Often, in linear optimization problems, we've estimated the data we're using in the problem, but it's subject to change.

Understanding how the solution changes when the data changes is called sensitivity analysis.

One way that the data could change is through marketing decisions.

Suppose that American Airlines' management is trying to figure out whether or not it would be beneficial to invest in marketing its fares.

They forecast that the marketing effort is likely to attract one more unit of demand, of each type, for every $200 spent.

So for the discount fare, the marketing cost per unit is $200, and for the regular fare, the marketing cost per unit is also $200.

We want to know how much this will increase our marginal revenue for each type of fare.

This graph shows our current feasible space and optimal solution.

What would happen if we increased the marketing for discount fares?

The demand for discount fares would increase.

But since we're not even meeting the current demand for discount fares with the optimal solution, this doesn't give us any extra revenue.

So we shouldn't add any marketing for discount fares.

Actually, American Airlines could decrease their budget to market discount fares, and even if the demand decreases, it wouldn't change our revenue.

The demand could go all the way down to 66 without affecting our decisions.

In sensitivity analysis like this, we're often concerned with the shadow price of a constraint.

For a discount demand constraint, this is the marginal revenue gained by increasing the demand by one unit.

In this case, the shadow price is 0 for demand greater than or equal to 66.

Now, let's look at what happens when we market regular fares.

If we increase the demand for regular fares, our revenue increases.

If we increase by 25 units of demand, our revenue increases to $86,883.

If we increase by another 25 units of demand, our revenue increases to $96,358.

So what's the shadow price in this case?

Remember that the shadow price is the marginal revenue for a unit increase in demand, in this case, of regular seats.

From 100 to 125, the revenue increased by $86,883 minus $77,408, which is equal to $9,475.

Since this was an increase of 25 units of demand, the shadow price is 9,475 divided by 25, which equals 379.

We can calculate that this is the same shadow price from 125 to 150.

So the marginal revenue for every extra unit of regular demand from 100 to 166 is $379.

So given this analysis, how can we help the marketing department make their decisions?

The forecast was an extra unit of demand for every $200 spent.

For discount fares, this isn't worth it, since the shadow price, or marginal revenue, is 0.

But for the regular fares, this is worth it, since the shadow price is $379.

So the marketing department should invest in marketing regular fares to increase the demand by 66 units.

Another sensitivity analysis question in our problem is whether or not it would be beneficial to allocate a bigger aircraft for this flight.

This would change the capacity constraint, which currently limits the capacity to 166.

With our current aircraft, the management knows that the cost per hour is $12,067.

So the total cost of the six-hour flight is $72,402.

With the 166 seats filled, we get a revenue of $77,408 from our optimal solution.

If we increase the capacity of the aircraft to 176 seats, the total cost would increase to $76,590.

But how much would this increase our revenue?

And if we increase the capacity of the aircraft to $218, the total cost would increase to $87,342.

But how much would this increase our revenue?

For our analysis, let's assume that the demand does not change.

If we increase our capacity to 176, the capacity constraint will move right.

And our optimal solution will move right too.

We now get a revenue of $79,788.

If we then increase the capacity to 218 seats, the capacity constraint will move right again, and our revenue will increase to $89,784.

So let's look at our extra profit from increasing the capacity to see if it's worth it.

With our current costs and revenue, the profit is $5,006.

If we increase the capacity to 176 seats, our profit actually decreases to $3,198.

And if we increase the capacity to 218 seats, our profit decreases even more to $2,442.

So even though our revenue is increasing, the cost increases too.

So it's not profitable for us to increase the capacity of our aircraft.

You can also see this by using LibreOffice, which we'll ask you to do in the next quick question.