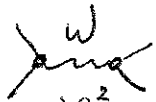
Over-view

We demand $M = \partial_\mu \epsilon^\mu \to \partial_\mu k^\mu = 0$ for external γ's

$$\left\{ \begin{array}{l} M = \partial_\mu \epsilon^\mu_\pm \to \partial_\mu k^\mu_\pm \\ \sigma\,(\lim k^0_t \to \infty) < \text{Unitary limit} \end{array} \right\} \text{ for external } W^\pm$$

and Q_W, $WW\gamma$ vertex, $Z^0(!)$, couplings of Z, Higgs, ...

Reactions	divergent as	outcome	
$e\nu \to \mu\nu$	$\frac{GS}{3\pi}$	 $\frac{i g_W^2}{s - m_W^2}$	$\frac{2 g_W^4}{3\pi} \frac{s}{(s - m_W^2)^2 + m_W^2 \Gamma_W^2}$
$\bar{d}u \to W^+\gamma$	$\frac{s}{m_W^2}$		$Q_{W^+} = Q_u - Q_d = +1$
$\bar{u}u \to W^+W^-$	$\frac{s^2}{m_W^4}$	$W^+W^-\gamma$ vertex, $Z^0(!)$	
$u\bar{d} \to W^+Z$	$\frac{s^2}{m_W^4}$	g_{WWZ} 	$Z \to f\bar{f}$ couplings
$d\bar{d} \to W^+W^-$	$\frac{s^2}{m_W^4}$		

$$m_W = \sqrt{\frac{\pi \alpha}{G\sqrt{2}}} \; \frac{1}{\sin\theta_W}$$

$ZZ \to W^+W^-$ $\qquad \frac{s^4}{m_W^8}$

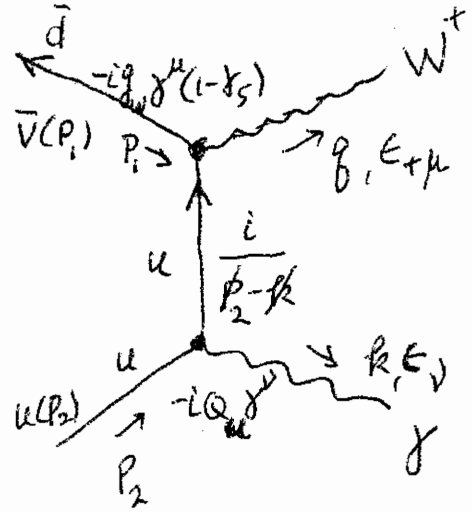
vertex, H and g_{HZZ}, g_{HWW}, $m_Z = \frac{m_W}{\cos\theta_W}$, $M_H < M_W \, 4 \sin\theta_W / \sqrt{\alpha} < 1 \text{ TeV}$

Details

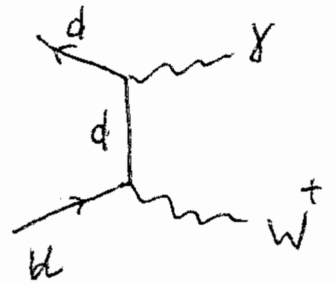
Consider an 'up' particle ($= \nu, u, c, \cdots$)

and a 'down' " ($= \ell, d, s, \cdots$) coupling to the W

Let $\bar{d}(p_1)\, u(p_2) \to W^+(q, \epsilon_+)\, \gamma(k, \epsilon)$



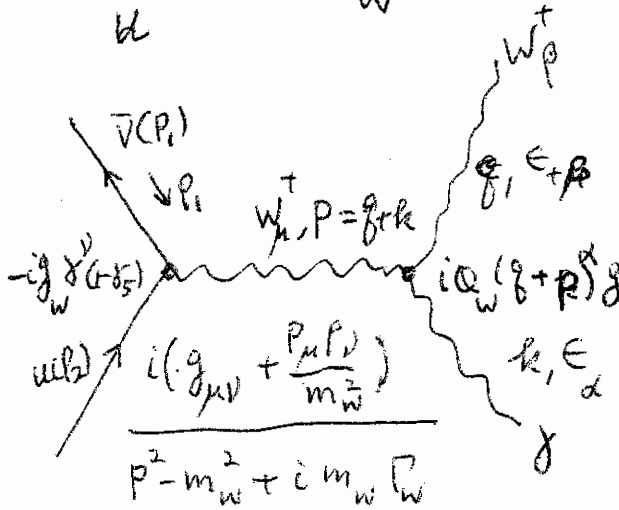
$$M_1 = -i\, Q_u\, g_w\, \bar{v}(p_1)(1+\gamma^5)\, \not\epsilon_+ \frac{\not p_2 - \not k}{(p_2-k)^2} \not\epsilon\, u(p_2)$$



$$M_2 = -i\, Q_d\, g_w\, \bar{v}(p_1)\, \not\epsilon \frac{\not k - \not p_1}{(k-p_1)^2} (1+\gamma^5) \not\epsilon_+ u(p_2)$$

if we try 

$= i\, Q_w (p_1 - p_2)^\alpha g^{\mu\nu}\, \epsilon_\alpha\, \epsilon_{+\mu}\, \epsilon_{-\nu}$



$$M_3 = i\, Q_w\, g_w\, \bar{v}(p_1)(1+\gamma^5) \not\epsilon_+ u(p_2) \frac{(2q+k)\cdot\epsilon}{(q+k)^2 - m_w^2}$$

$$M = M_1 + M_2 + M_3$$

Current conservation $M = \partial_\mu \epsilon^\mu \to \partial_\mu k^\mu = 0$

First simplify matrix elements using explicit Dirac algebra:

in M_1:
$$\frac{\not{P}_2-\not{k}}{(P_2-k)^2}\not{\epsilon}\, u(P_2) \xrightarrow[\epsilon^\mu\to k^\mu]{} \frac{(\not{P}_2-\not{k})\cdot\not{k}}{P_2^2+k^2-2P_2\cdot k} u(P_2) = \frac{\not{P}_2\cdot\not{k}}{-2(P_2\cdot k)} u(P_2)$$

$$= \frac{2(P_2\cdot k) - \not{k}\not{P}_2}{-2(P_2\cdot k)} u(P_2) = -u(P_2) \qquad \because \not{P}_2 u(P_2)=0$$

Similarly in M_2: $\bar{V}(P_1)\not{\epsilon}\frac{\not{k}-\not{P}_1}{(k-P_1)^2} \xrightarrow[\epsilon^\mu\to k^\mu]{} \bar{V}(P_1)\frac{(-\not{k}\not{P}_1)}{-2k\cdot P_1} = \bar{V}(P_1)$

in M_3:
$$\frac{(2q+k)\cdot\epsilon}{(q+k)^2-m_w^2} = \frac{2q\cdot\epsilon+\overset{0}{k\cdot\epsilon}}{2q\cdot k} = \frac{2q\cdot\epsilon}{2q\cdot k} \xrightarrow[\epsilon^\mu\to k^\mu]{} 1$$

So $M_1 \to i Q_u g_w \bar{V}(P_1)(1+\gamma^5)\not{\epsilon}_+ u(P_2)$

$M_2 \to -i Q_d g_w \bar{V}(P_1)(1+\gamma^5)\not{\epsilon}_+ u(P_2)$

$M_3 \to +i Q_{w^-} g_w \bar{V}(P_1)(1+\gamma^5)\not{\epsilon}_+ u(P_2)$

$\therefore\ Q_{w^-} = Q_d - Q_u = -1$ for w^-

Next let $\epsilon_+^\mu \to q^\mu$

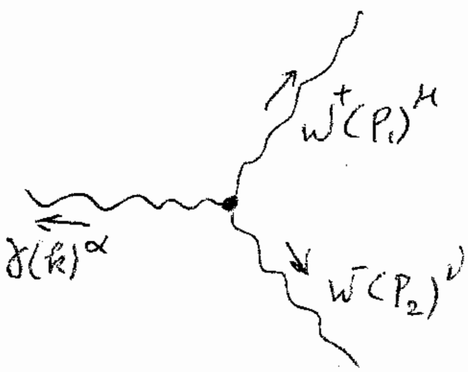
In M_1 $\quad \bar{v}(P_1)(1+\gamma^5)\not{\epsilon}_+ \dfrac{\not{P}_1+\not{q}}{(P_1+q)^2} \xrightarrow[\epsilon_+^\mu \to q^\mu]{} \bar{v}(P_1)(1+\gamma^5)\not{q}\dfrac{\not{P}_1+\not{q}}{(P_1+q)^2}$

$$= \bar{v}(P_1)(1+\gamma^5)\frac{\not{q}\,\not{P}_1}{2P_1\cdot q} = \bar{v}(P_1)(1+\gamma^5)\frac{2P_1\cdot q - \not{P}_1\not{q}}{2P_1\cdot q}$$

$$= \bar{v}(P_1)(1+\gamma^5)$$

$$\underset{\epsilon_+^\mu \to q^\mu/m_W}{M(\bar{d}u \to W\gamma)} = \frac{iQ_W g_W}{m_W} v(P_1)(1+\gamma^5)\gamma_\alpha u(P_2)\left\{\epsilon^\alpha - \frac{q\cdot\epsilon}{q\cdot k}k^\alpha\right\} \propto \frac{E}{m_W}$$

No cancellation possible! $WW\gamma$ vertex needs <u>symmetri</u>



$$iQ_W\left[(P_1-P_2)^\alpha g^{\mu\nu} + (P_2-k)^\mu g^{\alpha\nu} + (k-P_1)^\nu g^{\mu\alpha}\right]\epsilon_\alpha\,\epsilon_{+\mu}\,\epsilon_{-\nu}$$

$$= iQ_W V(P_1,\mu,P_2,\nu,k,\alpha)\,\epsilon_\alpha\,\epsilon_{+\mu}\,\epsilon_{-\nu}$$

Yang-Mills vertex !

Using Yang-Mills vertex we get

$$\mathcal{M}(u\bar{d} \to W^+\gamma)\Big|_{\epsilon^\mu \to k^\mu} = 0 \qquad \text{if} \quad Q_W = Q_d - Q_u$$

$$\mathcal{M}(u\bar{d} \to W^+\gamma)\Big|_{\epsilon_+^\mu \to \frac{q^\mu}{m_W}} = i\,\frac{Q_W g_W}{m_W}\,\bar{v}(P_1)(1+\gamma^5)\,\not{\epsilon}\,u(P_2) \times \left[1 - \frac{(q+k)^2}{(q+k)^2 - m_W^2}\right]$$

(refer to p. 57)

$$= i\,\frac{Q_W g_W}{m_W}\,\bar{v}(P_1)(1+\gamma^5)\,\not{\epsilon}\,u(P_2)\,\frac{-m_W^2}{2\,q\cdot k}$$

$$\sim \quad \sqrt{E} \qquad \sqrt{E} \qquad \frac{m}{E^2}$$

$$\sim \quad \mathcal{O}\left(\frac{m}{E}\right) \xrightarrow[\text{as } E \to \infty]{} 0 \quad \text{O.K.}$$