ANDREW LO: What I want to talk about is option pricing. But given that there's the midterm coming up, what I'd like to do is to actually skip the more technical part today. Today, what I was going to do was to describe a method for pricing options, a particular option-pricing formula.

Now, we have a course, 15.437, on options and futures. And that's really what I would recommend for those of you who are interested in derivatives. But we really can't let you leave MIT without understanding a little bit about the basics of option pricing.

And it's such a beautiful argument that it's important, I think, for all of you to see it at least once. But since I'd like you to focus on it and really absorb it, and I suspect that most of you are thinking about the mid-term, I'd rather postpone that till Monday, and then talk today about the very basics of option payoff diagrams, which is relatively straightforward. And then give you a little bit of a history of option pricing, and tell you a bit about how it came about. And ultimately, where the literature fits within the grand scheme of things.

So last time, if you recall, we talked about options as insurance. And we went through a very simple set of examples, where I described the put option as really being parallel to insurance in all of these different terms. But the differences are that a put option, first of all, can be used early. So you don't have to wait until you have an accident or wait until it expires. You can decide at any point in time that you want to exercise it.

Also, unlike insurance contracts, options can be bought and sold in organized exchanges. So you can buy a put option. You can sell a put option.

And then finally, dividends have an impact on options. And so most options have dividend protection, in the sense that if there's a dividend paid, then the strike price will be adjusted accordingly. Now, it's important to understand the differences between an option and an underlying.

Because they really have some very, very important distinctions, in terms of their payoffs. So the way that we try to emphasize that is by looking at a diagram that graphs the option value as a function of the underlying parameters that influence the option. And the most important parameter is, of course, the underlying price of the stock or asset on which the option is written.

So this is an example of a payoff diagram that plots the value of the option at maturity for a call option on an underlying stock. And the x-axis is the price of the stock. And the y-axis is the value or price of the option on the date of maturity or exercise.

So let's suppose that the option has a strike price of $20. That gives the holder of the option the right to purchase the stock for $20 at the maturity date. So it's a call option, meaning it gives you the right to call away or buy the stock. And the strike price is set at $20.

Now, if the actual price of the stock is below $20, you're never going to want to call the option. Rather, you're never going to want to call the stock. You're never going to want to exercise the call option.

Because if you did, you'd be buying something for $20 that would be worth less than $20. So if the true stock price is anything less than $20, this option, at expiration, is worth nothing to you. You would never use it.

Now, it's critical to understand that this payoff diagram is the value at maturity. Prior to maturity, if the value of the underlying stock is less than $20, the option could still have value. Typically it will have value. Because there's always a chance that the stock price goes above $20 at the maturity date.

So let's be clear that this is the value of the call option at maturity date. And if it turns out that the stock price is greater than $20, then the option has value. And the value increases, dollar for dollar, with the stock price above $20.

So the slope of this line is 45 degrees. It literally goes up in lockstep with the underlying stock price. To be clear, if the stock price is $25 and you get to buy it for $20, the option, that right to buy for $20 is worth $5.

Because the stock is really worth $25. So the way you can see that is you can buy the stock for $20, with this piece of paper that you own. And then you can turn around and sell that stock on the open market for $25. So you've made that $5 profit.

The important thing about this diagram, the blue line, is that the upside is unlimited. But the downside is very much limited, at 0. OK? So this is an example of a security that has an asymmetric payoff, asymmetric.

The upside is not the same as the downside. Remember the payoff of a stock, or of a futures contract. It's symmetric. It's that straight line. Here, this is not a straight line. It's kinked at the strike price, K. That's a very important feature.

Now, it looks like, from this diagram, this call option is one of these propositions that you hear on late-night TV, make a $1 million with no money down. Like, there's no way to lose.

How could that possibly be? How could we have come up with a security that has no downside? Wouldn't everybody want one? Yeah?

AUDIENCE: Well, it has value [INAUDIBLE].

ANDREW LO: Exactly. Yeah, there's no free lunch. So of course, everybody wants it if it's free. But of course, it's not free. So you have to pay for it. You have to pay something today in order to get access to this asymmetric payoff.

So the net payoff, that is, if you were buying the call option and paying a certain amount of money, then the net payoff to you would be given by the dotted line, which is the blue line. But you subtract from it the value of the premium that you pay. It's called an option premium. But it's just the price of the option whenever you bought it.

And then if you want to take into account the time value of money, you should take the future value of that price that you paid when you bought the option. So if you bought the option in the beginning of the month and it expires at the end of the month, you've paid something at the beginning of the month. If you want to find your net payoff, you could either, at the maturity date, subtract from the blue line the value of what you paid multiplied by the one-month interest rate factor, so that you subtract time t dollars from time t dollars.

Or you can do a present value, where you take the payoff and you move it back to the beginning of time. Typically what we do is we actually ignore the time value of money, just because it's a month's worth of interest. And people don't really worry about that too much. Yeah?

AUDIENCE: Can you make some inferences about the future price of the stock by looking at the price of the option?

ANDREW LO: Yes, absolutely, you can. And we're going to show you how to do that when I give you the asset pricing formula for it. But you're absolutely right. By looking at the option, that gives you information about what's going on.

Just like when I tell you for crisis management, if you look at T-bills today, you get a sense of how much demand there is for cash, putting money in your mattress. By looking at options, you actually get a sense of where markets are going to be going. So after I give you a pricing formula, next time, I'm going to show you the prices of options.

In particular, we're going to look at the price of a put option on the S&P 500 for the next month and for the next two months. And you're going to find a very, very big difference in those two. That's telling you something about where the market thinks volatility is going in the S&P 500 over the next couple of months. So yes, there'll be all sorts of wonderful things you'll be able to tell by looking at the prices.

But in order to do that, we do have to understand how these payoffs work. So getting back to this diagram-- I want to make sure everybody is with me now-- this dotted line shows you your net payoff and a net of the price you paid for this particular call option. And the neat thing about this net payoff is that it then describes to you the fact that this is not a surefire way to make money and not lose any.

You might lose money, because you paid something upfront for the call option. And so the only way you're going to come out ahead is if the stock price actually exceeds-- not this point, but actually something like this point. So the stock price has to go up by a little bit more than $20 in order for you to make money, net of what it cost you to buy that option.

Now, I want you to go back and think about the difference between an option and a futures contract. Remember a futures contract we said was no money down, 0 NPV when you get into the futures. That's not true with a call option. A call option is actually worth a positive amount of money on day one.

So if you want a call option, you've actually got to pay for it. And then there's an issue about whether you'll make money. Because it depends on whether the stock price exceeds this point. It's got to exceed not only the strike price, but the amount that you paid for that option.

Any questions about that? Or is that pretty clear? So this is important. So ask now if you don't quite get it. Because if you don't get this, you're going to get confused by what I'm going to say in a few minutes.

Let me give you another example, just to really fix ideas. Let's do the put option case. Now, the put option allows me to sell the stock for, let's say, $20, or before the exercise date. So with a put option, am I going to hope if I buy a put-- so I buy the right to sell the stock at $20. That's a little bit hard to keep track of.

I'm buying a piece of paper that gives me the right to sell the stock for $20. If I own that piece of paper, this put option, am I going to wish that the stock price goes up or down?

AUDIENCE: Down.

AUDIENCE: Down.

ANDREW LO: Down. I'm only going to get paid on the put option if the stock price goes below $20. Because then I have something valuable, right?

If it goes below $20, I get to sell the stock for $20. So I make the difference between what it's worth and $20. If the stock price is at $20 or above, then my put option expires, worthless. I'm not going to use it. Because it would be foolish for me to sell something for $20, when I can sell on the open market for $25.

So the payoff diagram is exactly the opposite of this. In fact, it looks like this. So now the blue line is the payoff of the option itself, the gross payoff. $20 and above, it's worthless.

But $20 and below, this is the 45-degree line. But unlike the call option, my upside for the put option is limited. Is limited to what?

AUDIENCE: 0.

ANDREW LO: Right, 0 or whatever that is, $10, in this case. If the stock price goes to $0, then my put option is worth a maximum of $10. So the upside is bounded by that $10 limit. And that's the gross upside.

If I look at the net, I subtract how much I pay. And then I get the dotted line. Any questions about the put options payoff?

Now, just to fix ideas, let me go back to the call option and show the difference between the stock return versus the call option return. If you take a look at the call option, again, it's going to look like this when you subtract from it the price. But the stock is going to look like that line there.

Meaning that the stock return is linear. But the option return is non-linear. And this is one of the most important and subtle ideas with this instrument.

Up until now, all of the instruments that we've looked at stocks, bonds, futures, and forwards, their payoffs have been relatively simple, in the sense that they're straight lines if you plot the underlying price and their payoffs. This is the first time we have analyzed the security that has a bizarre structure like this. And you might think it's straightforward because well, you understand the contractual terms of an option. But from a risk-and-return perspective, it's actually quite a bit more complicated than most people would appreciate.

One of the reasons that we are in a current financial crisis today is because of the complexity of the securities that have been created. And the complexities are really along the lines of these non-linearities. As I mentioned to you, insurance is a put option.

So you can actually use the theory of option pricing to value insurance contracts, like credit default swaps. In fact, the payoff of a credit default swap is not that different from something that looks like this. And what that means is that a portfolio of credit default swaps does not behave like a portfolio of stocks or a portfolio of bonds.

They have very important differences, both in terms of their risk, and in terms of their return. In this case, you can see the risk of a put option is bounded above. The upside is bounded above. The downside is bounded.

But the call option is unbounded above, in terms of its upside. Bounded below, in terms of its risk. What if, now, you decided you were going to sell somebody a call option? Or you were going to short a call option? Can anybody guess what the payoff would look like? Yeah?

AUDIENCE: If you're going to sell a call [INAUDIBLE] agreement that's your payoff.

ANDREW LO: Yeah.

AUDIENCE: As long as the stock doesn't go above the [INAUDIBLE] so someone can call you out. Then it's [INAUDIBLE].

ANDREW LO: So what is it going to look like, in terms of the diagram? How would I have to change this?

AUDIENCE: It would be flat, say, $2. And then it would go down.

ANDREW LO: That's right. It would be a mirror image of the blue line, where you reflect it along the x-axis, it would go this way. And what that means is that your downside is unlimited.

But your upside is very limited. Now, why would anybody want to do that? That seems like a terrible deal.

Well, the difference is that you are now getting paid to do that. In other words, if you flip this image-- let me draw it here. If you now have a call option that you've shorted, you go down here. This is $20. You will get paid for doing this. Meaning if you look at your net return, it's going to look like this.

So that means that as long as the stock price stays below a little bit extra than $20, you will actually get to keep that premium. But if the stock price goes up, your losses are unbounded. That's different. That's a different payoff structure than what we're used to with traditional instruments.

You can do all sorts of calculations. Long Call looks like that. Long Put looks like that. Shorter Call looks like this. And Shorting a Put looks like that.

And once you take all of these things and put them together, you can mix and match and get some really interesting payoff types of structures. So let me give you an example. This is just payoff tables that will show you when you get paid what.

So this is a very helpful exercise for you to go through, just to verify that these graphs are, in fact, what they should be. So I would ask you to go through this on your own. And there are all sorts of trade-offs that you can implement by looking at these various different payoffs and putting them together.

For example, you can buy a stock and a put, or buying a call with one strike and selling a call with another, or buying a call and a put with the same strike. Each of these portfolios of options gives you a different kind of a payoff diagram. And as a result, it allows you to make bets on market events that you otherwise wouldn't be able to make a bet on.

So let me give you an example of this. Let's see. Let's do something like, oh, I don't know. How about a call and a put? Suppose you decide to buy a call, and you buy a put with the exact same strike price.

So buying a call at a strike price of $50 will give you that left diagram. And then buying a put with the strike price of $50 will give you the right diagram. And your payoff for those two, at maturity, is going to look like a V.

Now, in fact, you have to subtract how much you paid in order to do this. So your net payoff will be this V, shifted down. Sorry, I didn't do the interactive graphics here.

But it's going to look like this, where what I've done is I've subtracted the amount of money it cost you to buy the put and the call. Yeah, question.

AUDIENCE: In this particular example, really all you're saying is, except for a short range in the stock price around the strike price, you will always make money.

ANDREW LO: That's right.

AUDIENCE: So is there a reason why you wouldn't do a lot of this, if you know that there is some movement that's going to happen in the stock?

ANDREW LO: So the question is, how much does it cost you to do that? When you say small, it's all relative. You've got to find out exactly what that is.

The smaller the range is, the more expensive it'll be for you to actually buy it. So there's a trade-off. It's all a matter of how much you pay for it.

But before I go there, let me just make sure everybody understands what this payoff is accomplishing. What are you doing when you are buying a portfolio with a payoff diagram that looks like this? What you're doing is saying that you're going to make lots of money if the stock price goes way up or way down.

The only way you're not going to make money, if the stock is not doing a whole lot, if it's staying around here. OK. So this is an example where you are making a bet. Not that markets are going to go up, not that markets are going to go down, but that markets are going to be wild.

That is, you're making a bet on volatility. Which may seem like a pretty good bet nowadays. But the problem is that there's a difference between this diagram and this diagram. And what is the difference? What determines how big or small this little tiny area is, where you don't make any money?

What determines that is how much you have to subtract and how far this V gets shifted down.

And you know what? Right now, it's shifted down a lot.

In other words, it costs a lot to buy a put and a call. Why does it cost a lot? Because volatility is very high.

And when you're buying a put, you're buying insurance. It's very, very expensive now to buy insurance. Because we're in the middle of a hurricane. And that's probably the worst time for you to buy hurricane insurance, is when you're actually in the middle of a hurricane.

So what that means is, that this thing has shifted down a lot. So that means that you have to have really, really volatile markets in order to make money. So it's shifted down enough so that supply equals demand, as you would expect. So there's no free lunch going on out there. It's priced fairly.

Now, even though it's priced fairly, if it turns out that you're the kind of person that really doesn't like a lot of risk and you believe there's going to be tons more volatility coming, then for you, it's worth it to do it. For somebody else who doesn't believe that there's going to be a lot more volatility coming, it's worth it to be on the other side of that trade. By the way, if I'm on the other side of the trade, what does my payoff diagram look like then? If I'm selling a put and a call, what will it look like?

AUDIENCE: The opposite.

ANDREW LO: Yeah, exactly, the opposite. We're going to flip it, flip this thing against the x-axis. So it'll be an upside-down V. But because we're shorting puts and calls, we get money upfront. So the upside-down V is going to be pushed up over the x-axis.

So it's going to look like the mirror image of this. And as long as stock prices are not more volatile than this range, we will make money. But our downside is unlimited in both directions.

So you got to be really confident that you know that markets aren't going to be any more volatile than they are now. Now, if you were Warren Buffett, and you bought Goldman Sachs three, four weeks ago, and you thought it was a great deal then, well, you would have lost money by now. Warren Buffett has lost money.

On the other hand, as you all know, Warren Buffett doesn't it make investments for the short term. He's thinking about this investment as a 10, 20-year investment. And over 10 or 20 years, I suspect it will be a very good deal.

But if you're looking at what's going on over the next few weeks, the question is, do you believe that markets will be less volatile or more volatile? If you do, you're going to be on one side or the other of that trade. The point is that this now allows us to make bets on volatility.

Whereas before, with a futures or a forward or a stock or a bond, you only could bet on it going up or going down, or mispricings because certain kinds of arbitrage relationships have been violated. This is the first time that we've been able to make a bet on wild swings. And that's a really amazing thing.

It's an extraordinary innovation to be able to do that. It allows individuals to engage in kinds of side bets that they otherwise wouldn't be able to. And more importantly, it allows other individuals to insure against certain kinds of eventualities that they'd never be able to do. Now you can buy insurance against volatility, which is a pretty remarkable thing to be able to do.

OK, so that's just one example of an option strategy, a very simple one. There are other examples that I've given you here. For example, this is kind of a fun one.

This is two calls. That should be a minus sign, sorry. Call1 minus Call2. So you basically buy a call option, and you short another one at different strike prices.

And so what this allows you to do, this is really interesting. This gives you upside from 50 up until 60. So you buy a call at 50. You're short a call at 60.

So that means you're going to get upside between 50 and 60. And then nothing after that and nothing before that. Now, this seems like a really ridiculous strategy to engage in. Why would you want to cut off your upside?

Because with a call, if you just bought a call, you'd basically get all of the upside. Right? Why would you ever want to do this? Anybody tell me what the logic for that is? Yeah.

AUDIENCE: Because it's cheaper.

ANDREW LO: Exactly. It's cheaper. It's cheaper because when you short the call at 60, you're getting money today. So that helps you finance the call at 50. It's cheaper, but it's not a free lunch. What you're getting in exchange for that extra premium is you're giving up any profits above and beyond the stock price going above 60.

So you're giving up the unbounded upside. And you're bounding it at 60. But the benefit of giving up that upside is that you now have some money to reduce the cost of getting that call at 50. OK?

And so you might use this if you think, well, I suspect that the stock has got some room to grow. I think it will bounce around between 50 and 60. But I can't possibly see the stock ever being worth more than 60. So I'm happy to give up that upside to other people who are more optimistic than me, and get some money for it and help me to finance my purchase of the stock at 50.

So it's cheaper. That's the bottom line. Another way of looking at it is, you have to move this whole diagram down by how much it costs. It turns out you move it down by less than if it were just the pure call option by itself.

So the way you can think of it is you buy the call option. You move it down by that much. And then you sell the other call and you move it up by the amount of that 60 call.

So you can do this. You can do this in reverse. You can bet on the downside, in that way. You can do something that's called a butterfly spread, where basically it looks like this.

So the payoff if it stays within a range, you get paid. But if it's really volatile, then you don't get paid. So you're willing to give up the upside on both ends because you think the stock is going to be self-contained. You're betting against volatility increasing, and you're using the ability to get rid of those unbounded gains to finance the positions.

And it turns out that with these kinds of payoffs, you can prove mathematically that it's possible to generate any other payoff in the world. There is a mathematical result that's actually related to this Taylor approximation and Fourier expansion that says that any possible security that you can come up with can be approximated by a sequence of calls and puts. That's a really powerful idea.

But in fact, from a practical purpose, you don't even need to use anything that fancy. If you have just a very small number of calls and puts, you can put together extraordinarily complex payoff diagrams that will get you whatever kind of a risk profile you're looking for. That's the power of option pricing.

OK, any questions about these payoff diagrams? I would urge you to work through a few examples just to make sure you really understand them. Because it's a easy thing to think that you understand. But unless you're forced to go through the exercise and draw these diagrams, you won't have an appreciation for how to do them and how important they are. Yeah, question.

AUDIENCE: I'm curious for a while, is there any implicit volatility, I mean implicit in the price of an option and a call and a put, is respective volatility of the market. But how much inside that price is it actually generating volatility itself? Do you see what I'm saying? The price, the call could also be a motor of volatility in the market.

ANDREW LO: Yes. . That's a great question. Let me repeat it. In fact, that question was asked shortly after Black and Scholes came up with their formula.

It created the whole literature, which was started by our very own former Dean, Dick Schmalensee. He wrote a paper with a fellow named-- I think it's Robert Trippi, small Schmalensee and Trippi. They wrote a paper on implied volatilities of options. So the ideal is that options are actually dependent on volatility.

And I'll show you that not this time, but next time. I'm going to go through a pricing model. And you're going to see how volatility actually plays a very concrete role.

So they came up with a brilliant idea. Let's take a look at an option price. And we know what the stock price is. We know what the strike price is. We know what the other parameters are.

Let's ask the question, given the price of an option, what is the volatility that is consistent with that market price? Because allowing you to invert the market price for the volatility gives you information about what's going on. It's exactly the question you asked, about information implicit in the market price.

It turns out that that's done all the time. And not only is it done all the time, but there is now an index that's been created by the Chicago Board Options Exchange called the VIX, which stands for the Volatility Implied Index. What they do is they look at options on the S&P 500.

And they ask the question, what is the volatility that is consistent with the option price on the S&P for at-the-money option. At-the-money means the strike price is equal to the current price of the stock, or the index, of this case. And that's an incredibly important concept.

Because that tells you something about where the market sees volatility going forward, not just looking backwards. But today, right now, what does the market think volatility should be? And if you look at the VIX over the last few weeks, you're going to be shocked.

We're going to take a look at it next time, next Monday. I'm going to do this in class, where I'll show you what that volatility looks like. Historically, the S&P 500 has had a volatility level of what? Does anybody know? What's the typical stock market volatility?

AUDIENCE: About 15%?

ANDREW LO: 15% to 20%. Yeah, it's bounced around there, on an annualized basis. Last week on an intradaily basis, the VIX index, which is the Implied Volatility, reached an interdaily high of 89% volatility, for the stock. And right now, I don't know what it is. I haven't checked today. But my guess is it's probably 60 to 70.

AUDIENCE: 71%.

ANDREW LO: Is it, what?

AUDIENCE: 71%.

ANDREW LO: 71%. OK, 71% annual volatility. Now, that's the forward-looking implied volatility for S&P options.

And what that tells you is that we're in for some turbulent times ahead. If you look at the implied volatility for the one year contract, it's going to be much lower. Because people are going to expect that the volatility of the S&P, going forward in time, is going to decline between now and a year from now.

At least, we hope so. Otherwise a lot of people are going to be needing zan-- zan-- what is it? Zantac and other kinds of pharmaceuticals. OK. So those are option diagrams.

And I want to mention one last thing before we go to the history of option pricing theory. I want to mention that one of the reasons option pricing theory has been so important in finance is because soon after the papers by Black and Scholes and Merton were published, it became clear that everywhere you looked, there were options to be found. That is, all other kinds of financial securities, when you looked more closely, they were actually options as well.

So let me give you an example I said before that stock prices were not like options. Well, as an approximation, that's true. But in reality, if you look carefully at what a stock is, in fact, a stock is an option. So let me see how that is.

Well, equity, the equity of a corporation is a claim on the corporation's assets. But if that corporation has any kind of debt financing, then actually the equity holders are second in line. The bondholders are first in line. So the equity only gets paid after the bondholders get paid off.

So in particular, if you think about the maturity date of the bonds, then on maturity date, the value of the equity is the maximum of either 0, or the value of the firm's assets minus the face value of the bond, or what the bond has to be paid off at. Because if the bondholders don't get paid, then the equity holders get nothing. Then the bondholders get the firm. All of the assets of the firm transfer to the bondholders through bankruptcy proceedings. At least, that's the theory.

So the value of the equity on the maturity date for the bonds is actually the maximum of 0, V minus B. Now, that should look very familiar to you. That should look like the payoff of a call option. Where the strike price is B, and the value of the underlying security is V, the value of the firm's assets.

So what that means is that equity holders can be viewed as owning an option on the firm's assets with a strike price of B. And the bondholders look like they have a put option. They've shorted a put on the firm.

But that's leveraged with a certain amount of debt. It's a protected levered put, is the way that people usually put it. So the debt is the minimum of V or B.

You either get the assets, or you get what you owed, which is smaller. And you can show that that's equivalent to B minus max of 0B minus V. That looks like a short put position mixed in with some borrowing.

And when you add the two, you see that the value of the firm is equal to the value of the debt and the value of the equity. The point of this example is that option pricing can be used to value the capital structure of a corporation as well. And within the last few years, a very active part of the hedge fund industry has been devoted to engaging in something called capital structure arbitrage.

Capital structure arbitrage says that this equation has to hold. But in practice, there is a discrepancy with what the market value for D is, and the market value for E. And using option pricing theory and models for credit risk, hedge funds have been able to make a play by either buying a company's equity and shorting their debt, or buying the debt and shorting the equity, whichever is cheaper or more expensive, and engaging in what seems like an arbitrage transaction.

Now, that presupposes that you've got the credit calculations done correctly. So in order to engage in those kind of trades, you have to have superior credit modeling capabilities, certainly better than what rating agencies were doing. And actually, there were cases where hedge funds were actively betting against rating agency models. Because they felt that rating agencies had mispriced some of their ratings based upon the models that they'd created, versus the ones the rating agencies were using.

Now, turns out that when you look more carefully at other securities, and even other kinds of opportunities, options are there as well. For example, when I started here at MIT, 20 years ago, I remember, distinctly, some of my senior colleagues referring to Assistant Professors as options.

[LAUGHTER]

Now, let me explain. You know that in academia, when you start out as a Assistant Professor, there's no guarantee for employment. You have, typically, a three-year contract.

And at the end of three years you either get renewed or you get fired. And at the end of the next three years, you come up for what's called a tenure review. Tenure review means that they send letters to 15 of the top people in your field across the country. Across the world, actually.

And they base their decision on whether to give you lifetime guaranteed employment, as to whether or not these 15 people say that you're the greatest thing since whatever. And if you don't get that kind of review, then you're asked to leave. I mean, you have to leave. There's no choice for continued employment.

So the idea behind hiring Assistant Professors were that each one of them was viewed as an option. Meaning that you could benefit from them for a while. But if they didn't work out, you could always get rid of them. But once you got tenure, that was it. There was no longer an option.

So what that suggested, from a hiring perspective, is what kind of Assistant Professor should you hire if you believe in option pricing, as applied to the labor market? Can you can you characterize the type of-- Yeah, what was that?

AUDIENCE: Take risks.

ANDREW LO: Take risks. You want to hire faculty that are extremely volatile. Not emotionally, hopefully, but intellectually. In other words, because you get all the upside, but you don't get any downside.

So what you want to do is you want to take risk. You want to take chances on faculty that may or may not work out. And that, in fact, has been the approach that we and others have used in hiring, based upon this kind of option pricing analysis. And it applies to all sorts of things.

When you think about getting an education, you can argue that getting an education is an option. You don't have to use your degree. You don't have to use your education. But you have it. It's an option.

And so thinking about value in education, you could actually use this framework, try to compute the flexibility it gives you, in order to take advantage of career opportunities. So there are lots of things that look like options. Yeah, Megan?

AUDIENCE: [INAUDIBLE] distressed debt manager, [INAUDIBLE]?

ANDREW LO: A distressed debt manager if they're holding distressed debt of a company, they would actually be holding a short put position. They're holding the debt. So they have a short put position.

So if they wanted to hedge it, they can either buy a put on the assets, which would then help them to hedge it out, Yeah, right. And as I was saying, there are all sorts of other examples of options and derivative securities. The field has exploded. There are literally many, many trillions of dollars of notional amounts.

Now, again, notional amounts can be a little bit misleading. Because you know that for every option seller, there's an option buyer. Options are zero net investment side bets, unlike equities, where companies that have real assets behind them issue pieces of paper called equity.

Options are issued by the Options Clearing Corporation for the Chicago Board Options Exchange. The Mercantile Exchange also has options. There's options traded everywhere. In fact, one of the largest exchanges is the International Securities Exchange. It was started up by Bill Porter, the fellow who started e-trade. And it is the most active options exchange in the world. It's all done electronically.

And these options are pure side bets. But they're not just for purposes of gambling. They're for purposes of hedging and engaging in insurance, of the kind that we talked about before.

So this has really exploded. And that's why we have an entire course, 15.437, devoted to just the pricing of options and futures. So we can't, obviously, cover all of it in this course. But I want to just give you a flavor of it.

Let me skip, now, this next section on valuation of options. Because as I said, this is a little bit more technical. I want to spend some time on it and make sure you all understand it. And then I will come back to this on Monday. When I want to do now is just to give you a little bit of a history of option pricing. Because it's kind of fun.

First of all, in order to figure out how to price options, we have to begin with figuring out what a particular model would be for the underlying stock. In order to price an option, you actually have to say something about how the underlying security behaves. So we have to start with that.

And we're going to start in the very early 16th century, with probably the first-known model for asset prices that ever existed in the world. And that was developed by a Italian mathematician by the name of your Gerolamo Cardano. Now, those of you who were on high school math team, I suspect you've heard of Cardano. Anybody tell me who Cardano was?

No math team geeks here? All right, Cardano was, it turns out, the second person to have come up with a solution for the cubic equation. You all know what the quadratic equation is, right?

You know, ax squared plus bx plus c equals 0. That's a quadratic equation. Anybody know what the solution of that is? Yeah, what is that?

AUDIENCE: [INAUDIBLE]

Great Great. All right, You get the pocket protector award.

[LAUGHTER]

Very good. It turns out that there is exactly the same kind of solution for the cubic equation. Of course, nobody remembers that. I won't ask you whether you know that. You might. But there is a formula for the cubic equation. It turns out that there are no more formulas beyond the cubic.

So there's something very special about the cubic equation. And this Italian mathematician, Cardano, was the first to publish it. The reason I say that he's the second person to come up with it, is that it turns out he stole the formula from a colleague, and a colleague who had actually come up with the solution.

And Cardano heard about it and said, well, please tell me what it is. And the other person said, I'm not going to tell you what it is. Because you're going to just write it up and claim credit. And Cardano says no, no, I promise I won't. And then the guy says, all right, here it is. He told him. And then Cardano did, in fact, rip him off.

So it's known as Cardano's formula, but it really shouldn't. And I'm embarrassed to say, I don't remember the guy who actually invented it. But Cardano, in addition to having come up with this solution, or stolen this solution, Cardano also wrote a book on gambling.

And this book, which is titled *Liber De Ludo Aleae,* The Laws of Gambling, he developed what was the precursor to the modern mathematical description of stock prices. And it was described in this way.

"The most fundamental principle of all in gambling is simply equal conditions, e.g. of opponents, of bystanders, of money, of situation, of the dice box, and the die itself. To the extent to which you depart from that equality, if it is in your opponent's favor, you are a fool, and if in your own, you are unjust." It turns out that what he was describing was, essentially, a 50/50 bet.

Or what we call a fair game, or what is now known as a martingale. A martingale simply says that expected winnings and losses is equal to 0. Or rather, your expected wealth next period is equal to whatever your wealth is today if you have a fair game that you're betting on.

It turns out that that simple model developed into what we now think of as the Random Walk Hypothesis. And the Random Walk was really the fundamental driver behind the option pricing model that Black and Scholes and Merton developed. Now, the reason the Random Walk holds a very special place in the hearts of financial economist is because most economists suffer from a psychological disorder that we call physics envy.

We all wish that we had these three laws that explains 99% of all behavior. In fact, economists have 99 laws that explain maybe 3% of economic behavior. But there's one example, only one, in the history of finance, where an economist actually came up with an idea before a physicist. And that was later adopted by a physicist.

And the idea I'm talking about is the Random Walk hypothesis, or in the continuous time realm, Brownian motion. In 1900, a student by the name of Louis Bachelier was writing a dissertation in Paris. He was a mathematics PhD student. But he was writing about pricing warrants that were trading on the Paris Bourse. So it was a finance thesis.

And in order to solve the problem, he had to come up with a mathematical description for the underlying price. And he came up with this notion of what we now call Brownian motion, of Random Walk. And he did it a full three years before a well-known physicist published a paper on that. Anybody know who that physicist was?

AUDIENCE: Was it Brown?

ANDREW LO: No. No, Brown was many years before. And he was a biologist. Yeah?

AUDIENCE: Einstein.

ANDREW LO: That's right, Albert Einstein, in 1903, actually published a paper on the photoelectric effect and Brownian motion. And if you take a look at what Baschelier did, he was working with the French mathematician by the name of Henri Poincare. Poincare was a very well-known mathematician who was the advisor to Baschelier, and who is renowned now for a variety of different contributions, including the theory of dynamical systems.

Baschelier wrote this thesis and developed the mathematics of Brownian motion. And when he was looking for a job, Poincare wrote a letter of recommendation. And this is what Poincare wrote about Baschelier. He said that "The manner in which the candidate obtains the law of Gauss is most original, and all the more interesting as the same reasoning might, with a few changes, be extended to the theory of errors. He develops this in a chapter which might at first seem strange. For he titles it 'Radiation of Probability.' In effect, the author resorts to a comparison with the analytical theory of the propagation of heat."

Now, remember this is a thesis on pricing warrants on the Paris Bourse. Fourier's reasoning is applicable almost without change to this problem. Which is so different from that for which it had been created." And of course, his adviser, at the end, always has to complain a little bit about his student, as we all do.

So he said, "It is regrettable that the author did not develop this part of his thesis further." What Poincare was mentioning, with regard to Fourier, was the theory of heat conduction. In physics, there is a very standard model that everybody that goes into advanced physics will cover.

And that is, how does heat get conducted through a solid medium? And in deriving the equation that ultimately is known as the heat equation, you actually use the same theory that Baschelier applied to pricing warrants on the Paris Bourse. He gets what's known as a partial differential equation.

And that's it right there. That's the equation that he used in his thesis. If you look at his thesis, you'll see it there.

That's the heat equation. It's the same equation that explains the conduction of heat in a solid medium. But he derives it for the purpose of pricing this financial security.

Now, it turns out that there was one slight mistake that Baschelier made in his thesis. It was a mathematical error that, ultimately, didn't really affect the results. But it became known. And when he came up for tenure, they wrote to all the various different big names. And he was ultimately turned down for tenure, because they found this mistake.

And he was blackballed. So he couldn't get a job except for a small teaching college, a women's teaching college in the south of France. Which frankly, sounds pretty good to me.

[LAUGHTER]

But you know, for him, it was not the way he would not want to end his career. But at the end of his career, it was discovered that this mistake was not as serious. And people wrote him a letter saying, gee, you're a great guy anyway.

Paul Samuelson, actually, was the person who discovered Baschelier's thesis when he was in Paris at the Sorbonne, reading through various different archives. So Paul Samuelson's responsible for resurrecting the reputation and the work of Louis Baschelier. You can see his thesis now. It's been republished and reprinted.

But the point of the thesis is that by assuming that the underlying stock price was a Random

Walk, and by developing the mathematics of the Random Walk, he was able to figure out what the price of an option was on that stock. And it turns out that the pricing of the option on the stock reduces to solving this heat equation. And that explains why there are, nowadays, so many physicists and mathematicians that are in finance.

It's because the whole body of knowledge that comes along with the physical interpretation for the heat equation can be applied, virtually identically and verbatim, to the pricing of options and other derivative securities. And so very quickly, we can see that the information that's contained in these market prices can be understood within a mathematical framework that we know.

So now going back to the history, it turns out that this was not known in the 1970s. It wasn't rediscovered by Paul Samuelson until later on. The folks that actually worked on option pricing, that tried to figure out the mathematical prices of options were quite a few. Kruizenga, who is an MIT PhD student in the 1950s-- oh, question? No? OK.

In the 1950s, there was an MIT PhD student of Paul Samuelson's who tried to work on this problem. And he actually has a thesis titled "Put and Call Options-- A Theoretical and Market Analysis." It's actually in the MIT archives, if you want to go take a look at it.

But he didn't quite get it. He didn't get the right solution, because he didn't have the mathematical machinery to be able to work out the final elements of it. K. Sprenkle, a student at Yale in 1961, wrote a thesis under Jim Tobin and Arthur Okun, titled "Warrant Prices As Indicators of Expectations and Preferences," and tried to price it as well.

But he wasn't able to come up with a pricing formula either. And there were a number of other attempts to try to come up with the appropriate pricing formula, including attempts by Samuelson in '65, where he had to make assumptions on individual preferences in order to get a price. That didn't work out. And then Samuelson and Merton in '69, they tried to come up with a pricing formula that was preference free. And they still couldn't do it.

Along came Black and Scholes. Fischer Black who, at that time, was a consultant working at Arthur D. Little. He wasn't even an academic. The Arthur D. Little building, that's the building that is right over there, the one that they won't let us tear down. Because it's supposed to be an architectural gem of sorts.

That was the Arthur D. Little Building. Fischer had his office there. Myron had his office in the next building over, Myron Sholes.

And they started talking about option pricing. And Fischer came up with an analysis that was very much along the lines of Baschelier. He basically got this formula,

but he couldn't solve it because he had never heard of the heat equation because his Fischer Black's background was in computer science, not in mathematics.

It was ironic, because Fischer Black actually had a PhD. Not in economics or finance, but in applied math. But he had never taken physics. So he was doing discrete math.

So he started talking to Myron Scholes and as legend would have it, Myron took that heat equation, went over to the math department here, and asked one of the mathematics professors, have ever seen this thing? And a math person looked at him and said, oh yeah, that's just heat equation. Here, you solve it like this.

And so Myron apparently took it back to Fischer Black. And Fischer said, hmm, this is interesting. We can now write a paper. And they wrote a paper on this. At the same time, Bob Merton was working on another direction that was trying to come up with a solution.

Ultimately, he came up with the same solution. They didn't know it because they had actually not communicated to each other. But ultimately, Myron and Fischer, they sent their paper to something like five economics journals.

Every single one of them rejected the paper saying this is too specialized. It's not really economics. It's not finance. We don't know what it is, but go away.

And it was only until they were able to change the title of the paper from option pricing to the pricing of options and corporate liabilities that they finally-- so it was exactly this-- well, I'll show you next time. They changed it to start focusing more on corporate finance. They ultimately got their paper published.

It turns out that Merton used a very different approach but got to the same point. And so Black and Scholes got their paper, ultimately, accepted into *The JPE.* Merton got his paper accepted *The Bell Journal,* both in the same year.

In fact, Merton got his paper published first. But he argued that the paper should be delayed because he wanted Fischer Black and Myron Scholes have their paper come out in the same year. He felt that he derived so much intuition for what Black and Scholes were doing, that he didn't want to get there first, because it was not fair to them.

That was one of the most extraordinary acts of professional ethics in the profession. Because it was pretty clear to both of them what was at stake. This was a huge problem that took an enormous amount of time to solve.

And of course, the rest is history. They were awarded the Nobel Prize in 1997, Myron and Bob. Unfortunately, Fischer Black had died of cancer the year before. But it was very clear in the Nobel address, both on the participants' part, as well as the Nobel Committee, that Black should have received it as well.

So that's the history and the heritage of option pricing. You can see why MIT is rightly proud of it. And given that we're out of time, let me stop here. And then next time, what we're going to do is to take up where we left off, and focus on the actual pricing formula.

I'm going to derive it for you. Not the Black-Scholes formula, but a simpler version. And you'll see it, and you'll be able to take a look at it and play with it. We'll go on from there. OK, I'll see you on Wednesday for the mid-term exam.