8.871

Solutions to problem set #2

1.

Bosonic Massless Fields:	ϕ	$B^{(2)}$	$C^{(0)}$	$C^{(2)}$	$C^{(4)}_{+}$
Electrically Charged BPS Branes:	-	F1	D(-1)	D1	D3
Magnetically " " " :	-	NS5	D7	D5	D3

The D7 is the magnetic dual of the D(-1)

2.

a) For type IIA, we can either start from 11d supergravity or use the algebra directly. We'll do both.

In 11d supergravity the algebra (schematically) looks like:

$$\{Q, Q\} = P_\mu \Gamma^\mu + Z^{(2)} + Z^{(5)}$$

where P_μ is the momentum, $Z^{(2)}$ and $Z^{(5)}$ are two- and five-forms respectively, all space-time indices are appropriately contracted with gamma matrices and spinor indices are omitted.

Upon reduction on a circle, P_μ gives a vector (10d momentum) and a scalar (D0 charge), $Z^{(2)}$ gives a two-form (D2 charge) and a vector (F1 charge) and $Z^{(5)}$ gives a five form (NS5 charge) and a four-form (D4 charge). The charges for the D6 and D8 branes are missing.

If we look directly at the 10d theory there are two spinors of opposite chirality, 16 and 16'. The possible central charges correspond to the antisymmetric forms that appear in the decomposition of the product of these spinors.

$$16 \times 16' = \underset{D0}{[0]} + \underset{D2}{[2]} + \underset{D4}{[4]} \qquad \text{(twice)}$$

$$(16 \times 16)_s = \underset{F1}{[1]} + \underset{NS5}{[5]_+} \quad , \quad (16' \times 16')_s = \underset{F1}{[1]} + \underset{NS5}{[5]_-}$$

Again, D6 and D8 are missing. One can check that the total number of d.o.f. on the right hand side of these equations is $528 = \frac{32 \times 33}{2}$ as we expect for a theory with 32 supercharges.

b) For type IIB we have two spinors of the same chirality, 16_1 and 16_2.
Working as above:

$$16_1 \times 16_2 = \underset{D1}{[1]} + \underset{D3}{[3]} + \underset{D5}{[5]_+} \quad \text{(twice)}$$

$$(16_1 \times 16_1)_s = \underset{F1}{[1]} + \underset{NS5}{[5]_+} \qquad (16_2 \times 16_2)_s = \underset{F1}{[1]} + \underset{NS5}{[5]_+}$$

The charges for D(-1), D7, D9 branes are missing. Again, the total number of components comes out right (528)

c) In Heterotic theories, we have one spinor, 16.

$$(16\times 16)_s = \underset{F1}{[1]} + \underset{NS5}{[5]_+}$$

Total number of components: $\frac{16\times 17}{2} = 136$

BPS bound: The operators $\{Q, Q\}$ are positive definite by construction. In the rest frame of a p-brane, the right hand side of the SUSY algebra can be written $\int dV_p\,(T_p \pm Q_p)$ where dV_p is the p-dim volume form, T_p is the p-brane energy per unit volume (tension) and Q_p the charge per unit volume. Thus,
$T_p \geq |Q_p|$. This is the BPS bound. For a supersymmetric state, $Q|\Omega\rangle = 0 \Leftrightarrow T_p = |Q_p|$. Supersymmetric branes are said to 'saturate' the BPS bound.

3.

Under S-duality of Type IIB string theory:

$$g_s \xrightarrow{S} g_s' = \frac{1}{g_s} \qquad \text{or} \qquad g_s = \frac{1}{g_s'}$$

$$l_s \xrightarrow{S} l_s' = l_s\, g_s^{1/2} \qquad \text{or} \qquad l_s = l_s' \cdot g_s'^{1/2}$$

(a) $T_{D5} = \dfrac{1}{g_s l_s^6} = \dfrac{g_s'}{(l_s' g_s'^{1/2})^6} = \dfrac{1}{g_s'^2 l_s'^6} = T'_{NS5}$

(b) $T_{NS5} = \dfrac{1}{g_s^2 l_s^6} = \dfrac{g_s'^2}{(l_s' g_s'^{1/2})^6} = \dfrac{1}{g_s' l_s'^6} = T'_{D5}$

(c) $T_{D3} = \dfrac{1}{g_s l_s^4} = \dfrac{g_s'}{(l_s' g_s'^{1/2})^4} = \dfrac{1}{g_s' l_s'^4} = T'_{D3}$

(d) $T_{D7} = \dfrac{1}{g_s l_s^8} = \dfrac{g_s'}{(l_s' g_s'^{1/2})^8} = \dfrac{1}{g_s'^3 l_s'^8}$ scales $\sim \dfrac{1}{g_s'^3}$

This $\frac{1}{g_s^3}$ scaling does not correspond to any known object in string theory. In fact, the problem arises because the tension of the D7 is ill-defined. The D7 is a codimension 2 object that creates a conical deficit angle in spacetime. See [hep-th/9812028, 9812209] for a detailed treatment of D7 branes.

4. [See solutions to pset #1]

a) F1 ending on Dp.

The endpoint of F1 is the source for a two-form field strength $F^{(2)} = dA^{(1)}$

The gauge invariant combination is

$$F^{(2)} - B^{(2)}$$

where the gauge transformations are

$\delta B^{(2)} = d\Lambda^{(1)}$
$\delta A^{(1)} = \Lambda^{(1)} \Rightarrow \delta F^{(2)} = d\Lambda^{(1)}$

Note that this is different from the usual gauge invariance of a U(1) gauge field $\delta A^{(1)} = d\Lambda^{(0)}$.

b) Dp ending on NS5

Type IIA

$p=2$. The endpoint of the D2 couples electrically to $A^{(2)}$. The field strength is $F^{(3)} = dA^{(2)}$

Gauge variation: $\delta A^{(2)} = \Lambda^{(2)} \Rightarrow \delta F^{(3)} = d\Lambda^{(2)}$
$\delta C^{(3)} = d\Lambda^{(2)}$

Invariant : $F^{(3)} - C^{(3)}$

p=4: The endpoint of the D4 is a vortex that couples magnetically to a zero-form $A^{(0)}$. The correspon ding field strength is $F^{(1)} = dA^{(0)}$.

Gauge variation: $\delta *_6 F^{(1)} = d\Lambda^{(5)}$

↳ NS5 W.Y.

$$\delta C^{(5)} = d\Lambda^{(5)}$$

Invariant: $*_6 F^{(1)} - C^{(5)}$

Type IIB

p=1. The endpoint of a D1 couples magnetically to a vector potential. The field strength is $F^{(2)} = dA^{(1)}$.

Gauge variation: $\delta A^{(1)} = \Lambda^{(1)} \Rightarrow \delta F^{(2)} = d\Lambda^{(2)}$

$$\delta C^{(2)} = d\Lambda^{(2)}$$

Invariant: $F^{(2)} - C^{(2)}$

p=3. The endpoint of the D3 couples magnetically to the same 1-form as above.

Gauge variation: $\delta *_6 F^{(2)} = d\Lambda^{(3)}$, $\delta C^{(4)} = d\Lambda^{(3)}$

Invariant: $*_6 F^{(2)} - C^{(4)}$

$p=5$. The endpoint of the D5 is a domain wall. It couples magnetically to a zero form $F^{(0)}$

Gauge variation: $\delta *_6 F^{(0)} = d\Lambda^{(5)}$
$\delta C^{(6)} = d\Lambda^{(5)}$

Invariant : $*_6 F^{(0)} - C^{(6)}$

c) M2 ending on M5

The endpoint of the M2 couples electrically to a two-form $A^{(2)}$

Gauge variation: $\delta A^{(2)} = \Lambda^{(2)} \Rightarrow \delta F^{(3)} = d\Lambda^{(3)}$
$\delta C^{(3)} = d\Lambda^{(3)}$

Invariant : $F^{(3)} - C^{(3)}$

d) D_p ending on $D_{(p+2)}$.

The endpoint of the Dp couples magnetically to the same 1-form that an F1 ending on $D_{(p+2)}$ couples to electrically.

Gauge variation: $\delta *_{p+3} F^{(2)} = d\Lambda^{(p)}$, $\delta C^{(p+1)} = d\Lambda^{(p)}$

Invariant : $*_{p+3} F^{(2)} - C^{(p+1)}$