$(\varepsilon_i)_\alpha$ Polarization vectors of W, massive, spin = 1

$$[-k^2 g^{\mu\nu} + k^\mu k^\nu + m_W^2 g^{\mu\nu}](\epsilon_i)_\nu = 0$$

$$k^\mu k^\nu \qquad (\epsilon_i)_\nu = 0$$

or $\qquad k^\nu (\epsilon_i)_\nu = 0 \qquad$ with $\varepsilon_i^\mu \varepsilon_{j\mu} = -\delta_{ij}$

we have 3 polarization vectors for $\vec{W}$;

For $k^\mu = (k^0, 0, 0, |\vec{k}|)$

$$\left.\begin{array}{l} \varepsilon_1^\mu = (0, 1, 0, 0) \\ \varepsilon_2^\mu = (0, 0, 1, 0) \end{array}\right\} \text{Transverse}$$

& $\varepsilon_3^\mu = \left(\frac{|\vec{k}|}{m_W}, 0, 0, \frac{k^0}{m_W}\right)$ longitudinal polarization

When $\frac{k^0}{m_W} \to \infty$, $\varepsilon_3^\mu = \frac{k^\mu}{m_W} + \theta\left(\frac{m_W}{k^0}\right) \longrightarrow \infty$

$$\sum_{\text{spin}} MM^* \Rightarrow \sum_i (\varepsilon_i)^\mu (\varepsilon_i)^\nu = G^{\mu\nu} = \begin{pmatrix} -1+\frac{k^{02}}{m_W^2} & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1+\frac{k^{02}}{m_W^2} \end{pmatrix}$$

i.e. $G^{00} = \frac{|\vec{k}|^2}{m_W^2} = -1 + \frac{k_0^2}{m_W^2}$

$G^{11} = G^{22} = 1$

$G^{3} = \frac{k^{02}}{m_W^2} = 1 + \frac{|\vec{k}|^2}{m_W^2}$

$\therefore \sum_i \varepsilon_i^\mu \varepsilon_i^\nu = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2} \qquad$ Compare with $S^{\mu\nu}$!

Let $k^\mu = (k^0, 0, 0, |\vec{q}|)$

$$\sum_{\lambda=1}^{4} \varepsilon_\mu^{\lambda *} \varepsilon_\nu^{\lambda} = \sum_{\lambda=2}^{3} \varepsilon_\mu^{\lambda *} \varepsilon_\nu^{\lambda} + \varepsilon_\mu^{L *} \varepsilon_\nu^{L} + \varepsilon_\mu^{S *} S_\nu$$

$(M \neq 0)$

$$= \begin{pmatrix} 0 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 0 \end{pmatrix} + \frac{1}{M^2} \begin{pmatrix} P^2 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & E^2 \end{pmatrix}$$

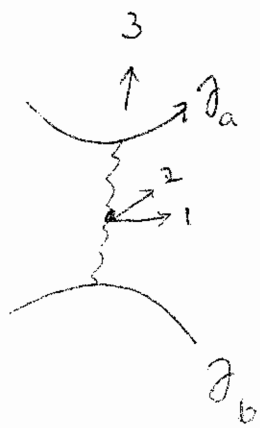
$$= \left(g_{\mu\nu} - \hat{k}_\mu \hat{k}_\nu \right) + \begin{pmatrix} -1 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & 1 \end{pmatrix} + \frac{1}{M^2} \begin{pmatrix} E^2 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & p^2 \end{pmatrix}$$

$$= -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}$$

$(M \to 0)$

$$= \begin{pmatrix} 0 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 0 \end{pmatrix} + \begin{pmatrix} 0 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & 1 \end{pmatrix} + \underbrace{\begin{pmatrix} -1 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & 0 \end{pmatrix}}_{(-g_{\mu 0}\, g_{\nu 0})}$$

$$= \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} = -g^{\mu\nu}$$



Coulomb Interaction

$\to 0$ as $q^0 \to |\vec{q}|$

$$M \sim J_\mu^a \frac{-g^{\mu\nu}}{q^2} J_\nu^b$$

$$= \Big(\underbrace{J_1^a J_1^b + J_2^a J_2^b}_{\text{Transverse}} + \underbrace{J_3^a J_3^b - J_0^a J_0^b}_{\text{Longitudinal + scalar}} \Big) / q^2$$

$\because \partial^\mu J_\mu = 0$

$\therefore q^\mu J_\mu = q^0 J_0 - |\vec{q}| J_3 = 0$

$\therefore J_0 = J_3$ if $q^0 \to |\vec{q}|$

$\therefore$ For real γ, $\sum \varepsilon_\mu^{\lambda *} \varepsilon_\nu^{\lambda} \longleftrightarrow -g_{\mu\nu}$

Important for γ

As $m \to 0$

$$\epsilon_1^\mu = (0,1,0,0) \qquad \epsilon_2^\mu = (0,0,\cancel{1},0)$$

but $\epsilon_3^\mu = \frac{k^\mu}{m} + \theta\left(\frac{m}{k^0}\right) \to \infty$!!

For any process with an external γ we need

$$M = M_\mu \epsilon^\mu \qquad \text{with} \quad M_\mu k^\mu = 0$$

In Fourier transform, $\partial_\mu M^\mu = 0$ current conservation

In any Lorentz frame,

$$\begin{array}{lll} \text{Frame 1} & k^\mu = (k^0, \vec{k}) & \varepsilon_{Tr}^\mu = (0, \vec{\varepsilon}_{Tr}) \\ \downarrow \text{boost} & \downarrow & \downarrow \\ \text{Frame 2} & k'^\mu = (k^{0\prime}, \vec{k}') & \varepsilon'^\mu = (\varepsilon'^0, \vec{\epsilon}') \end{array}$$

$$\varepsilon'^\mu = \left(\varepsilon'^0 \frac{k'^0}{k'^0}, \vec{\epsilon}'\right)$$

$$= \left(\underbrace{\varepsilon'^0 \frac{k'^\mu}{k'^0}}_{\uparrow}, \underbrace{\vec{\epsilon}' - \frac{\varepsilon'^0}{k'^0} \vec{k}'}_{\epsilon'_{Tr}}\right)$$

Eliminated due to Gauge transformation

$$A'_\mu \to A_\mu + \partial_\mu i a^{-ikx}$$

$$\varepsilon'_\mu \to \varepsilon_\mu + a k_\mu$$

Useful Formulae

$$\sigma_J \leq (2J+1)\,\frac{8\pi}{s}$$

$$\hbar = 6.58 \times 10^{-25}\ \text{GeV sec} = 1$$
$$\hbar c = 0.197\ \text{GeV F} = 1$$
$$(1\ \text{GeV})^{-2} = 0.389\ \text{mb}$$
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

$x^\mu = (t, \mathbf{x}),$ $\quad p^\mu = (E, \mathbf{p}) = i\left(\frac{\partial}{\partial t}, -\nabla\right) = i\partial^\mu$

$p \cdot x = Et - \mathbf{p}\cdot\mathbf{x},$ $\quad p^2 \equiv p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$

$(\square^2 + m^2)\phi = 0,$ $\quad (i\gamma^\mu\partial_\mu - m)\psi = 0.$

In an electromagnetic field, $i\partial^\mu \to i\partial^\mu + eA^\mu$ (charge $-e$)

$j^\mu = -ie(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*),$ $\quad j^\mu = -e\bar{\psi}\gamma^\mu\psi$

γ-Matrices

$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \quad \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0.$

$\gamma^{0\dagger} = \gamma^0, \quad \gamma^0\gamma^0 = I; \quad \gamma^{k\dagger} = -\gamma^k, \quad \gamma^k\gamma^k = -I, \quad k = 1,2,3.$

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0, \quad \gamma^{5\dagger} = \gamma^5.$

(Trace theorems on pages 123 and 261)

Standard representation:

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \boldsymbol{\gamma} \equiv \beta\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spinors

$$(\not{p} - m)u = 0 \qquad \bar{u} = u^\dagger\gamma^0$$
$$\bar{u}(\not{p} - m) = 0 \qquad \not{p} \equiv \gamma_\mu p^\mu$$

$u^{(r)\dagger}u^{(s)} = 2E\delta_{rs}, \quad \bar{u}^{(r)}u^{(s)} = 2m\delta_{rs}, \quad \sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = \not{p} + m = 2m\Lambda_+$

$\frac{1}{2}(1-\gamma^5)u \equiv u_L, \quad \frac{1}{2}(1+\gamma^5)u \equiv u_R.$

If $m = 0$ or $E \gg m$, then u_L has helicity $\lambda = -\frac{1}{2}$, u_R has $\lambda = +\frac{1}{2}$.

Kinematics

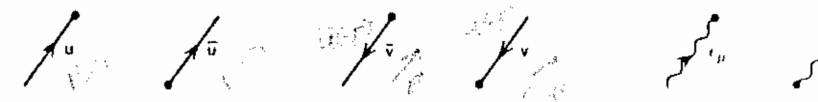
Lorentz invariant phase space ($P \to p_1 + \cdots p_n$)

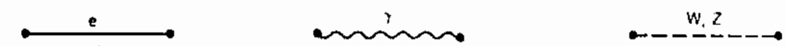
$$dQ = (2\pi)^4\delta^4(P - p_1 - \cdots p_n)\prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

Scattering: $\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathfrak{M}|^2$

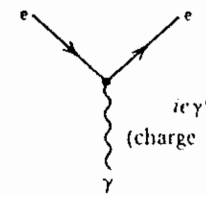
Decay: $d\Gamma(A \to 1 + \cdots n) = \frac{|\mathfrak{M}|^2}{2m_A}dQ.$

Feynman Rules for $-i\mathfrak{M}$

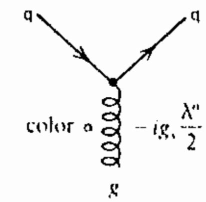




$\frac{i}{(\not{p} - m)}$ $\qquad \frac{-ig_{\mu\nu}}{p^2}$ $\qquad \frac{-i(g_{\mu\nu} - p_\mu p_\nu/M^2)}{p^2 - M^2}$



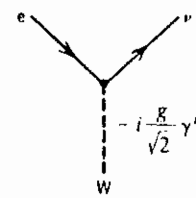
$ie\gamma^\mu$ (charge $-e$)



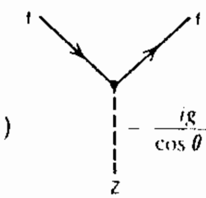
color a $\quad -ig_s\frac{\lambda^a}{2}\gamma^\mu$

$$\begin{cases} \alpha_s = \dfrac{g_s^2}{4\pi} \\ \quad = \dfrac{12\pi}{(33 - 2n_f)\log(Q \end{cases}$$

Z^0



$-i\frac{g}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)$



$-\frac{ig}{\cos\theta_W}\gamma^\mu\frac{1}{2}(c_V^f - c_A^f\gamma^5)$

$$\begin{cases} c_V^f = T_f^3 - 2\sin^2\theta \\ c_A^f = T_f^3 \end{cases}$$

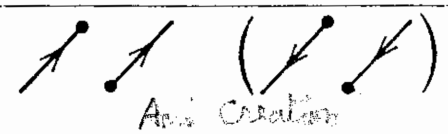
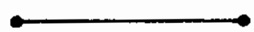
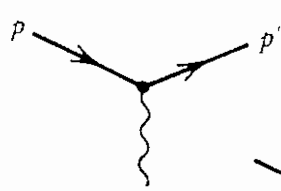
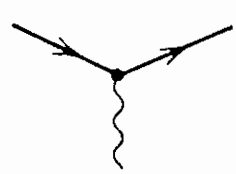
f	Q_f	$(T_f^3)_L$	$(T_f^3)_R$
u, c, t	$+\frac{2}{3}$	$\frac{1}{2}$	0
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	0
ν_e, ν_μ, ν_τ	0	$\frac{1}{2}$	
e, μ, τ	-1	$-\frac{1}{2}$	0

$\sin^2\theta_W = 0.23,\ g\sin\theta_W = e,\ G = \frac{\sqrt{2}g^2}{8M_W^2} = 1.17 \times 10^{-5}$ (

$u\bar{d} \to W^+\gamma$:

$i g_{ZWW}\left[(P_1 - P_2)^\alpha \epsilon^\mu \epsilon^\nu + \quad + \right] = i g$

TABLE 6.2
Feynman Rules for $-i\mathfrak{M}$

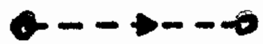
		Multiplicative Factor
• **External Lines**		
Spin 0 boson (or antiboson)		1
Spin $\frac{1}{2}$ fermion (in, out)		u, $\bar{u}$
antifermion (in, out)		$\bar{v}$, v
Spin 1 photon (in, out)		ε_μ, ε_μ^*
• **Internal Lines—Propagators (need $+i\varepsilon$ prescription)**		
Spin 0 boson		$\dfrac{i}{p^2 - m^2}$
Spin $\frac{1}{2}$ fermion		$\dfrac{i(\not{p} + m)}{p^2 - m^2}$
Massive spin 1 boson		$\dfrac{-i\left(g_{\mu\nu} - p_\mu p_\nu/M^2\right)}{p^2 - M^2}$
Massless spin 1 photon (Feynman gauge)		$\dfrac{-ig_{\mu\nu}}{p^2}$
• **Vertex Factors**		
Photon—spin 0 (charge $-e$)		$ie(p + p')^\mu$
Photon—spin $\frac{1}{2}$ (charge $-e$)		$ie\gamma^\mu$

Loops: $\int d^4k/(2\pi)^4$ over loop momentum; include -1 if fermion loop and take the trace of associated γ-matrices

Identical Fermions: -1 between diagrams which differ only in $e^- \leftrightarrow e^-$ or initial $e^- \leftrightarrow$ final e^+

F.1.2 Propagators

Spin-0



$$= \frac{i}{p^2 - m^2 + im\Gamma}$$

Spin-$\frac{1}{2}$

$$= \frac{i}{\not{p} - m} = i\frac{\not{p} + m - i\Gamma/2}{p^2 - m^2 + im\Gamma}$$

Photon

$$= \frac{i}{k^2}\left(-g^{\mu\nu} + (1 - \xi)\frac{k^\mu k^\nu}{k^2}\right),$$

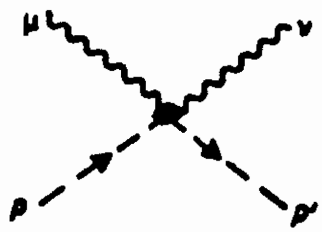
for a general ξ gauge. Calculations are usually performed in Lorentz or Feynman gauge with $\xi = 1$ and photon propagator

$$= i\frac{(-g^{\mu\nu})}{k^2}.$$

F.1.3 Vertices

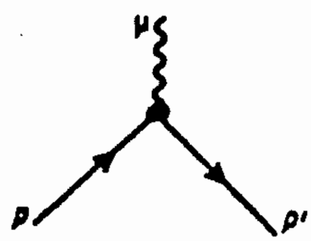
Spin-0

$-ie(p+p')_\mu$
(for charge $+e$)



$2ie^2 g_{\mu\nu}$

Spin-$\frac{1}{2}$



$-ie\gamma_\mu$
(for charge $+e$)

F.2 QCD: rules for tree graphs

F.2.1 *External particles*

Quarks. The SU(3) colour degree of freedom is not written explicitly: the spinors have 3(colour) × 4(Dirac) components

ingoing: $u(p, s)$ or $v(p, s)$
outgoing: $\bar{u}(p', s')$ or $\bar{v}(p', s')$

as for QED.

Gluons. Besides the spin-1 polarisation vector, external gluons also have a 'colour polarisation' vector a^{α} ($\alpha = 1, 2, \ldots, 8$) specifying the particular colour state involved:

G_μ^α

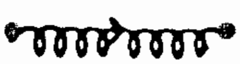
ingoing: $\varepsilon_\mu(k, \lambda)a^\alpha$
outgoing: $\varepsilon_\mu^*(k', \lambda')a^{*\alpha}$.

F.2.2 *Propagators*

Quark

 $$= \frac{i}{\not{p}-m} = i\frac{\not{p}+m}{p^2-m^2}.$$

Gluon

 $$= \frac{i}{q^2}\left(-g^{\mu\nu} + (1-\xi)\frac{q^\mu q^\nu}{q^2}\right)\delta^{\alpha\beta}$$

for a general ξ gauge. In Feynman gauge this reduces to

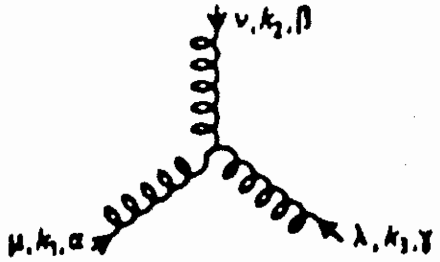
 $$= \frac{i}{q^2}(-g^{\mu\nu})\delta^{\alpha\beta}$$

which is usually the most convenient form.

$\lambda_{1,3} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdots \lambda_8 = \begin{pmatrix} 1 & & 0 \\ & +1 & \\ 0 & & -2 \end{pmatrix}/\sqrt{3}$

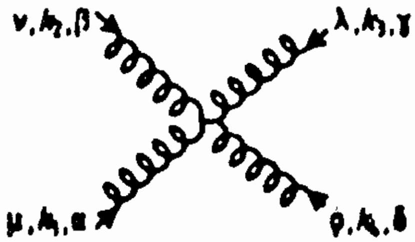
F.2.3 *Vertices*

$$-ig_s\frac{\lambda^\alpha}{2}\gamma_\mu$$



$$-g_s f_{\alpha\beta\gamma}[g_{\mu\nu}(k_1-k_2)_\lambda + g_{\nu\lambda}(k_1-k_3)_\mu + g_{\lambda\mu}(k_3-k_1)_\nu]$$

$$[\lambda_\alpha, \lambda_\beta] = 2i \sum f_{\alpha\beta\gamma} \lambda_\gamma$$



$$\begin{aligned}-ig_s^2[&f_{\alpha\beta\eta}f_{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) \\ &+ f_{\alpha\delta\eta}f_{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}) \\ &+ f_{\alpha\gamma\eta}f_{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]\end{aligned}$$

It is important to remember that the rules given above are only adequate for tree diagram calculations in QCD (see Chapter 14.4)

F.3 The standard model of electroweak interactions: rules for tree graphs.

F.3.1 External particles

Leptons and quarks

Ingoing:	$u(p, s)$	or	$v(p, s)$
Outgoing:	$\bar{u}(p', s')$	or	$\bar{v}(p', s')$.

$g \cdot \varepsilon_\lambda = 0$ for each λ

Vector bosons

Ingoing: $\varepsilon_\mu(k, \lambda)$

Outgoing: $\varepsilon^*_\mu(k', \lambda')$.

& $\sum_\lambda \varepsilon^{\mu *}_\lambda \varepsilon^\nu_\lambda = -g^{\mu\nu} + \dfrac{k^\mu k^\nu}{k^2}$

take $\vec{k} \parallel \hat{z}$, $\lambda = \pm 1$, $\varepsilon_\pm = \mp\sqrt{\tfrac{1}{2}}(0, 1, \pm i, 0)$

$\lambda = 0$, $\varepsilon_0 = (|\vec{k}|, 0, 0, E)/M$

F.3.2 Propagators

Leptons and quarks

$$\longrightarrow \quad = \frac{i}{\not{p} - m} = i\frac{\not{p} + m}{p^2 - m^2}.$$

Vector mesons (U gauge)

$$W^{\pm}, Z^0 \quad = \frac{i}{k^2 - M_V^2}\left(-g^{\mu\nu} + k^\mu k^\nu / M_V^2\right) \qquad -M_V^2 \Rightarrow -M_V^2 + iM_V\Gamma$$

where the mass M_W of the charged W bosons is given by $\qquad \Rightarrow -M_V^2 + i\frac{s\Gamma}{M_V}$

$$\frac{G_F}{2^{1/2}} = \frac{g^2}{8M_W^2}$$

with $g \sin\theta_W = e$ (where, in our convention, $e > 0$) so that

$$M_W = \frac{\sqrt{(1+\Delta r)}\, e(m_e)}{2^{5/4} G_F^{1/2} \sin\theta_W} \simeq \left(\frac{37.3}{\sin\theta_W}\right) \mathrm{GeV}/c^2 \sqrt{(1+\Delta r)}$$

The mass of the neutral Z boson is related to that of the charged W bosons by

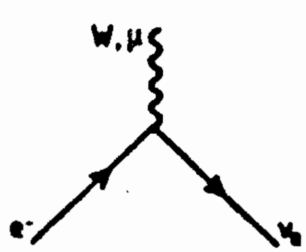
$$M_Z = M_W / \cos\theta_W.$$

Higgs scalar

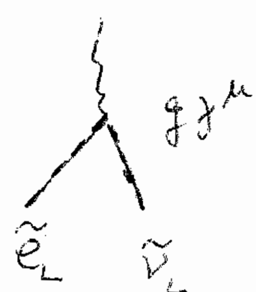
$$- - - \blacktriangleright - - - \quad = \frac{i}{p^2 - \mu^2}$$

F.3.3 Vertices

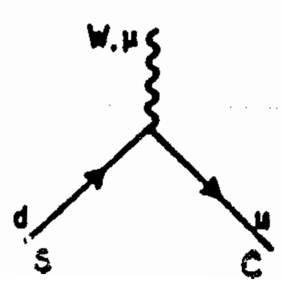
Charged current weak interactions



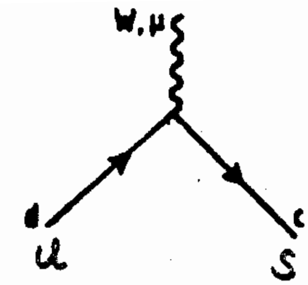
$$-i\frac{g}{2^{1/2}}\gamma_\mu \frac{1-\gamma_5}{2}$$



Boson pair



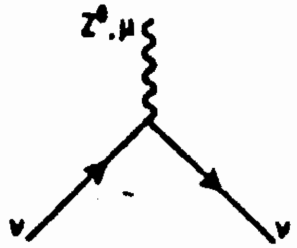
$$-i\frac{g}{2^{1/2}}\cos\theta_C \gamma_\mu \frac{1-\gamma_5}{2}$$



$$-i\frac{g}{2^{1/2}}(\pm\sin\theta_C)\gamma_\mu\frac{1-\gamma_5}{2}$$

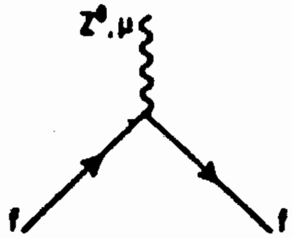
Neutral current weak interactions

Massless neutrinos



$$\frac{-ie}{\sin\theta_W\cos\theta_W}\frac{1}{2}\gamma_\mu\frac{1-\gamma_5}{2}$$

Massive fermions



$$\frac{-ie}{\sin\theta_W\cos\theta_W}\gamma_\mu\left(\frac{c_L}{2}\frac{1-\gamma_5}{2}+\frac{c_R}{2}\frac{1+\gamma_5}{2}\right)$$

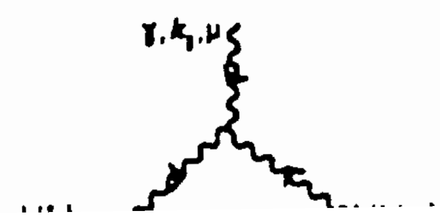
where $\qquad C_L/2 \qquad\qquad C_R/2 \leftarrow$ notations in H/M.

$c_L = -\frac{1}{2}+\sin^2\theta_W$,	$c_R = \sin^2\theta_W$,	for e^-, μ^-
$c_L = +\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$,	$c_R = -\frac{2}{3}\sin^2\theta_W$,	for u, c
$c_L = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$	$c_R = \frac{1}{3}\sin^2\theta_W$,	for d, s

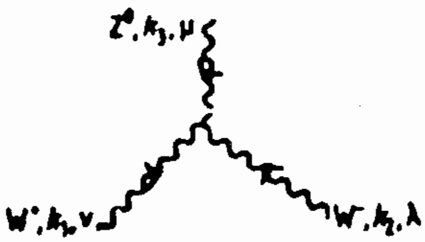
(massless neutrinos have $c_{L/2} = \frac{1}{2}$; $c_R = 0$).

Vector boson couplings. (a) Trilinear couplings

γW^+W^- vertex

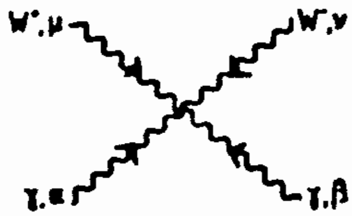


$$ie[g_{\nu\lambda}(k_1-k_2)_\mu+g_{\lambda\mu}(k_2-k_\gamma)_\nu+g_{\mu\nu}(k_\gamma-k_1)_\lambda]$$

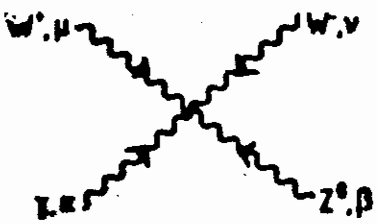


$$i\frac{e\cos\theta_W}{\sin\theta_W}\left[g_{\nu\lambda}(k_1-k_2)_\mu + g_{\lambda\mu}(k_2-k_3)_\nu + g_{\mu\nu}(k_3-k_1)_\lambda\right]$$

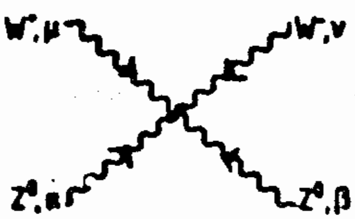
(b) Quadrilinear couplings



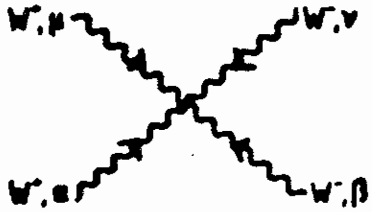
$$-ie^2(2g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})$$



$$-ie^2\cot\theta_W(2g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})$$



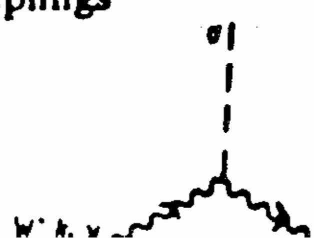
$$-ie^2\cot^2\theta_W(2g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})$$



$$\frac{ie^2}{\sin^2\theta_W}(2g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta})$$

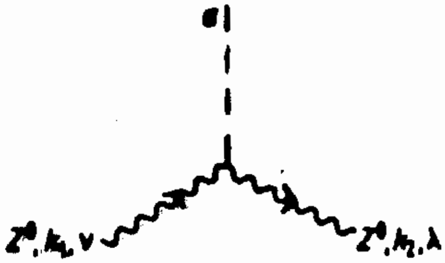
Higgs couplings. (a) Trilinear couplings

σW^+W^- vertex



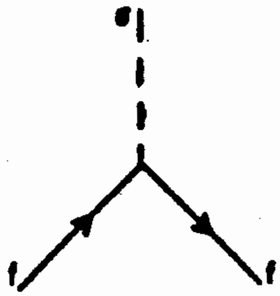
$$\frac{ie}{\sin\theta_W}M_W g_{\nu\lambda}$$

σZ⁰Z⁰ vertex



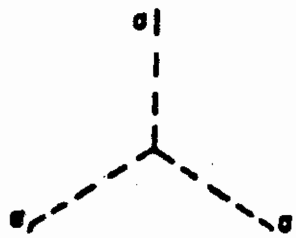
$$\frac{ie}{\sin 2\theta_W} M_Z g_{\nu\lambda}$$

Fermion Yukawa couplings (massive fermions, mass m_f)



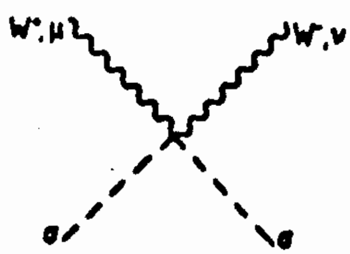
$$-\frac{ie}{2\sin\theta_W}\frac{m_f}{M_W}$$

Trilinear self-coupling



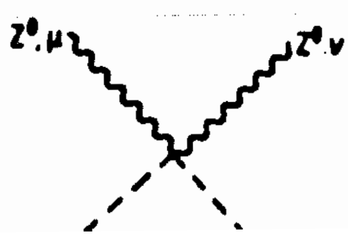
$$-i\frac{3\mu^2 e}{2M_W\cos\theta_W}$$

(b) Quadrilinear couplings
$\sigma\sigma W^+W^-$ vertex



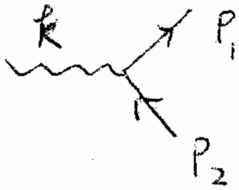
$$\frac{ie^2}{4\sin^2\theta_W} g_{\mu\nu}$$

$\sigma\sigma ZZ$ vertex



$$\frac{i\,e^2}{2\sin^2 2\theta_W} g_{\mu\nu}$$

$W^{\pm}$ decays & width Γ_W & propagator



$$M = i\, g_W\, \bar{u}(p_1)\, \gamma^{\mu}(1-\gamma_5)\, v(p_2)\, \varepsilon_{\mu}$$

$$\sum_s |M|^2 = g_W^2\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right) \mathrm{Tr}\left(\not p_1 \gamma^{\mu}(1-\gamma_5) \not p_2 \gamma^{\nu}(1-\gamma_5)\right)$$

$$= 8\, g_W^2\, m_W^2$$

$$\langle |M|^2 \rangle = \frac{1}{2s+1} \sum_s |M|^2 = \frac{8}{3} g_W^2 m_W^2$$

$$\therefore\ \Gamma(W \to e\bar{\nu}) = \frac{1}{16\pi m_W} \langle |M|^2 \rangle = \frac{1}{6\pi} g_W^2 m_W$$

$$\Gamma_W = \frac{g_W^2 m_W}{6\pi} \cdot N_f \qquad N_f \approx 9\ (e\bar{\nu}, \mu\bar{\nu}, \tau\bar{\nu}, 3d\bar{u}, 3s$$

$\therefore$ $W^{\pm}$ is unstable!

For stable particles at rest:

$$i\partial_t \Psi = H\Psi = E\Psi = m\Psi \to \Psi(t) = e^{-imt}\Psi(0)$$

$$\text{Probability} = |\Psi(t)|^2 = |\Psi(0)|^2$$

For unstable particles at rest, if created at $t=0$:

$$|\Psi(t)|^2 = |\Psi(0)|^2 e^{-\Gamma t} \qquad \Gamma = \frac{1}{\tau} \text{ where } \tau = \text{lifetime}$$

$$\therefore\ \Psi(t) = e^{-imt}\Psi(0)\, e^{-\frac{\Gamma}{2}t} = \Psi(0)\, e^{-it(m - i\frac{\Gamma}{2})} \qquad ; \quad m - i\frac{\Gamma}{2} \Rightarrow \text{comp} \atop \text{mas}$$

Fourier transformation $\Psi(E) = \int_0^{\infty} dt\, e^{-iEt}\Psi(t)$

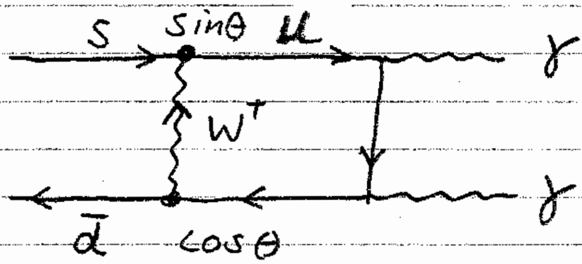
$$= \int_0^{\infty} dt\, e^{it(E - m + i\Gamma/2)}\, \Psi(0) \quad \frac{i}{\qquad} \quad \frac{2m}{\qquad} \quad (E_m$$

Divergence of crossections

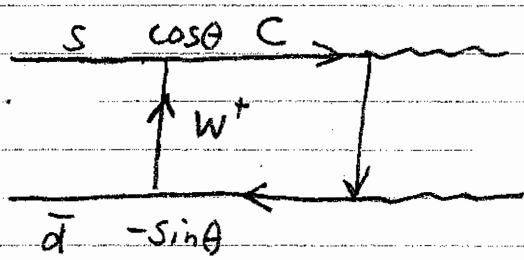
$\Longrightarrow$ introduce new particles!

Examples:

② $K^0_L \to \mu^+\mu^-$ or $\gamma\gamma$

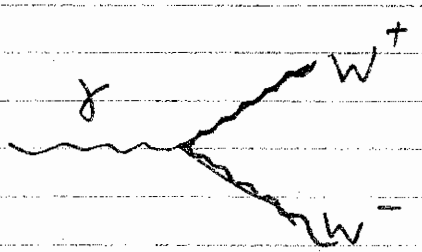


$M = \sin\theta\cos\theta$



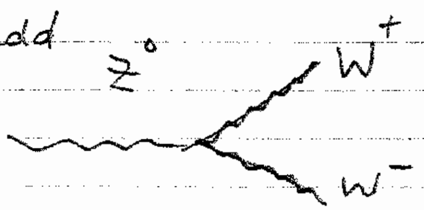
$$\begin{pmatrix} d \\ s \end{pmatrix}' = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix}$$

③



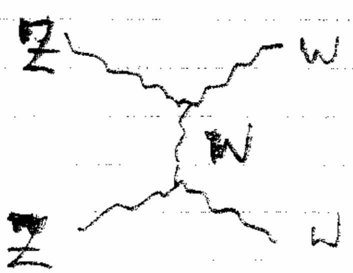
$\sigma \to \infty$

add

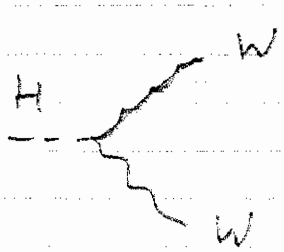


finite

④



$+$



finite