but given mean variance preferences, which is not an unreasonable assumption, and given that we've assumed these parameters are stable over time, that's where we are. Rami?

AUDIENCE: Somebody might have said this, but you assume all fees are trading fees?

ANDREW LO: Forget about fees. There are fees no matter what you do. So for now, I'm going to forget about fees. I'll put fees back in later, and if I do that, then it's going to look even more compelling for you to want to invest in mutual fund M, versus n stocks. I don't know how many of you have traded individual stocks, but if you ever try to manage a portfolio of 1,000 stocks, it's actually fairly time consuming, right?

And by the way, there are more than 1,000 securities. I mean the S&P 500 you can think of as being M, but that's an approximation, right? There's probably 7,000 or 8,000 securities that trade today. Probably only 2,000 or 3,000 that you would really take seriously, and probably only 1,500 that you really need from a diversification perspective. 1,500 stocks. Would you want to trade in that, or would you want to trade in one mutual fund? Yeah?

AUDIENCE: Can I ask, I mean knowing that with M you're trying to get on that tangent portfolio? And you said, for example, Warren Buffet beats it the whole time. Why don't you just buy one share of Berkshire Hathaway, and you'd have a higher Sharpe ratio?

ANDREW LO: Because Warren Buffett beat it in the past, do you think he's going to beat it in the future?

AUDIENCE: I would [INAUDIBLE] it.

ANDREW LO: I don't know. That's right. Good question, good question. I mean, if you're thinking about Warren Buffett as a 10 year investment, I think I might short that. I mean, you know, he seems healthy, but you know those Cherry Cokes have to have an impact. I'm sorry. You know, you eat enough steaks at that Omaha restaurant, I don't know what it is, and those Cherry Cokes, I don't know.

OK, so fine. Let's not do Warren Buffett, let's do somebody else. Fine. You tell me who that is? Tell me who the next Warren Buffett is? Can anybody tell me? I'll be happy to do that, I'll be happy to invest in them. Who is it?

AUDIENCE: Andrew Lo.

ANDREW LO: Thank you, but those who can't do teach, those who can't teach, teach gym. And at least I don't teach gym.

The point is that we don't know who the next Warren Buffett is going to be, and I don't want to have to figure that out. I mean, that's a pretty tall order to tell an investor that they've got to figure out who the next investment genius is. If they knew, they wouldn't have to ask them to invest. They'd invest themselves, right?

So what I'm showing you is a simple way of investing that may not be as good as Warren Buffett, but it's certainly better than trying to pick the next Warren Buffett if you don't know what you're doing. Jen?

AUDIENCE: Is it easier to kind of figure out the future covariances of the different than it is to pick the next Warren Buffet?

ANDREW LO: Thank you, that's another way of looking at it. If you ask the question, is it easier to try-- is the historical covariances and variances and expected returns more predictive of the future than your ability to find the next Warren Buffett, then yes, that's another good argument. That in other words, this framework relies on less ability to forecast. It doesn't completely rule it out because, as I said, these parameters, they change over time. And you have to think about that impact. So it's not totally trivial, but from the theoretical perspective, it seems like it's a very internally consistent approach.

Now, let me go on for a little while longer because if it were just this, then this would be an interesting rule of thumb. But this is not a theory of financial markets just yet. I haven't really done anything truly astounding because you're still left with the question of, what's the appropriate risk-reward trade-off? What should I use for my discount rate? A lot of financial decision making is not just picking stocks and making good investments. But it's whether or not should I invest in nanotechnology as a corporate officer of a particular tech company, or should I invest in green technologies? What discount rate should I use? How should I engage in capital budgeting or project financing? All of these questions seem like they have nothing to do with investments. So I don't want to make this course into an investments course. There's a lot about corporate financial management that relies on being able to understand these markets.

So let me show you where we go next, because we're very close now to the big payoff. We've already identified the tangency portfolio as being special. I'm going to call that portfolio M, and I'm going to argue that everybody in their right minds are going to be indifferent between picking among these two investment opportunities, T-Bills and M, versus the n plus 1 investment opportunities of all stocks, plus T-Bills.

It turns out that portfolio M, therefore, has to be a very specific portfolio. And it turns out that that portfolio is the portfolio of all assets in the entire economy, in proportion to their market capitalizations. Now what I just said is an incredibly deep result, so I don't expect you to just get it. Let me say it again. First of all, I want you to understand it, and then I'm going to try to give you the intuition for it.

If it's true that everybody, not only in this room, but in the world, if everybody in the world is indifferent between investing in those n plus 1 securities, and in two, then we can argue that those two securities play a very special role. In particular, think about what that mutual fund M has to be. Everybody in the world wants to hold M.

So, let's make the leap of faith that everybody does hold M. So in other words, now we're in a world where everybody is already mean variance optimizers, and they already hold two assets in their portfolio. The treasury bill asset, and the mutual fund M. So you hold M, you hold M, you hold M, you hold M, you hold M, you hold M, you hold M, everybody holds M. We hold different amounts of it, so as a hedge fund manager, you're holding a large amount of M. In fact, you're holding twice as much M as your wealth allows, and you're borrowing T-Bills to do so. Somebody who's very conservative is holding a very tiny little bit of M. Mostly, that person is invested in T-Bills.

But the point is that every single person's portfolio you look at, when you look at their portfolio, it's M. If that's true, if what I just said is true, what portfolio does M have to be? There's only one that it can possibly be. And that is the portfolio of all equities in the marketplace, held in proportion to their market value. Do you see the beauty of that?

Now, let me try to explain it. I hope you understand it, let me explain it. Why does that have to be? This has to do with supply equaling demand. Now, I'm going to make an argument about equilibrium. I haven't done so up until now. Up until now, I haven't said anything about supply equaling demand, but I'm about to do so.

If everybody is holding this portfolio M, that's the demand side, right? Everybody is demanding M. On the supply side, I'm assuming that all stocks that are being supplied are held. If all stocks that are being supplied are held by somebody, but if everybody in the world is holding the same portfolio M, when you aggregate all of the demands. So I'm going to add up your demand, and your demand, and your demand, and you're. We're going to go through the class, and go through the world, we're going to add up everybody's demand. In every single case, your weights are identical. You're holding the same portfolio M.

So when I aggregate the entire world, and I get the portfolio M, what does it have to equal? It can only equal the sum total of all assets in the world, right? Supply equals demand. And therefore, when I aggregate all of your holdings of M into one big fat M, that big fat M can only be equal to one thing, which is all the equities in the world. And the weightings are just simply their market caps, right? There's only so much of General Motors. Take the entire sum total of that, that's the global investment in General Motors. And then you do that for every single stock, and you divide that by the total market capital of all stocks, you get the market portfolio, M.

So this shockingly, simple, but extraordinarily powerful result is due to Bill Sharpe. Harry Markowitz came up with portfolio optimization. He applied mean variance analysis to portfolio optimization and argued that everybody has to be on the line. Bill Sharpe looked at this and said, aha. If everybody's on that line, that means that everybody's going to be either holding M or T-Bills, or both, and therefore, the only thing that M could possibly be is the market portfolio.

And now we have a proxy for the market portfolio, the Russell 2000. Or the S&P 500. Both of those are very well diversified stock that have lot-- they don't have everything in it-- but they have a lot of things in it, that proxy for everything. The Russell 2000 has 2,000 stocks weighted by market cap. That's as close as you're going to get to everything that you care about. So now, you'll see we're benchmarking is coming from, but I'm going to get back to that in more detail.

So this equilibrium result that says supply equals demand, identifies this portfolio M. And what it says is that if everybody does this, if everybody takes finance here and learns how to do this, it's not going to kill the idea. It's going to lead to a very well-defined portfolio M. Now, let me take it one step farther, then I want to ask you to ask questions. If I know what that portfolio M is, then I've got an equation for this line. I can write down a relationship between the expected return and risk of a portfolio on this line.

And this is it. The expected rate of return of an efficient portfolio, by efficient I mean a portfolio that's on that line. Anything that's not on that line, if it's below that line, it's inefficient, right? You're not getting as much expected return per unit risk, and you're not reducing your risk as much as you can, per unit of expected return. The expected return of an efficient portfolio is equal to the risk-free rate, plus the ratio of the standard deviation of that portfolio, divided by the standard deviation of the tangency portfolio, or the market, multiplied by the excess return of the market portfolio.

This result is a risk-reward trade-off between risk and expected return. You see, what it says is really something quite astounding. It's telling you that, here's the risk-free rate. That's the base return for your portfolio. And what this is telling you is that what you should expect for your portfolio is that base return, plus something extra. And the extra is the market's excess return, multiplied by a factor. And the factor is simply how risky your portfolio is relative to the market.

Let's do a simple example. Suppose that your portfolio is the exact same risk as the market. Well, if that's the case, then what is your expected rate of return?

AUDIENCE: The market.

ANDREW LO: It's the market. So it's the risk-free rate, plus the market excess return, which, when you add it together, is just the market. Suppose you're holding a portfolio that's more risky than the market. Is your rate of return greater or less than the rate of return of the market?

AUDIENCE: Greater.

ANDREW LO: Greater. Suppose that your portfolio has no risk. Suppose that sigma p is 0, then what's your rate of return?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Exactly. Makes sense, right? This is very intuitive. What this tells us, now, is that we can figure out what the fair rate of return is for an efficient portfolio. For any portfolio on this line, I can tell you what my fair rate of return is, and it's an objective measure. It's not just theory now. Now, I can go into the marketplace, I can measure the expected return of the market. You know what that is, historically? Not including the last few months.

AUDIENCE: 7%.

ANDREW LO: It's about 7%, historically. Over the last 100 years, 7%, the expected rate of return of the market. Sorry, the expected risk premium, the excess rate of return. About 7%. What about the volatility of the market? It's been about 15% historically. So according to this relationship, I've already figured out what this number is. It's like 7%. I've already figured out what this number is. It's like 15%. So now, you should be able to get a benchmark for what to expect when you've got a particular level of risk in an efficient portfolio. You've got all the ingredients.

What about risk-free rate? Well, it depends on what risk-free rate, but let's talk about over a one year period. Right now we're looking at somewhere between, I don't know, 1%, 2%, 3%, depending on what day of the week you're looking at. So one year T-Bill rate is about 1% or so, yeah?

AUDIENCE: I think it was the last class we talked about unsystematic risk.

ANDREW LO: Yes.

AUDIENCE: Is that defined by [INAUDIBLE] in this case?

ANDREW LO: No, the unsystematic risk is risk that is not measured by sigma p, so we're going to come back to that. Let me hold off on that for now, because I want to come back to it after I finish developing this. There's going to be a connection between systematic and unsystematic risk that's going to come right out of this relationship. Yeah, Brian?

AUDIENCE: So if you take the S&P 500 as M here, the market portfolio, and the capitalization is the weight, so you've got non-zero weights for all the different stocks there. Does that imply that there's no stocks in the S&P 500 that are Southeast of any others?

ANDREW LO: No, no, there could be.

AUDIENCE: Why would you have them, because we said those are strictly non-preferred?

ANDREW LO: Well, that's if you're looking at a pairwise comparison. If, now, you're trying to create an entire collection of these portfolios of securities, that's a different story. That's why I answered in response to Justin's question. Justin said, why not just trade off those two? Why not? It's because you can do far better by using all of them in this way.

You see, by looking at pairwise, you can no doubt do better. But if I use all of them, I get this entire line. And you can't get that entire line just from looking at two of these stocks, you need all of them.

AUDIENCE: So in this portfolio of three, if you kick GM over to the right a little bit, and made it strictly non-preferred to IBM, then you still might have a positive portfolio weight on GM?

ANDREW LO: You might, but more likely, it'll be a negative portfolio weight. It'll be negative, and you'll be shorting it somewhere along the line here. However, the tangency portfolio, by assumption, if it's the market portfolio, cannot have negative weights. And so there, what will happen, is that all of the stocks will change in their relationship based upon various different kinds of equilibrium, so that you won't get into a lot of those situations where you're going to be shorting these negative stocks. Yeah?

AUDIENCE: So basically, according to the Sharpe theory, every stock that the market, the capital is not 0 is worth holding in some portfolio?

ANDREW LO: That's right.

AUDIENCE: Diversifying your portfolio.

ANDREW LO: That's right. Every stock has some benefit in adding to this particular risk-reward trade-off, and the sum total benefit is summarized by this line. That's the ultimate objective.

AUDIENCE: If I don't hold a specific stock in the market and I gain a diversification [INAUDIBLE]?

ANDREW LO: Sorry? If you hold a specific stock?

AUDIENCE: If I don't hold it, because it has market [INAUDIBLE].

ANDREW LO: Oh, if you put 0 weight. Yes. What Sharpe would argue, based upon this theory, is that you want to hold as many stocks as you can to get the most diversification. Now, that's the theory. In practice, it may well be that the benefits do not outweigh the costs, because when you hold multiple stocks, you have to manage them, and so it may cost more. So a mutual fund that has 3,000 stocks may have a higher expense ratio than a mutual fund with 500. It may not. Nowadays, actually, the technology is so good that probably it doesn't. But 15 years ago, that was not true. But apart from the transactions cost, the theory suggests more is better. Because it will always give you more opportunities, and it can never hurt you because you could always put a 0 weight on them if you don't like them.

Now, it turns out that this is a trade-off between the expected return of an efficient portfolio, and the risk of that portfolio. In other words, this applies only to portfolios on that tangency line. What if you want to know what the expected rate of return is for Wal-Mart? We just said that no individual stock is going to be likely to be on that efficient frontier. And therefore, no individual stock is likely to be on this line.

So this is great if what you're talking about is investing in efficient portfolios, but how does that help the corporate financial officer that's trying to figure out how to do capital budgeting for a particular pharmaceutical project? It turns out, it doesn't. It doesn't help. This doesn't answer that question. It turns out, you need to have an additional piece of theory that allows you to derive the same results, not just for the efficient portfolios here, but for any portfolio. And this is another innovation of Bill Sharpe. This is actually why Bill Sharpe won the Nobel Prize. It was not for this little picture here, but it was for this equation right here.

What Bill Sharpe discovered is after computing the equilibrium relationships among various different securities, he's demonstrated that there has to be a linear relationship between any stock's expected return and the market risk premium. Just like here, where you've got the risk-free rate, plus some extra premium. So this is the premium, the second term. But what Bill Sharpe showed was that if this portfolio is not an efficient portfolio, if it's not on that line, the linear relationship still holds. But it turns out that this particular multiplier is no longer the right one to use. It turns out that the right parameter to plug in there, is something called beta.

Now, you've heard all about beta, I'm sure. But now, I'm telling you exactly what beta is. Beta is the multiplier that is defined by the covariance between the return on the market and the return on the individual asset, divided by the variance of that market return. If the portfolio happens to be on that efficient frontier, then this beta reduces to this previous measure. So this is a special case of the more general relationship where beta is used as the multiplier.

So let me repeat what beta is. Beta is the ratio of the covariance between the return on the particular asset or portfolio, that may or may not be efficient, it's any asset, with the return on the market portfolio. So this numerator is a measure of the covariability between the particular asset that you're trying to measure the expected return of, and that tangency portfolio, divided by the variance of that tangency portfolio.

Beta, it turns out, is the right measure of risk, in the sense that it is the beta that determines what the multiplier is going to be on the market risk premium, which is to be added to your asset's expected rate of return, above and beyond the risk-free rate. That's how the cost of capital is determined for your asset.

So I think you all saw how I derived this, but I didn't derive this. I'm just telling you this is really where Sharpe's ideas became extraordinarily compelling. And in order to understand how to derive that, I'm going to refer you to 433, because in that investment's course, we really delve into the underpinnings of that kind of calculation. It's a little bit more involved, it involves some matrix algebra. But it's not terribly difficult or challenging, and certainly be happy to give you references if any of you are interested. I believe it's in Brealey, Myers, and Allen.

But the bottom line is that this gives you an extraordinarily important conclusion now to the several weeks that we've been working towards this goal. Which is now, finally, after eight or nine weeks, I can tell you how to come up with the appropriate discount rate for various NPV calculations. The answer is the expected rate of return, the appropriate fair rate of return, or the market equilibrium rate of return, is simply given by the beta of that security, multiplied by the expected excess return on the market portfolio.

So now, this has a lot of assumptions, granted. We're going to talk about those assumptions over the next couple of lectures. But what we've done today is move the theory forward by quite a bit, because we've identified a particular method for coming up with the appropriate cost of capital as a function of the risk. Where the risk is measured, not by volatility anymore, but by the covariance between an asset and the market portfolio.

And next time, I'm going to try to give you some intuition for why this should be, why this makes sense, and why, in a mean variance efficient set of portfolios, why it reduces to something that we know and love.

Any questions? OK. I'll stop here, and I'll see you on Wednesday.