15.072
Homework Assignment 3

Given: March 15, 2006 **Due:** March 24, 2006

Problem 1 *Give an example of a queueing system and performance measure such that conservation law holds for admissible scheduling policies, but does not hold if the scheduler knows the service times of the jobs in the queue in advance (and thus the policy is not admissible).*

HINT: work with queueing system with just one class.

Problem 2 *Suppose X is a random variable distributed according to distribution function $F(x)$ and belongs to γ-MRLA class. Namely*

$$\int_t^\infty \mathbb{P}(X > \tau)d\tau \le \gamma \mathbb{P}(X > t) \tag{1}$$

for every $t \ge 0$.

A. *Show that*

$$\frac{\mathbb{E}[X^2]}{2\mathbb{E}[X]} \le \gamma.$$

HINT: integrate both parts of (1). You may assume that $\mathbb{E}[X^2] < \infty$.

B. *Suppose that interarrival times A_n in G/G/1 system belong to γ-MRLA class. Argue from this that the idle period I in a G/G/1 queueing system satisfies*

$$\frac{\mathbb{E}[I^2]}{2\mathbb{E}[I]} \le \gamma.$$

You may assume that I has finite second moment: $\mathbb{E}[I^2] < \infty$.

Problem 3 *Establish the following identity corresponding to the Region IV for G/M/m queueing system. Let $j < m < i + 1$. Then*

$$P_{ij} = \int_0^\infty \binom{m}{j} e^{-j\mu t} \Big[\int_0^t \frac{(m\mu y)^{i-m}}{(i-m)!} (e^{-\mu y} - e^{-\mu t})^{m-j} m\mu dy \Big] dA(t).$$

HINT. As in other cases condition on the duration of the interarrival time.