Chapter 2: Deriving AdS/CFT

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Lecture 13

Important equations for this lecture from the previous ones:

1. The Dirichlet condition (D-condition) for the ends of open string ($\sigma_{end} = 0, \pi$) in spatial direction, mentioned after equation (28) in lecture 10:

$$\delta X^i = 0 \to X^i(\sigma_{end}, \tau) = \text{const} \quad . \tag{1}$$

2. The Neumann condition (N-condition) for the ends of open string ($\sigma_{end} = 0, \pi$), mentioned after equation (28) in lecture 10:

$$\partial_\sigma \delta X^\mu = 0 \quad . \tag{2}$$

2.1.3: D-BRANES

There are many nonperturbative objects in string theories (or cannot be seen easily from perturbative string theory, but can be guessed from the consistency of interactions or the dualities network), such as the D-branes or the NS-branes. For the interests of this course, we will need a closer look at D-branes in bosonic string theory.

Let's start with the D-condition, given in equation (1). The physical interpretation is that the end point is restricted at a hypersurface or a p-dimensional surface, a "spacetime defect" where open strings can end, which seemingly is not a degrees of freedom from perturbative string point of view. Such object is called a D-brane, and Dp-brane is a D-brane with p spatial direction:

1. It should be noted that D-condition can also be in the time direction, such as the case with D(-1)-branes (which is stuck at a single position in spacetime), but first one has to analytically continue the spacetime to Euclidean signature first (imaginary time). In Lorentzian spacetime, however, there's no D(-1)-brane (in the sense that it's not a stable object, only appears for an instant in time). Because time in the target space cannot standstill, so the condition $X^0 = \text{const}$ doesn't happen.

2. If in all direction the end of open string has N-condition, one can interpret that this means the open string can end anywhere, which can be thought as there's a space-filling brane (D25-brane, as the number of spatial dimensions is 25 in bosonic string theory) and there's interesting dynamics associated with that nonperturbative objects. One way to think about it is that strings must naturally exist as closed strings, and they can only break open at some special places in spacetime, where D-branes are located. Open strings can be thought as the quantized fundamental fluctuation on the surface of D-branes.

3. There can be many different D-branes of different dimensions in spacetime, and an open string can have its ends on different D-branes.

A Dp-brane in a general spacetime dimension D breaks translational T^D and Lorentz symmetries $SO^+(1,D)$ (Poincare symmetry in D dimensions) into:

$$\left(T^p \otimes SO^+(1,p)\right) \otimes SO(D-1-p) \tag{3}$$

The Poincare symmetry along the Dp-brane is preserved, while in the directions perpendicular to the brane (the traverse directions) only the spatial rotation symmetry is unbroken.

Consider a single Dp-brane (N-condition for direction $\alpha = 0, 1, .., p$ and D-condition for direction $a = p+1, p+2, ..., D-1$), then the classical solution gives:

$$X^\alpha(\sigma,\tau) = x^\alpha + 2\alpha' p^\alpha \tau + X_R^\alpha(\tau-\sigma) + X^\alpha(\tau+\sigma) \tag{4}$$

$$= x^\alpha + 2\alpha' p^\alpha \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\alpha}{n}e^{-in\tau}\cos(n\sigma) \quad \Rightarrow \quad X_R^\alpha = X_L^\alpha \ ; \tag{5}$$

$$X^a(\sigma,\tau) = b^a + 2\alpha' p^a \tau + X_R^a(\tau-\sigma) + X_L^a(\tau+\sigma) \ , \quad X^a(0,\sigma_{end}) = b^a \tag{6}$$

$$= b^a + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^a}{n}e^{-in\tau}\sin(n\sigma) \quad \Rightarrow \quad p^a = 0 \ , \quad X_L^a = -X_R^a \tag{7}$$

The quantization in D-directions is exactly the same as in N-directions. In other words, the commutation relations for α_n^a is exactly no different to before, and the D-condition only constrains the center of mass motion as strings still oscillate the same in all directions. In particular, the zero-point energy on the worldsheet remains the same ($X_{L,R}$ are still periodic functions with 2π period). It should be noted that if the boundary conditions of the open strings are mixed, such as ND or DN. The open string spectrum ends on a single Dp-brane:

$$\left(\alpha_{-n_1}^{\mu_1}\right)^{m_1}\left(\alpha_{-n_2}^{\mu_2}\right)^{m_2}...\left(\alpha_{-n_k}^{\mu_k}\right)^{m_k}|0,p^\alpha\rangle \ , \quad M^2 = -p^\alpha p_\alpha \tag{8}$$

The particles are in (p+1)-dimensional spacetime. They live in the worldvolume of the Dp-brane and their states should fall into the representations of the group given in equation (3). The massless states (now use $i = 2, 3, ..., p+1$ as $\alpha = +, -, i$):

$$\alpha_{-1}^i|0,p^\alpha\rangle \ \ , \alpha_{-1}^a|0,p^\alpha\rangle \tag{9}$$

The first is a massless vector in (p+1)D while the later describes $D-1-p$ scalar in (p+1)D. These particles give rise to a vector field A_α and $D-1-p$ scalar fields Φ^a in the worldvolume of that D-brane. The low energy effective action of these modes:

$$S_{eff} \sim \int d^{p+1}x\left(\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}\partial^\alpha\Phi^a\partial_\alpha\Phi^a + ...\right) \tag{10}$$

The number of massless scalar fields is the same as the number of traverse directions of the Dp-brane, since their physical interpretation is the description of D-brane fluctuation in the perpendicular directions. While this intuitively makes sense, one can see it mathematically from the following aspect:

1. At stringy worldsheet level, having excitations of constant Φ^a modifies the D-condition from $X^a = b^a$ to $X^a = b^a + \Phi^a$.

2. Working out the precise prefactor in equation (10) gives:

$$S = -T_p\int d^{p+1}x\left(1 + \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}\partial^\alpha\Phi^a\partial_\alpha\Phi^a + ...\right) \tag{11}$$

From the dualities network or conventional CFT arguments, one derives the Dirac-Born-Infeld (DBI) effective action to describe the perturbative fluctuation dynamics (the derivatives of $F_{\alpha\beta}$ and $\partial_\alpha\Phi^a$ are very small) of Dp-branes:

$$S_{Dp} = -T_p\int d^{p+1}x\sqrt{-\left|G_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right|} \ , \quad G_{\alpha\beta} = \partial_\alpha X^\mu\partial_\beta X^\nu g_{\mu\nu} = g_{\alpha\beta} + \partial_\alpha\Phi^a\partial_\beta\Phi^a \tag{12}$$

At leading order one arrives at this effective action with the exact value of T_p (can be found by matching between 2 Dp-branes the perturbative string interaction of annulus topology with the interaction described by QFT's propagation with vertex weight given from the DBI effective action). Note that excluding the unfluctuated part (with no field involves), the rest can be found exactly from just the scattering of strings. The unfluctuated part gives the mass of the Dp-brane:

$$M_{Dp} = T_p \int d^p \vec{x} = T_p V_p \tag{13}$$

With $A_\alpha = 0$ and $\Phi^a = \Phi^a(t)$, then at leading order:

$$S = -\int dt M_{Dp} \Big(1 + \frac{1}{2} \dot{\Phi}^a + .. \Big) \tag{14}$$

This action is similar to that of a relativistic classical object, with Φ^a can be associated with the center of mass position of the Dp-brane.

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