The sum over i runs over all particles in the final state, and the direction $\vec{n}_T$ is called the thrust axis. To fully understand this equation, let's first ignore the $\max_{n_T}$ and pick a fixed direction $\vec{n}_T$. Thrust is then defined by summing the absolute value of the projections of the momenta of all particles onto the thrust axis, and divide by the sum over the magnitude of all momenta. In the situation where the momenta of all particles are aligned (or anti-aligned) exactly with the thrust axis, the magnitude of the projection onto the thrust axis is exactly equal to the magnitude of the momentum itself, such that one obtains $T = 1$. Thus, energetic particles that are collinear or anti-collinear to the thrust axis give $T \approx 1$. Soft particles with vanishing momentum do not contribute to the the thrust, since their contributions vanish in the numerator and denominators. Thus, events with $T \approx 1$ only contain particles which are either collinear or anti-collinear to the thrust axis, or are usoft, and are therefore 2-jet like and can be described by SCET$_{\rm I}$. For later convenience we will often choose the variable

$$\tau = 1 - T \tag{9.2}$$

instead of T itself. In this case the 2-jet case corresponds to $\tau \to 0$, while τ away from zero corresponds to three or more jets.

To make the connection of thrust with SCET even more obvious, we will define the two four-vectors

$$n^\mu = (1, \vec{n}_T), \qquad \bar{n}^\mu = (1, -\vec{n}_T) \tag{9.3}$$

Using this definition, we can write

$$
\begin{aligned}
T &= \frac{Q - \sum_{i \in R} n \cdot p_i - \sum_{i \in L} \bar{n} \cdot p_i}{Q} \\
\Rightarrow \tau &= \frac{\sum_{i \in R} n \cdot p_i + \sum_{i \in L} \bar{n} \cdot p_i}{Q}
\end{aligned}
$$

9.2   Factorization

The thrust distribution in the full theory is given by summing over all final states in the event, and projecting each event onto its value of thrust, defined by (9.1)

$$\frac{d\sigma}{d\tau} = \frac{1}{2Q^2} \sum_X \left| M(e^+e^- \to X) \right|^2 (2\pi)^4 \delta^4(q - p_X)\delta(\tau - \tau(X)). \tag{9.4}$$

Here $M(e^+e^- \to X)$ is the full QCD matrix element to produce the final state X from the collisions of an e^+e^- pair.

To obtain the expression in SCET, we need to match the full QCD matrix element onto operators in SCET. As was already discussed in Section Since we only consider final states with energetic particles collinear to either the direction n^μ or $\bar{n}^\mu$, the appropriate operator in SCET is

$$O_{n\bar{n}} = \bar{\chi}_{\bar{n}} \Gamma \chi_n \tag{9.5}$$

where χ_n is the gauge invariant quark jet field introduced in (6.20) and Γ is a Dirac structure that describes the production of a $q\bar{q}$ field from a γ/Z boson. Thus, the matching from full QCD onto SCET can be written as

$$M(e^+e^- \to X) = C_{n\bar{n}} \langle 0 | O_{n\bar{n}} | X \rangle \tag{9.6}$$

such that we can write

$$\frac{d\sigma}{d\tau} = \frac{1}{2Q^2} \sum_n |C_{n\bar{n}}|^2 \sum_X |\langle 0 | O_{n\bar{n}} | X \rangle|^2 \ (2\pi)^4 \delta^4(q - p_X)\delta(\tau - \tau(X)) \tag{9.7}$$

8.851 Effective Field Theory
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