8.821 String Theory
Fall 2008

8.821 F2008 Lecture 03: The decoupling argument; AdS/CFT without string theory, a discovery with hindsight

Lecturer: McGreevy

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Today:
1. Backreaction and decoupling
2. A bold assertion
3. Hints, lore, prophesy

Recall from last time that:

• D-branes are subspaces where open strings can end. Nomenclature: Dp-branes have p spatial dimensions.

• Light open strings are localized at the brane, and represent the fluctuations of the brane.

• D-branes carry RR charges (which saturate Dirac quantization condition; this is necessary since we have both electric and magnetic charges). This can be seen by computing the amplitude for D-branes to emit RR gauge bosons; it is described by worldsheets as in the figure below.

• The world volume theory on the brane is a YM theory.

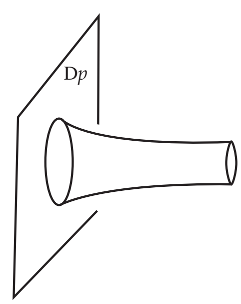

Figure 1: The disk amplitude for the emission of a closed string by a D-brane.

The disk amplitude for the emission of a graviton gives the tension of the D-brane and is proportional to

$$g_s^{-2+2h+b} = \frac{1}{g_s}. \tag{1}$$

The backreaction on the geometry from N coincident D-branes is determined from Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_{\rm N} T_{\mu\nu}^{\rm brane} \propto g_s^2 \cdot \frac{1}{g_s} \cdot N = g_s N = \lambda, \tag{2}$$

where we used the fact that

$$G_{\rm N} \propto \text{closed string coupling} \propto g_s^2. \tag{3}$$

At small λ, the backreaction is negligible and the physics is decribed by closed strings everywhere (these are excitations of empty space) plus open strings (YM $+\alpha' F^4 + \cdots$) localized on the branes (these are excitations of the branes) *in flat space.*

At large λ, the D-branes will gravitationally collapse into an RR soliton (black hole with the same charge). For $p > 0$, the black hole is not a point particle, but is also extended in p spatial dimensions.

The case of $p = 3$

The 3-brane will fill $3 + 1$ dimensions (e.g., $x^{\mu=0,1,2,3}$). We can put the brane at $y^1 = \cdots = y^6 = 0$, i.e, the $\mathbb{R}^{3,1}$ is at a point in the transverse $\mathbb{R}^6$. Let $r^2 = \sum y_i^2$ be the distance from the brane, then the metric of the brane will take the form

$$ds^2 = \frac{1}{\sqrt{H(r)}} \overbrace{\eta_{\mu\nu} dx^\mu dx^\nu}^{\mathbb{R}^{3,1}} + \sqrt{H(r)} \overbrace{\sum_{i=1}^{6} dy_i^2}^{\mathbb{R}^6}, \tag{4}$$

and

$$H(r) = 1 + \frac{L^2}{r^4}, \qquad dy^2 = dr^2 + r^2 d\Omega_5, \tag{5}$$

where $H(r) \to 1$ for large r and L is like an ADM mass. The RR F^5 satisfies

$$\int_{S^5 \text{ at constant } r} F_5 = N. \tag{6}$$

The solution looks like an RN black hole (Figure 2). Far away (large r with $H(r) = 1$), the solution will asymptote to $\mathbb{R}^{9,1}$. Near the horizon ($r \ll L$ with $H(r) = L^2/r^4$), the metric will take the form of an $\mathrm{AdS}_5 \times \mathrm{S}^5$

$$ds^2 = \underbrace{\frac{r^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + L^2 \frac{dr^2}{r^2}}_{\mathrm{AdS}_5} + \underbrace{L^2 \frac{r^2}{r^2} d\Omega_5}_{\mathrm{S}^5}. \tag{7}$$

The ultra-low energy excitations near the brane can't escape the potential well (throat) and the stuff from infinity can't get in. The absorption cross section of the brane goes like

$$\sigma \sim \omega^3 L^8. \tag{8}$$

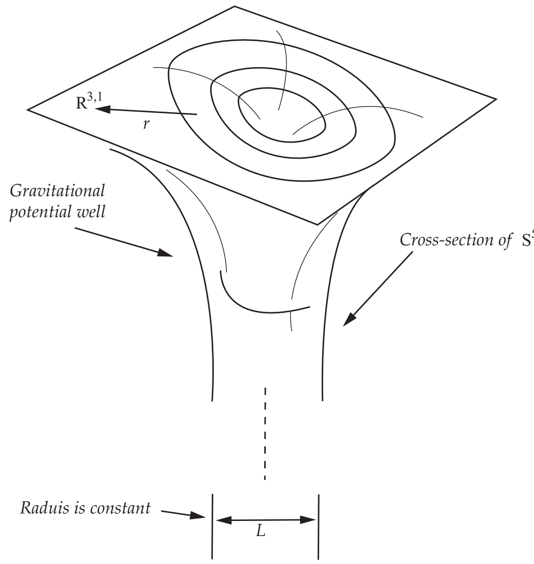

Figure 2:

The above conclusion can be reached in another way by noting that for low ω, the wavelength of the excitations will be too large to fit in the throat which has fixed size.

Comparison of low energy decoupling at large and small λ ($\lambda = Ng_s$)

Large λ		Small λ
Throat states = IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$	$\xrightarrow{\text{related by variation of } \lambda\text{, i.e, dual}}$	4D $\mathcal{N} = 4$, SU(N) YM
+		+
Closed strings in $\mathbb{R}^{9,1}$		Closed strings in $\mathbb{R}^{9,1}$

Maldacena: Subtract "Closed strings in $\mathbb{R}^{9,1}$" from both sides.

Matching of parameters

The parameters of the gauge theory are $\mathrm{g}_{\mathrm{YM}}^2 = g_s$ (we will see that it is really a parameter in $\mathcal{N} = 4$ YM) and the number of colors N. By Gauss's law

$$\int_{\mathrm{S}^5} \star F_5 = \int_{\mathrm{S}^5} F_5 = N \qquad \text{(quantized by Dirac).} \tag{9}$$

The supergravity equations of motion relate L (size of space) and N (number of branes) as follows:

$$R_{\mu\nu} = G_{\mathrm{N}} F^5_{\mu\ldots} F^{5\ldots}_{\nu}, \tag{10}$$

where

$$G_{\mathrm{N}} \propto g_s^2 \alpha'^{\,4}, \qquad \text{and } F^5_{\mu\ldots} F^{5\ldots}_{\nu} \propto N^2. \tag{11}$$

Dimensional analysis then says $R_{\mu\nu} \propto L^8$. This gives

$$\frac{L^4}{\alpha'^2} = g_s N = \lambda \text{ ('t Hooft coupling).} \tag{12}$$

In terms of gravitational parameters, this says

$$G_\mathrm{N} \sim g_s^2 (\alpha')^4, \quad L = N^{1/4} G_\mathrm{N}^{1/2} \longrightarrow G_\mathrm{N} \sim \frac{1}{N^2} \qquad \text{(in units of AdS raduis, } i.e. \text{ at fixed } \lambda). \tag{13}$$

A picture of what has happened here which can sometimes be useful is the Picture of the Tents. We draw the distance from the branes as the vertical direction, and keep track only of the size of the longitudinal and transverse directions as a function of this radial coordinate. Flat space looks like this: (Hence, tents.) The picture of the near-horizon limit looks instead like:

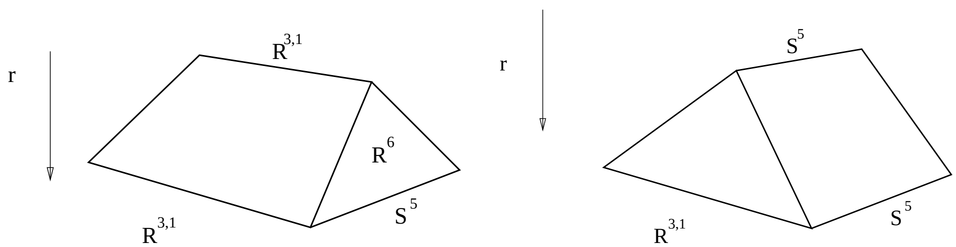

Figure 3: Left: Flat space. Right: $AdS_5 \times S^5$. Now it's the Minkowski space $\mathbb{R}^{3,1}$ which shrinks at $r = 0$, and sphere stays finite size.

A Bold Assertion

Now we back up and try to understand what is being suggested here, without using string theory. We will follow the interesting logic of:
Reference: Horowitz-Polchinski, gr-qc/0602037

Assertion: Hidden inside any non-abelian gauge theory is a quantum theory of gravity.

What can this possibly mean??
Three hints from the Elders:

1) At the least it must mean that there is some spin-two graviton particle, that is somehow a composite object made of gauge theory degrees of freedom. This statement seems to run afoul of the Weinberg-Witten no-go theorem, which says:
Theorem (Weinberg-Witten)**:** A QFT with a Poincaré covariant conserved stress tensor $T^{\mu\nu}$ forbids massless particles of spin $j > 1$ which carry momentum (*i.e.* with $P^\mu = \int d^D x T^{0\mu} \neq 0$).

GR gets around this because the total stress tensor (including the gravitational bit) vanishes by the metric EOM $\frac{\delta S}{\delta g_{\mu\nu}} = 0$. (Alternatively, the 'matter stress tensor' which doesn't vanish is not general-coordinate invariant.)

Like any good no-go theorem, it is best considered a sign pointing away from wrong directions. The loophole in this case is blindingly obvious in retrospect: the graviton needn't live in the same spacetime as the QFT.

2) Hint number two comes from the Holographic Principle ['t Hooft, Susskind]. This is a far-reaching consequence of black hole thermodynamics. The basic fact is that a black hole must be assigned an entropy proportional to the *area* of its horizon in planck units (much more later). On the other hand, dense matter will collapse into a black hole. The combination of these two observations leads to the following crazy thing: The maximum entropy in a region of space is its area in Planck units. To see this, suppose you have in a volume V (bounded by an area A) a configuration with entropy $S > S_{BH} = \frac{A}{4G_N}$ (where S_{BH} is the entropy of the biggest blackhole fittable in V), but which has *less* energy. Then by throwing in more stuff (as arbitrarily non-adiabatically as necessary, *i.e.* you can increase the entropy) you can *make* a black hole. This would violate the second law of thermodynamics, and you can use it to save the planet from the Humans. This probably means you can't do it, and instead we conclude that the black hole is the most entropic configuration of the theory in this volume. But its entropy goes like the *area*! This is much smaller than the entropy of a local quantum field theory, even with some UV cutoff, which would have a number of states $N_s \sim e^V$ (maximum entropy = $\ln N_s$) Indeed it is smaller (when the linear dimensions are large!) than any system with local degrees of freedom, such as a bunch of spins on a spacetime lattice.

We conclude from this that a quantum theory of gravity must have a number of degrees of freedom which scales like that of a QFT in a smaller number of dimensions.

This crazy thing is actually true, and AdS/CFT is a precise implementation of it.

Actually, we already know some examples like this in low dimensions. One definition of a quantum gravity is a generally-covariant quantum theory. This means that observables (for example the effective action) are independent of the metric:

$$0 = \frac{\delta S_{eff}}{\delta g_{\mu\nu}} = T^{\mu\nu}.$$

We know two ways to accomplish this:
1) Integrate over all metrics. This is how GR works.
2) Don't ever introduce a metric. Such a thing is generally called a topological field theory. The best-understood example is Chern-Simons gauge theory in three dimensions, where the variable is a gauge field and the action is

$$S_{CS} = \int_M \text{tr } A \wedge dA + ...$$

(where the dots is extra stuff to make the nonabelian case gauge invariant); note that there's no metric anywhere here. With either option (1) or (2) there are no local observables. But if you put the theory on a space with boundary, there are local observables which live on the boundary. Chern-Simons theory on some manifold M induces a WZW model (a 2d CFT) on the boundary of M. We will see that the same thing happens for more dynamical quantum gravities.

3) A beautiful hint as to the possible identity of the extra dimensions is this. Wilson taught us that a QFT is best thought of as being sliced up by length (or energy) scale, as a family of trajectories of the renormalization group (RG). A remarkable fact about this is that the RG equations for the

behavior of the coupling constants as a function of RG scale z is *local* in scale:

$$z\partial_z g = \beta(g(z))$$

– the RHS depends only on physics at scale z. It is determined by the coupling constant evaluated at the scale z, and we don't need to know its behavior in the deep UV or IR. This fact is not completely independent of locality in spacetime. This opens the possibility that we can associate the extra dimensions raised by the Holographic idea with energy scale.

Next we will make simplifying assumptions in an effort to find concrete examples.