8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids II **Problem Set 4** **Due March 10, 2008**

1. Consider a tight-binding model on a lattice with hopping matrix element t. Add an on-site disorder potential V_i, where V_i is a random variable distributed uniformly between $\pm\frac{W}{2}$. Consider the one-dimensional case with N sites.

   (a) For a given realization of V_i, consider the eigenvalues for periodic boundary conditions, i.e., $V_N = V_1$ and $\psi_N = \psi_1$ where ψ_1 is the wavefunction on site i. The eigenvalues are solved by diagonalizing an $(N-1) \times (N-1)$ matrix. Set up the form of the matrix.

   (b) Now consider a twisted boundary condition, i.e.,$V_N = V_1$ and $\psi_N = \psi_1 e^{i\phi}$. How is the matrix modified from (a).

   (c) Show that the eigenvalues E_α in (b) are equivalent to a problem with complex hopping, i.e., t is replaced by $te^{i(\phi/(N-1))}$ and with periodic boundary conditions. This is the problem of a ring with $N-1$ sites with a magnetic flux through the ring. What is the value of the flux in units of the flux quantum hc/e.

   (d) Diagonalize the matrix numerically for a given realization of disorder. Choose $W/t = 2.0$ and $N = 20$. Plot the energies of the 10 levels near $E = 0$ as a function of ϕ. Now increase N and observe how the picture changes.

   (e) For the values of W/t chosen in part (d) calculate the dimensionless conductance of the sample using the Thouless formula

   $$G = \frac{E_T}{\Delta}$$

   where

   $$E_T = \overline{\frac{d^2 E_\alpha}{d\phi^2}}$$

   and Δ is the average energy level spacing. Calculate E_T and Δ by averaging over the 10 levels near $E = 0$ and by averaging over a number of realizations of the random potential. Check the dependence of G as a function of sample size N.

(f) *Optional.* If you are interested, you may repeat the problem for a two-dimensional square lattice and contrast the behavior. Compare $W/t = 4$ and 9.