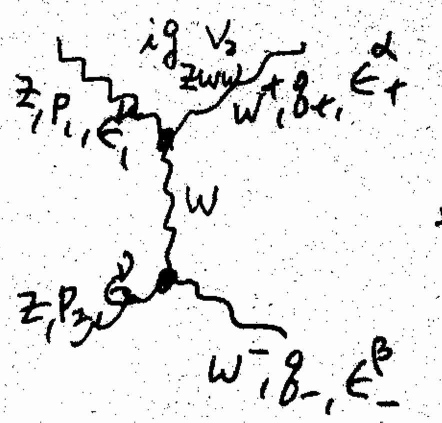
4 vector vertex, Higgs, Couplings & Mass of Higgs

Consider

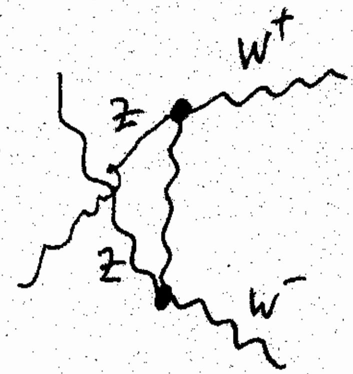
$$Z(P_1, \epsilon_1) + Z(P_2, \epsilon_2) \to W^+(q_+, \epsilon_+) + W^-(q_-, \epsilon_-)$$



$$M_1 = i g_{ZWW}^2 V_3(q_+, \epsilon_+, P_1 - q_+, \alpha, -P_1, \epsilon_1)$$

$$* \frac{g_{\alpha\beta} - \frac{1}{m_W^2}(P_1 - q_+)_\alpha (P_- - q_+)_\beta}{(P_1 - q_+)^2 - m_W^2}$$

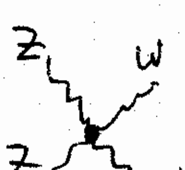
$$* V_3(P_2 - q_-, \beta, q_-, \epsilon_-, -P_2, \epsilon_2)$$



$$M_2 = M_1(P_1, \epsilon_1 \rightleftarrows P_2, \epsilon_2)$$

$$M_1 + M_2 \xrightarrow[\epsilon_+ \to \frac{q_+}{m_W}]{} \frac{i g_{ZWW}^2}{m_W}\Big[2(q_+ \cdot \epsilon_-)(\epsilon_1 \cdot \epsilon_2) - (q_+ \cdot \epsilon_1)(q_- \cdot \epsilon_2)$$

$$- (q_+ \cdot \epsilon_2)(q_- \cdot \epsilon_1)\Big]\left(1 + \frac{m_Z^4}{E^2 m_W^2}\right)$$



4-boson vertex is introduced to cancel the leading term

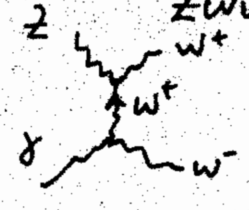
$$Y_4(\alpha, \beta, \mu, \nu) = \{2g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\}$$

which is to be multipled by $\epsilon_{+\alpha}\, \epsilon_{-\beta}\, \epsilon_{1\mu}\, \epsilon_{2\nu}$ to be M_3.

Four-vector vertices:

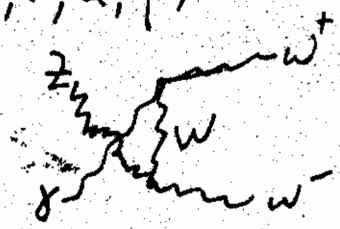
1) $ZZ \to WW$

$$-i g^2_{ZWW} \, Y_4(\mu,\nu,\alpha,\beta)$$

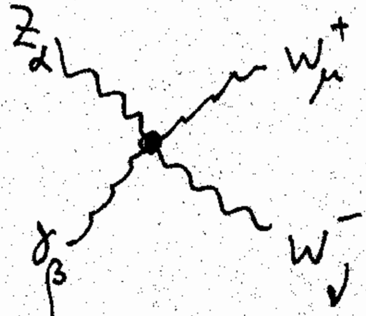


$i g_{ZWW} V_3$

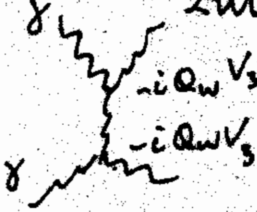
$i Q_W V_3$



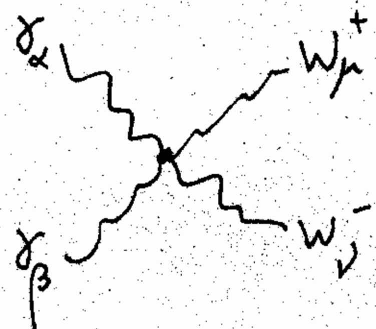
2) $Z\gamma \to WW$



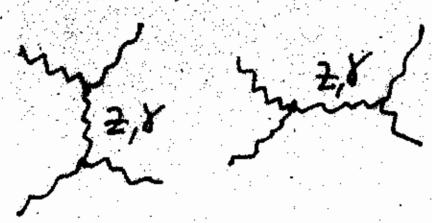
$$-i g_{ZWW} \, Q_W Y_4(\mu,\nu,\alpha,\beta)$$



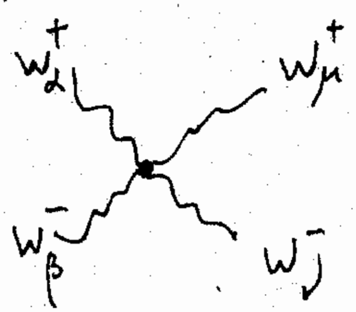
3) $\gamma\gamma \to WW$



$$-i Q_W^2 \, Y_4(\mu,\nu,\alpha,\beta)$$



4) $W^+W^- \to W^+W^-$



$$+i(\underbrace{g^2_{ZWW} + Q_W^2}_{e^2/s_w^2}) \, Y(\mu,\nu,\alpha,\beta)$$

$ZZ\gamma$, $Z\gamma\gamma$, $ZZ\gamma\gamma$ are not needed thus

Not Introduced $\to$ Decoupled forces!

For $ZZ \to W^+W^-$, take all 4 bosons longitudinal

$$\epsilon_i \to q_i/m_i \qquad q_+ \cdot q_- = E^2 + P^2 = 2E^2$$

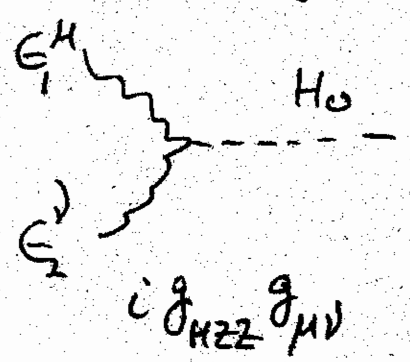
$$M_1 + M_2 \to i\, g^2_{ZWW} \frac{E^2 E^2}{m_W^2 m_Z^2} \left(1 + \frac{m_Z^4}{E^2 m_W^2}\right)$$

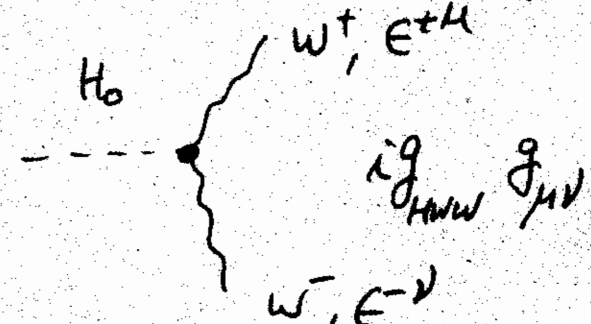
where $E^2 = s/4$.

Include the ZZWW vertex,

$$M_1 + M_2 + M_3 \to i g^2_{ZWW} E^2 \frac{m_Z^2}{m_W^4} + \mathcal{O}(1)$$

Still divergent as $s \to \infty$. Introduce the Higgs





$$M_4 = -i g_{HZZ}\, g_{HWW} (\epsilon_1 \cdot \epsilon_2) \frac{1}{s - m_H^2} (\epsilon_+ \cdot \epsilon_-)$$

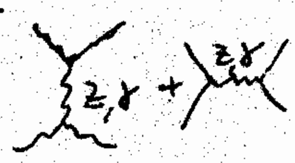
$$\xrightarrow[\epsilon_i \to \frac{q_i}{m_i}]{} -i g_{HZZ}\, g_{HWW} \frac{s^2}{4 m_W^2 m_Z^2 (s - m_H^2)}$$

So for $M_1 + M_2 + M_3 + M_4 \longrightarrow 0$ as $s \gg m_H^2$

$$\therefore\; g^2_{ZWW} \frac{m_Z^4}{m_W^2} = g_{HWW}\, g_{HZZ}$$

Determination of g_{HWW}, g_{HZZ} & m_W/m_Z

We derive 3 unitarity constraints:

from reactions	constraints
1) $ZZ \to W^+W^-$	$g_{HZZ}\, g_{HWW} = g^2_{ZWW} \dfrac{m_Z^4}{m_W^2}$
2) $W^+W^- \to W^+W^-$ 	$g^2_{HWW} = g^2_{ZWW}(4m_W^2 - 3m_Z^2) + 4Q_W^2 m_W^2$
3) $ZH \to WW$	$g_{HZZ} = g_{HWW}\, m_Z^2/m_W^2$

There is a unique solution

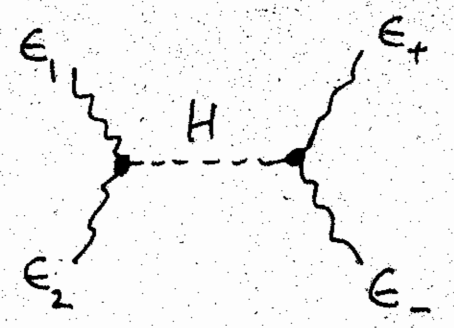
$$g_{HWW} = -e\,\frac{m_W}{S_W}$$

$$g_{HZZ} = -e\,\frac{m_Z}{S_W C_W}$$

$$m_W = m_Z\, C_W$$

Constraint on the Higgs Mass M_H using unitarity

For $Z_L Z_L \to W_L^+ W_L^-$, the Higgs contribution is



$$M_H = -i\, g_{HZZ}\, g_{HWW} \frac{(\epsilon_1 \cdot \epsilon_2)(\epsilon_+ \cdot \epsilon_-)}{s - m_H^2}$$

$$= -i \frac{e^2 m_W m_Z}{S_W^2 C_W} \frac{s}{2m_Z^2} \frac{s}{2m_W^2} \frac{1}{s - m_H^2}$$

The full amplitude is:

$$M = i \frac{e^2}{4m_W^2 S_W^2} \left(\frac{s^2}{s - m_H^2} - s \right) = -i \frac{\pi\alpha}{S_W^2} \frac{m_H^2}{m_W^2} \frac{s}{s - m_H^2}$$

The cross section

$$\sigma = \frac{1}{16\pi s} |M|^2 = \frac{1}{16\pi} \frac{\pi^2 \alpha^2}{S_W^4} \left(\frac{m_H}{m_W} \right)^4 \frac{s}{(s - m_H^2)^2}$$

$\leq \frac{16\pi}{s}(2J+1)$ unitarity limit for $J=0$, $2J+1 = 1$

no spin average, $\because$ only longitudinal polarization contributes.

$$\therefore \quad \frac{m_H}{m_W} \leq \frac{4 S_W}{\sqrt{\alpha}} \sim \mathcal{O}(1\,\text{TeV}) \text{ unitarity limit}$$

exactly like what we found from

$\bar{\nu}_e e \to W \to \mu \bar{\nu}_\mu \qquad m_W < 1.6$ GeV!

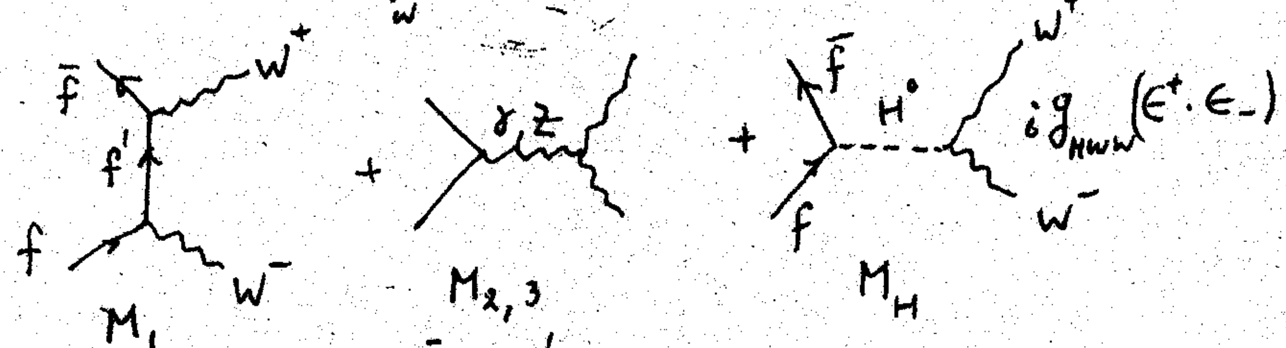
Higgs-fermion-fermion Couplings

Reconsider $\bar{f} f \to W^+ W^-$ but now with both W^+ & W^- approaching their longitudinal limit:

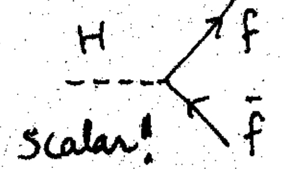
$$M_{1,2,3} \Rightarrow -2 i g_W^2 \, \bar{v}(p_1) \, u(p_2) \, \frac{m_f}{m_W^2} +$$

$\epsilon_+^\mu \to g_+^\mu$

$\epsilon_-^\nu \to g_-^\nu \sim \frac{E m_f}{m_W^2}$ · divergent!



now introduce $Hf\bar{f}$ vertex



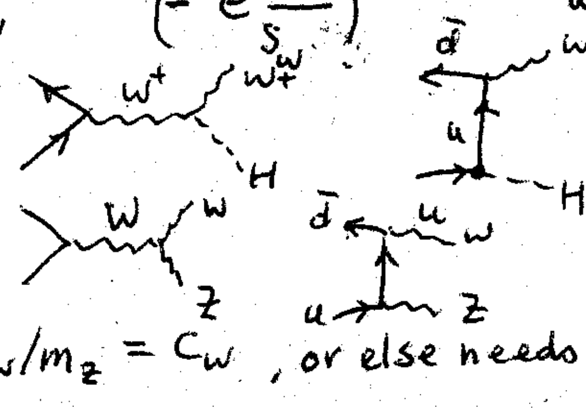
$= i g_{Hf\bar{f}}$ $\qquad g_{Hf\bar{f}} = \frac{e}{2 S_W} \frac{m_f}{m_W}$

$$M_H = -i \, g_{Hf\bar{f}} \, g_{HWW} \, \frac{\bar{v}(p_1) \, u(p_2)}{s - M_H} (\epsilon_+ \cdot \epsilon_-), \quad \epsilon_+ \cdot \epsilon_- \to E^2 + p^2 = \frac{s}{2}$$

$$g_{Hf\bar{f}} = \frac{-2^2 g_W^2 \frac{m_f}{m_W^2}}{g_{HWW}} = \frac{-4 \left(\frac{e^2}{S_W^2 \cdot 8}\right) \frac{m_f}{m_W}}{\left(-e \frac{m_W}{S_W}\right)} = \frac{e}{2 S_W} \frac{m_f}{m_W}$$

Note $\quad u\bar{d} \to W^+ H$

$\quad u\bar{d} \to W^+ Z$



convergent if $m_W / m_Z = c_W$, or else needs H^+