Optimization Methods in Management Science

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Recitation 9
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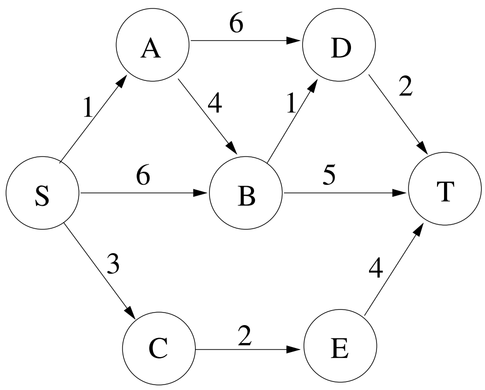
Problem 1

We apply Dijkstra's algorithm to the network given below to find the shortest path between S and T. (Arc labels represent distances.)



One *iteration* of Dijkstra's algorithm includes the selection of a permanent node as well as the scanning of arcs out of the node. After 2 iterations, we have the following list of `permanent` nodes, and the following distance labels.

`permanent`	
S	X
A	X
B	
C	
D	
E	
T	

Distance Label	
d[S]	0
d[A]	1
d[B]	5
d[C]	3
d[D]	7
d[E]	∞
d[T]	∞

Carry out one more iteration: indicate which nodes are `permanent` and the distance label for each node.

Problem 3

Each year, Data Corporal produces as many as 400 computers in Boston and 300 computers in Raleigh. Los Angeles customers must receive 400 computers, and 300 computers must be supplied to Austin customers. Producing a computer costs \$800 in Boston and \$900 in Raleigh. Computers are transported by plane and may be sent through Chicago. The costs of sending a computer between pairs of cities are shown in the following table.

	To		
From	Chicago	Austin	Los Angeles
Boston	80	220	280
Raleigh	100	140	170
Chicago	–	40	50

(a) Formulate a Minimum Cost Network Flow problem that can be used to minimize the total (production + distribution) cost of meeting Data Corporal's annual demand.

(b) How would you modify the Part A formulation if at most 200 units could be shipped through Chicago?

(c) How would you modify the network if 500 computers could be produced in Raleigh instead of 300? This illustrates a general strategy for dealing with excess supply.

Problem 3

Short answer (True or False).

1. For a Minimum Cost Network Flow Problem, a supply of -1 for a node is equivalent to a demand of 1.
2. For a Minimum Cost Network Flow problem, if the specified supply is not equal to the demand (that is, the sum of the b_i's does not equal 0), the corresponding Linear Program with flow balance equality constraints must be infeasible.
3. It is possible for an undirected graph to have an Eulerian Path but not an Eulerian Cycle.
4. Suppose all arcs in a network have different lengths. Then the network has a unique shortest path.

Problem 5

Multiple answer

1. Suppose we delete an arc of length k from a graph. Which of the following possibilities can happen to the shortest path from s to t?

   (a) It could increase by any number between 0 and k.

   (b) It could increase by a number greater than k.

   (c) It could possibly become infinite.

   (d) It could decrease.

2. Let $G = (N, A)$ be a network with$n$ nodes and m arcs.

   (i) The maximum number of distinct s-t paths can be greater than $2m$.

   (ii) If $m < n - 1$, then G is not connected.

   (iii) If m is odd, then the graph can not contain an Eulerian Cycle.

   (iv) If m is even, then every s-t cut has an even number of arcs.