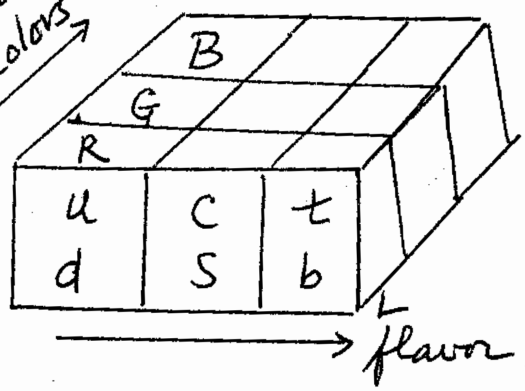
Building blocks & their interactions

Carrier of forces → W with coupling g_2

8 gluons, colors

$Q = \frac{2}{3}$

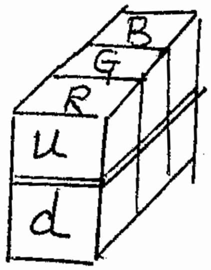
$-\frac{1}{3}$



$S=\frac{1}{2}$ fermions

$B=\frac{1}{3}$, $L=0$

$\uparrow I_3$



Carrier of force B with coupling g_1

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{matrix} Q=0 \\ =1 \end{matrix} \qquad B=0,\ L=1 \qquad \begin{matrix} \nu \\ e^- \end{matrix}$$

∴ Q is not diagonized

Also $M_{W^0} \neq M_Z$ & $g_2 \neq g_{Z^0}$

∴ Z^0 is not W^0, γ is not B,

g_1 interaction is not QED

Need a Quantum # independent of isospin famil.

Thus take hypercharge

$$Y \equiv 2(Q - I_3) = -1 \text{ for } \begin{pmatrix} \nu \\ \ell^- \end{pmatrix}_L \qquad = 0 \text{ for } \nu_R$$

$$= -2 \quad \ell_R$$

$$= \frac{1}{3} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad = \frac{4}{3} \quad u_R$$

$$= -\frac{2}{3} \quad d_R$$

$S=\frac{1}{2}$

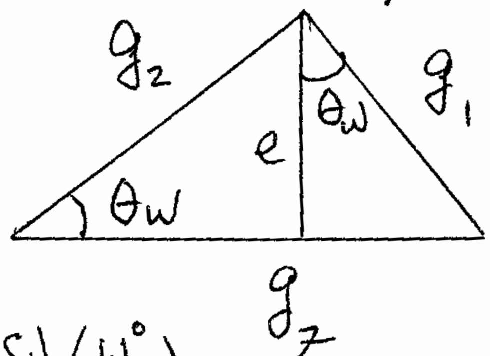
all obey Dirac Eq.

carriers of forces Spin = 1

color strong I.	SU_3	g_3	8 gluons
Isospin	SU_2	g_2	$W^{\pm}, W^0$
Y	U_1	g_1	B

Spin = 1 , all obey (massive) Maxwell Eq.

$$\alpha_s = \frac{g_3^2}{4\pi} = 12\pi / [(33-2n_f) \ln Q^2/\Lambda^2 + \text{2nd order}]$$



$$g_2 = \frac{e}{\sin\theta_W}$$

$$g_1 = \frac{e}{\cos\theta_W}$$

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W}$$

$$\begin{pmatrix} Z \\ \gamma \end{pmatrix} = \begin{pmatrix} C_W & -S_W \\ S_W & C_W \end{pmatrix} \begin{pmatrix} W^0 \\ B \end{pmatrix}$$

θ_W : mixing angle

$C_W \equiv \cos\theta_W$

$S_W \equiv \sin\theta_W$

between Weak & EM interactions.

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137} \quad \text{or } e = \sqrt{4\pi\alpha}$$

with $\hbar c = 1973$ eV-Å $\Rightarrow 1$

Fermion masses: $m_f \langle \psi | \psi \rangle = [\langle \psi_L \psi_R \rangle + \langle \psi_R \psi_L \rangle] m_f$

ψ_L transform like SU_2 doublet

ψ_R U_1 singlet

Not gauge invariant, thus arbitary values

Thus $m_f \equiv 0$!

H°, Higgs, invented to generate masses.

$(S=0,\ Q=0,\ B=L=0)$

H° obeys K-G Eq. since $s=0$

H° is also required, to prevent

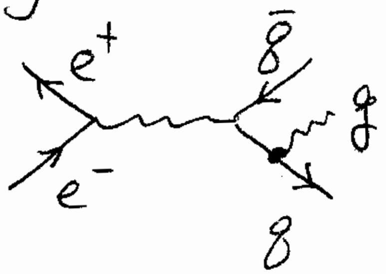
$\sigma(W^+W^- \to Z^\circ Z^\circ)$ from divergent !

<u>Unitarity</u> !

Running Coupling Constants

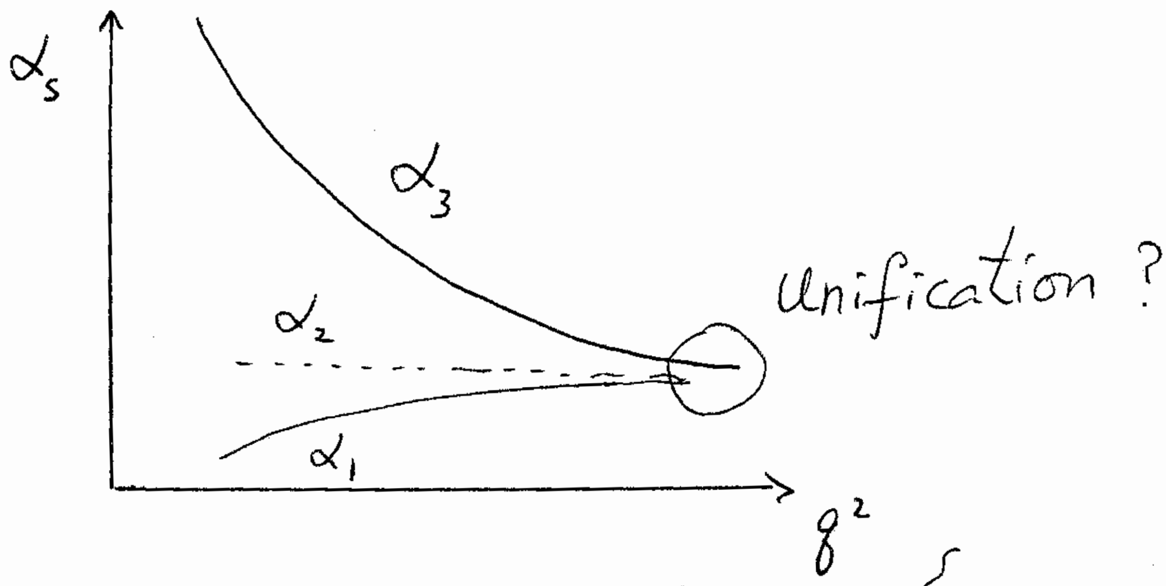
$$\alpha_s = \frac{g_3^2}{4\pi} = \frac{12\pi}{(33-2n_f)\ln\frac{q^2}{\Lambda^2} + \text{2nd order}}$$

$\Lambda \sim 0.1$ GeV determined from gluon emmision.

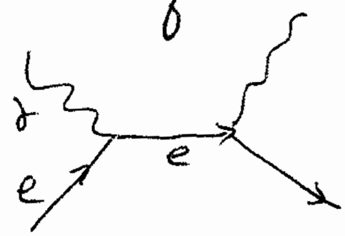


$\alpha_s \to \infty$ as $q^2 \to (0.1\,\text{GeV})^2$

$\alpha_s \sim 0.1$ at $q^2 \sim (100\,\text{GeV})^2$



em α is determined from



$$\sigma_{th} = \frac{8\pi}{3}\left(\frac{\alpha}{m_e}\right)^2 = \frac{2}{3}\alpha^2(4\pi R_e^2)\ ; \quad \sigma_T(\pi p) = \alpha_s^2(4\pi R_p^2)$$

$$R_p \sim \frac{1}{m_p}$$

α_2, θ_W determined from μ, Z^0 decays.

α: $\quad \sigma_{Th}(\gamma e \to \gamma e) = \frac{8\pi}{3}\left(\frac{\alpha}{m_e}\right)^2 = \frac{2}{3}\alpha^2(4\pi R_e^2)$

$$\boxed{\alpha^{(m_e)} = \frac{e^2}{(4\pi)\hbar c} = \frac{1}{137.0360}} \qquad R_e = \frac{\hbar}{m_e c}$$

$$\boxed{\alpha(m_Z) = \frac{1}{128}}$$

g_2^2: $\quad \mu$ decays

$$\tau = \frac{192\pi^3}{G_F^2 m^5} \qquad \therefore\ G_F = \frac{10^{-5}}{m_N^2} = \frac{\sqrt{2}\, g_2^2(m_\mu)}{8\, m_W^2}$$

$$= 1.16632 \times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$\pm .00002$$

g_3: $\quad \tau \to \pi + \nu$

$$\frac{\sigma(3\ \text{jets})}{\sigma(2\ \text{jets})} \qquad \boxed{\begin{array}{lll} \alpha_s = 10 & \longrightarrow & 0.1 \\ E = 0.1 & & 100\ \mathrm{GeV} \end{array}}$$

Gravity $\quad \frac{K M_p^2}{(4\pi)\hbar c} = 4.6 \times 10^{-40}$

S.I. Carriers of S.I. are:

gluons: double colors

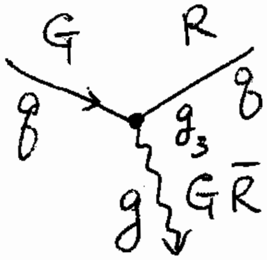
$$\overline{SU_3} \otimes SU_3 \quad \text{or} \quad \begin{pmatrix} \bar R \\ \bar G \\ \bar B \end{pmatrix} \otimes (R, G, B)$$

8 gluons

$$\bar R G,\ \bar G B,\ \bar R B,\quad \bar G R,\ \bar B R,\ \bar B G$$

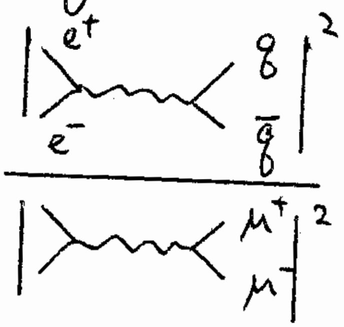
$$\frac{\bar R R - \bar G G}{\sqrt{2}}, \qquad \frac{\bar R R + \bar G G - 2\bar B B}{\sqrt{6}}$$

Plus the ninth colorless $\frac{\bar R R + \bar G G + \bar B B}{\sqrt{3}}$ = whit.



$$g_3 = \sqrt{\alpha_s 4\pi}$$

Why color = 3?

2) 

$$= \frac{n_c \sum Q_i^2}{1} \left(1 + \frac{\alpha_s}{\pi} + 1.4\frac{\alpha_s^2}{\pi^2} + 64.8\frac{\alpha_s^3}{\pi^3} + \cdots\right)$$

1) Why color? ground state $L = S = I = 0$

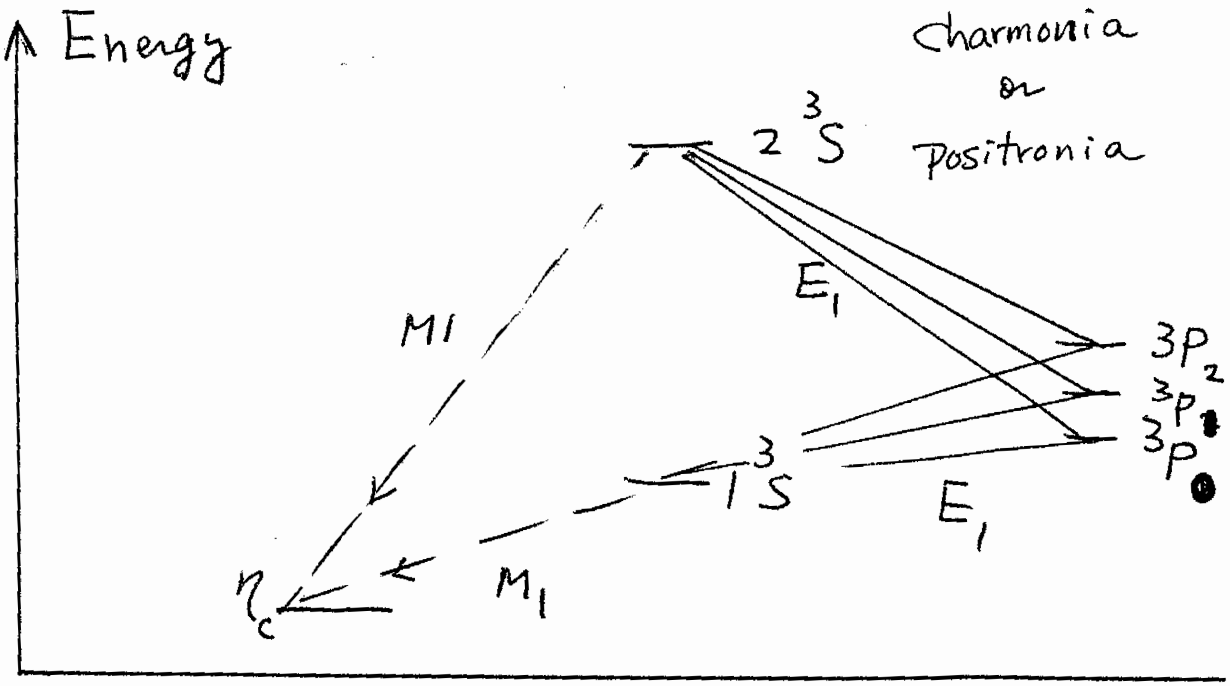
$$\psi_{q_1 q_2} = +\psi_{q_2 q_1} (-1)^L (-1)^{S+1} (-1)^{I+1} = \psi_{q_2 q_1} \quad \text{if } q_1 = q_2$$

5) violates Pauli exclusion Principle!

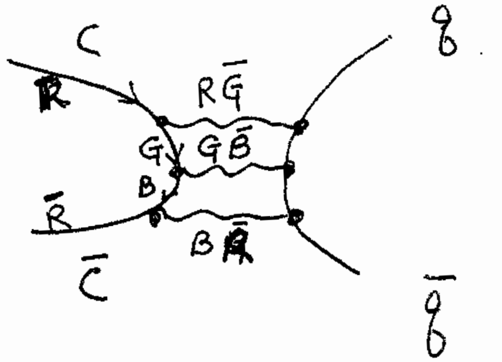
Mesons are $q\bar{q}$ system $\qquad P = (-1)^{L+1}$

$$(-1)^{L+1}(-1)^{S+1}\,C = +1$$

$$\therefore\ C = (-1)^{l+S}$$



$$J^{PC} = \quad 0^{-+} \qquad 1^{--} \qquad 2,1,0^{++}$$



$$\Big|\ \cdots\ \Big|^2 \propto \alpha_s^6(M) << 1$$

That is why $J(1^{--})$ is long lived!

6)

Weak charged Interaction changes I_3 .

$$\begin{pmatrix} u \\ d \downarrow \end{pmatrix} \quad \begin{pmatrix} c \\ s \downarrow \end{pmatrix} \quad \begin{pmatrix} t \\ b \downarrow \end{pmatrix} g_2$$

Big bang $\rightarrow H \rightarrow t\,\bar{t}$ $\hookrightarrow \bar{b} + \cdots$
$\hookrightarrow b + \cdots$

Mass eigenstates are mixtures of the 3 flavor families

Mixing Matrix

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_{\text{Mass}} \approx \begin{pmatrix} 0.97 & 0.22 & 10^{-2} \\ -0.22 & 0.97 & 5\times10^{-2} \\ -10^{-2} & -5\times10^{-2} & 0.99 \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{\text{flavor}}$$

$n^2 = 9$ complex #'s $\Rightarrow$ 18 parameters $2n^2$

unitary $\Rightarrow -9$ constraints $-n^2$

arbitary phases $\Rightarrow -5$ $(2n-1)$ $-(2n-1)$

$\therefore$ q's : 3 angles + 1 phase = 4 parameters 4
(cp violation)

l's : 4 mixing parameters 4

couplings: g_1, g_2, g_3, θ_W, λ (Higgs P.E.) 5

Masses : $6 m_q$, $6 m_l$, m_H or m_W 13

$\therefore$ $u \swarrow$ … d, $c \downarrow s$, $t \downarrow b$

total 26
free parameters

7)