Effective Field Theory (8.851) Spring 2013
Homework 2

Do problems 1 and 2. Problem 3 is optional and will not be graded, but is recommended for those people who have not previously seen an $\overline{\text{MS}}$ calculation with matching through thresholds.

Problem 1) Mixing and 4-quark operators

a) Compute the divergent part of the one-loop graphs that renormalize the operators for $b \to c\bar{u}d$ and derive the matrix Z_{ij} discussed in class.

b) Verify that in the case $\mu \to e\nu_\mu\bar{\nu}_e$ the four fermion operator is not renormalized at order e^2. Explain why.

Problem 2) Chiral Perturbation Theory and Decay Constants in SU(3)

Feel free to use a computer algebra package like Mathematica for parts of this problem.

a) Work out the tree level Feynman rules for the four meson interactions with $SU(3)$ chiral perturbation theory and the leading order Lagrangian. (You may use either the octet basis $M = \pi^a\lambda^a/\sqrt{2}$, or the charged particle basis.) You may start with the expression given in Lecture for the four boson interaction Lagrangian.

b) Work out the tree level Feynman rules in chiral perturbation theory for the left-handed SU(3) octet current $J^a_{L\mu} = (-if^2/4)\text{tr}[\lambda^a\Sigma\partial_\mu\Sigma^\dagger]$ (which was $\bar{\psi}\gamma^\mu P_L\lambda^a\psi$ in QCD) with one and three external mesons.

c) Write down the loop diagrams and terms in the chiral Lagrangian that are needed to give the decays constants at order p^4 (where $p \sim m_\pi \sim m_K$ so this also means order $m_\pi^4 \sim m_q^2$ etc.).

d) Determine the explicit contribution of the order p^4 Lagrangian terms to f_π and f_K taking $m_u = m_d = \hat{m}$, but $\hat{m} \neq m_s$.

e) Calculate the loops in part c) using dimensional regularization. Combine your result with d) to derive the full expression for f_π and f_K/f_π at this order (still taking the isospin limit $m_u = m_d$).

f) Extract a value for the relevant low energy contants, L_i, using the data $f_K/f_\pi = 1.23 \pm .02$, and discuss whether your result agrees with naive dimensional analysis.

Problem 3) Running Quark Masses (Optional Problem)

Consider QCD with n_f flavors.

a) Compute the quark mass anomalous dimension at one-loop in $\overline{\text{MS}}$. Solve to get the running $\overline{\text{MS}}$ mass $\bar{m}(\mu)$. (If you want to remind yourself how different schemes are related, you should also compute the relation between the pole mass and $\overline{\text{MS}}$ mass at one-loop.)

b) The Higgs decay rate for $H \to c\bar{c}$ is proportional to m_c^2, making it quite sensitive to the charm mass. Suppose we know the $\overline{\text{MS}}$ mass $\bar{m}_c(\mu = \bar{m}_c) = 1.4\,\text{GeV}$, and want to know what value to use for the Higgs decay at $\mu = m_H = 500\,\text{GeV}$. By matching and running in effective theories at LL order determine $\bar{m}_c(m_H)$. Why is this choice, $\mu \simeq m_H$, appropriate?

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