

Problem set #12

1) The string coupling and vector multiplets

- a) Consider a Dp - $D(p+4)$ system for $p=-1..4$. Recall and write down the relation between the string coupling and the gauge coupling on the Dp brane
- b) In the absence of the Dp brane the theory has 16 supercharges. Recall the multiplet structure in theories with 16 supercharges. In what supersymmetry multiplet does the string coupling transform?
- c) Under a reduction of supersymmetry from 16 to 8 recall how the different multiplets decompose.
- d) Show that the gauge coupling on the Dp brane is a part of a background vector multiplet in $p+1$ dimensions.
- e) repeat the analysis for $p=5$. Now $O9^-$ is present as well. In what multiplet does the gauge coupling transform in this case?

2) Effective gauge couplings at one loop -- Dp - $D(p+4)$ system

- a) Look up in books or papers the precise relation between the gauge coupling on the Dp brane and the string coupling up to factors of 2 and π .
- b) Write down the Laplace equation that the string coupling obeys up to factors of 2 and π
- c) Solve the Laplace equation with a delta function source (Poisson equation) up to factors of 2 and π and fix the sign.
- d) Write down the solution for the effective gauge coupling in $p=3$ (4 dimensions). Compare this to the one loop beta function equation and show that there is precise agreement up to factors of 2 and π and, most importantly, the sign.
- e) What is the sign of the beta function for the $D3$ $D7$ system? Is the theory IR free or UV free?
- f) What is the sign of the beta function for the $D3$ $D7$ $O7^+$ system? Is there any dependence on the number of $D3$ branes k and the number of $D7$ branes n ?
- g) What is the sign of the beta function for the $D3$ $D7$ $O7^-$ system? Find the dependence on k & n if at all.
- h) Generalize the result to any p in the range -1 to 4 and find the generalization of the beta function in other dimensions.

3) Give a list of all asymptotically free theories that one can construct using the k $D3$ n $D7$ $O7$ system. Do you find potential problems?