PROFESSOR: Well, we are in the middle of something really interesting. We are talking about squeezing. We are talking about non classical light. And today, I sort of want to wrap it up. And I think it will really be an exciting class.

But before I continue with the material, I want to address a question, which actually came up in discussions with several of the students. And this is, I realize that some people said, OK. Everything makes sense. But what are we plotting? What is really squeezed?

Are we squeezing in the spatial domain? Are we squeezing in the temporal domain? So the plots look wonderful, with these ellipses and the circles. But, what is it really we are doing here? So let me address that.

First of all, we are talking about a single harmonic oscillator. We showed that the [INAUDIBLE] equations can be reduced to a bunch of harmonic oscillator equations. One for each mode. And now we are talking, today and in the previous few classes, about one single mode, about one harmonic oscillator.

And the harmonic oscillator has canonically variables of momentum and position. But this is just to make a connection for you with something you have learned. What we are talking about is a single harmonic oscillator, which is one single mode of the electromagnetic field. So maybe let me draw a cartoon for that.

So let's assume we have a cavity. We have an electromagnetic wave. There is propagation, where it's e to the ikz. There is transverse confinement. Maybe there is a Gaussian e to the minus x squared plus y squared over sum [INAUDIBLE] parameter.

All of that is simply the spatial mode. And we just take that for given because we're not solving the spatial differential equation. All we are doing is we are looking at this one mode.

And the two degrees of freedom is that this mode can have a certain number of photons. It's the amplitude. And the second one is you can see the temporal phase. It can be a cosine omega t. it can be a sine omega t. It can be a superposition.

But whatever we are talking about is in this mode. There is nothing happening in the spatial domain. They're just asking, what is the oscillation in this mode? The whole mode does what it should. It has a prefect, of which is the amplitude. And it has a temporal effect of which we factor out. And this is what we are talking about.

Let me be a little bit more specific and say, that when we are plotting things, we are plotting the Q representation, the phase space representation of the statistical operator, which is simply describing this single mode of the harmonic oscillator. And by performing the diagram matrix elements, we obtain the Q distribution.

In that case, we have the vacuum. We have a displaced vacuum, which is in coherent state. And our x's are, from the very definition, in the complex plane with a real part of the imaginary part of alpha.

However, we can also define [? the Veetner ?] distribution, which is another phase based distribution. It's almost the same as the Q distribution. It's just this little bit smeared out by h bar because of some commutators. But nothing you have to worry about.

In that case, the projection of the W function on the vertical axis, on the y-axis, is the momentum wave function squared. On the x-axis, it is the spatial wave function squared of the harmonic oscillator.

So therefore, we may sometimes think, when we have a distribution here, we project it. And we see what is the momentum distribution. Or what is the spatial distribution of the mechanical harmonic oscillator. Which is analogous, which is equivalent, to the one mode of the electromagnetic field we are using.

I know it may help you to some extent to think about the P and Q. But it may also be misleading because it gives you the impression something is moving with a momentum P, in real space. Let me therefore emphasize what are the normalized forms of P and Q.

If I do the symmetric and anti-symmetric combination of the annihilation and creation operator, I over squared root 2. I call those a1 and a2. And they are nothing else than Q and P normalized by the characteristic momentum of spatial coordinates of the harmonic oscillator.

So what is important here is that a1 and a2-- forget about P,Q now. They are equivalent. But for the electromagnetic field, a1 and a2 have a very direct interpretation. They are called the two quadrature operators.

And what I mean by that becomes clear when I use the Heisenberg representation for the electric field. And I'm here using the formula which is given in the book of Weissbluth, in page 175. Some pages copied from this book have been posted on the website. So we have our normalization factor, which is related to the electric field of a single photon. We have the polarization factor.

But now we have an expression which involves the quadrature operators, a1 and a2. Just to be specific, we are not in a cavity here. Therefore, we have propagating waves cosine (kr) sine (kr). But you can also immediately use a similar expression for the case of a cavity.

Let us specify that r equals 0. And then we realize what the two quadrature operators are. a2 is the operator which creates and annihilates an electromagnetic field, so to speak, photons, which have an electric field. Which oscillates as cosine omega t. And a1 is the quadrature operator for the sine omega t component.

So therefore, if you simply analyze the electric field, what is cosine omega 2 is related to the a2 quadrature operator. The sine omega 2 oscillation is related to the a1 quadrature operator. So life would be easier, but more boring, if you could create a pure cosine, or pure sine oscillation of the electromagnetic field.

But you can't because a1 and a2 do not commute. And there is an uncertainty relation that delta a1 times delta a2 is larger or equal to 1/2. And we have the equal sign for coherent states alpha.

So therefore, if we look at the electric field, you know, everything moves around periodically in r because it's a traveling wave and t because it's an oscillating wave. So let's not confuse things with simply peak r equals 0. We've already done that. But now let's peak t equals 0.

That t equals 0. The sine omega t is 0. And therefore, the distribution for a2, the expectation value, and the variance for the quadrature operator, a2, can be simply read off by looking at the electric field.

So, in other words, at t equals 0, the electric field, which is obtained by projecting our quasi probabilities on the y-axis, gives the expectation value for a1 and the variant, delta a1 squared. Somebody says 2. That t equals 0. Yes. Yes.

And now, if you want to see what the other quadrature component is, well. We just wait a quarter period until the sine, which was 0, is maximum. And the cosine omega t is 0. So therefore, it's pi over 2 omega, using the projection on the y-axis, gives us a1. And delta a1 squared. Or, alternatively, we don't need to wait. We can do t equals 0. And we can project onto the x-axis.

Let me just throw few things into this diagram. If you had a classical motion, which would simply be cosine omega t. Then that would mean, if you had a motion which where only cosine omega t-- yes, it would be a point on the y-axis.

However, classically we can never have something which is just cosine omega t. The point has to blurred into a circle. This is the coherent state.

So this coherent state has now-- let me just call it 1, for the sake of the argument-- the point would be the classical oscillator. It is just cosine omega t. Everything is deterministic. No uncertainty. No nothing.

Of course, it means that the time of evolution, it goes in a circle. But this is what everything does in an harmonic oscillator when time evolves into the e to the i omega t factor. So let's not get confused with it. Let's just look at t equals 0.

And we input [INAUDIBLE] if r equals 0, the classical oscillator is one point. But now, we have a spread here. This says that trying to mechanically the amplitude of the cosine omega t term is not entirely false. It's not [INAUDIBLE] sharp.

There are fluctuations. And in addition, we have ellipse in this direction, which we project onto the x-axis. And this tells us what the distribution in our ensemble, in our kind of mechanical ensemble, is for the amplitudes of the sine omega t motion.

So the best we can do is try to mechanically-- if you want to have something, which is really just cosine omega t. We have to squeeze it, that the cosine omega t amplitude is now extremely sharp. But the sine omega t amplitude in the ensemble is completely smeared out. So this is what we're talking about.

Now, what I think has confused some of you is what I thought was a wonderful example. The classical squeezing experiment. I mean, these are visuals which will be in your head forever, when you saw Professor Pritchard with a circle pendulum he's just pulling. And then the circle squeezes into an ellipse. And it seems that something here is squeezed in real space.

But this is actually wrong. But how you should have looked at the experiment, and I made a comment about it, but maybe not emphatically enough. You should have really thought about a single pendulum. And this single pendulum, if it has an [INAUDIBLE] phase, is in a superposition of sine omega t and cosine omega t.

And if you pull on the string, if you shorten and lengthen the pendulum, it's sine 2 omega t. You will amplify the prefactor in front of sine omega t. And you will exponentially de-amplify the factor in front of cosine omega t.

So therefore, what will happen if this pendulum oscillates-- and let me say with a phase, well, sine omega t plus delta, if delta is 90 degrees. Cosine if delta is 0. It's sine. And let's say this pendulum oscillates at 45 degrees. Sine omega t plus 45 degrees.

If you now parametrically derive it with squeezing action, it would now mean that you, let's just make it a sine convention. You de-amplify the cosine. You amplify the sine. And after a while, instead of oscillating with sine omega t plus 45 degrees, it will oscillate with an amplified amplitude at sine omega t. This is what you have done.

And this is a mechanical analogy. There is, of course, no squeezing in any way because in a classical pendulum, we start with one definite value. If you prepare the system well. And then, we just change the motion. We amplify. We pick out a phase and that's what we are doing.

Now the true ways how classical squeezing can come in. One it is if the motion of the pendulum is-- maybe there is an uncertainty. Maybe Professor Pritchard did experiments with an ion trap. And actually, 20 years ago, he published a [INAUDIBLE] [? letter ?] on classic squeezing. And you think, how can you publish [INAUDIBLE] [? letter ?] on classical squeezing?

Well, he had developed the world's most accurate ion trap, measuring atomic masses with 10 and 11 digit positions. And what was actually one limiting factor was for Kelvin, the thermal distribution of harmonic oscillator modes. And so, he didn't have just one clean amplitude. The sine omega t amplitudes had a spread because of the thermal distribution he started from.

And so what he then did is, by simply classical squeezing by doing classically with the ion trap exactly what he did with the pendulum, derive sine 2 omega t, he could now take this classic distribution. This is a classic distribution. In one axis, it's a distribution of amplitudes of the cosine motion.

And here it's the classic distribution of the amplitudes of the sine motion. And he was squeezing it into this direction. So he had a narrower definition of the coefficient for the cosine omega t motion. And as I will tell you today, you can now do a homodyne measurement, which is reducing the noise.

So essentially, he prepared, quote unquote "Effectively a [? code ?] ensemble by squeezing the uncertainty in the cosine prefactor." Or at the expense of increasing the prefactor, the uncertainty, the variance, in prefactor of the sine omega t motion. Finally, you all saw something visually. You saw how a circular motion became a linear motion.

So what was going on here? Well, I mentioned to you that the circular pendulum actually has two modes. These are two modes of the harmonic oscillator. And I'm not talking about two modes of the harmonic oscillator. Everything we're discussing here is about one mode of the harmonic oscillator.

The circular motion of the pendulum was just a nice visualization trick that, if the pendulum moves in a circle, you have a degenerate harmonic oscillator. One is excited with sine omega t. The other one is excited with cosine omega t.

And instead of doing two experiments, if you would start with sine omega t, and you parametrically drive it, you amplified it. If you start with cosine omega t, you could bring the pendulum to a stop. But instead of doing two experiments, Professor Pritchard just did one.

And he showed that the sine omega t motions became larger. And the orthogonal cosine omega t motion shrank. And therefore, you saw that the circular motion, which was a superposition of sine and cosine, became [? pure ?] sine motion.

But the fact that there was something we could see in the spatial domain was simply due to the fact that we had two experiments in one. Two versions of the same harmonic oscillator, one in x and one in y. And then, when we did the experiment, we saw something visually in the spatial domain.

So that's why we saw squeezing in the spatial domain. But you should really think about it. What the whole action is, is it's an interplay of de-amplifying prefactors of cosine amplifying prefactors of sine. And if the prefactor has a distribution, by de-amplifying it, you also shrink the reach of the distribution. And this is what we call squeezing. Yes, [INAUDIBLE]?

AUDIENCE: Stupid question. So the operators, a1 and a2 here, right? You use those instead of a and a dagger because you use cosine and sine rather than [? e to the i omega t ?] [INAUDIBLE]? Because they should contain [INAUDIBLE], right?

PROFESSOR: Let me go back to the definition. They are actually exactly they correspond exactly to position and momentum of the mechanical harmonic oscillator.

AUDIENCE: Oh. That makes sense. Another thing is, technically speaking, we could call the [? Basco ?] electric field e cos kr minus omega t as maximum squeezed if we only had the cos components?

PROFESSOR: Well, not if I use it-- it depends how I define squeezing. So you would now give a definition of squeezing which says that the variance in a1 is now unequal to the variance in a2. So the classical oscillator is the point. It has 0 variance in a1. 0 variance in a2.

But as I said, you can actually apply all the way to a classical oscillator if you add technical noise or thermal noise. Then your system is prepared, not with a sharp value, but with a distribution which is simply may be [? false ?] distribution, due to the preparation. So recall, it's squeezing when the noise in the amplitude of the sine motion is not equal to the noise in the amplitude of the cosine motion.

Some people say, if it's a little bit narrower, they apply squeezing to the situation that we are uncertainty limited. And then we squeeze. But of course, you can always reduce the noise in your system by just preparing the system. By cooling the system. By selecting the system for measurements, until you reach the quantum limit.

So you can get a smaller delta a1, a smaller delta a2, without squeezing, just better preparation. Or by selecting your ensemble. So squeezing in a narrower sense only makes sense when you hit the limit of what quantum mechanics allows you.

And now you want to distribute the variance unequally between a1 and a2 because then you can get something in delta a1 or delta a2. Which is better then 1 over square root 2. And this is now really quantum mechanically squeezed. But they're both definitions. Classical squeezing exists. It's just not as common as quantum mechanical squeezing.

Other questions? Yes?

AUDIENCE: I remember that you said that in the classical squeezing, you are attenuating one amplitude. And you were amplifying the other amplitude. So, in this picture then, shouldn't we have the ellipse come down from the circle on the y-axis?

PROFESSOR: OK.

AUDIENCE: [INAUDIBLE] not just changing delta a, but also changing b [INAUDIBLE].

PROFESSOR: Let me just get a sketch up here. So this is a2. This is a1. a2 is for the cosine omega t. And a1 for the sine omega t. So just to be specific. So you want to prepare an harmonic oscillator, which is just sine omega t. This is a point here.

If we are now parametrically-- so this has a value at t equals 0. Our a1 is not. So if you are now squeezing your classical harmonic oscillator, you would have a situation where a1 of t is s naught times e to the plus or minus t, depending on whether you do the parametric [? drive ?] at sine 2 omega t or cosine 2 omega t.

So therefore, what would happen is this point will be amplified. That would mean it would just move out on the x-axis. So this would be for the plus sign. Or, for the other case, you would damp the motion to 0. And this is a minus sign here.

AUDIENCE: OK. And you were also saying the variance--

PROFESSOR: A point does not have variance.

AUDIENCE: [INAUDIBLE].

PROFESSOR: So if you want to build a variance, you need, let's say, three points. One is the average value. One is the left outlier. One is the right outlier. And what happens is now, as you amplify the motion, you would also amplify, magnify the distance between the points.

And if you de-amplify it with a minus sign, the distance between the points would shrink because all the three points converged to 0. So pretty much what I just told you for the three point ensemble. You can now use it and construct any initial condition you want. And see what happens due to squeezing. Other questions? Good.

So the question now is, how to measure squeezing. How to take advantage of squeezing. So the situation we are facing is the following.

Let's assume we have done a nice squeezing job. And that means that we have a sharp value. We have created a narrow distribution of the cosign omega 2 coefficients. So the cosine omega 2 motion is rather sharp. But we also know that the electric field itself is sharp at this moment. But since this ellipse rotates, the electric field will have enormous uncertainty a [INAUDIBLE] period later.

So if we want to take advantage that we have squeezed the electromagnetic field, they are now a couple of ideas which we can use. One is we could just measure the electric fields stroboscopically. We would just make a setup, where we look at this system in a [INAUDIBLE] measurement process. Only, we only measure the electric field when the ellipse is like this or is like this. And therefore, we have a sharp value of the electric field.

But instead of doing a stroboscopic measurement, we can do something else. Remember, we have a distribution of cosine omega t and sine omega t. And we have a distribution of co efficiency and s. And we know we are interested in the cosine omega t.

So how to pick that out has actually been solved in early [? radius ?]. You do a homodyne detection. In other words, you take a reference oscillator, which is strong. B of t is b naught times cosine omega naught plus delta.

And if you if you multiply the two signals, your signal you are interested in, or at least you're interested in one comportment, you multiply it with your local oscillator. And then you [? indicate ?] over time. Then, of course, when you peak the phase delta to be 0, cosine omega t times cosine omega [? dt ?] gives cosine squared omega t. It averages to 1/2. So you would times average where cosine omega t times sine omega t averages to 0.

So for delta equals 0, you project out the cosine component. And for delta equal to 90 degrees, you peak out the sine component. So therefore, you can have a measurement, it's a phase sensitive measurement, by multiplying your signal with the local oscillator, where you're only sensitive to the component you have squeezed. And therefore, your measurement uncertainty has now been reduced by the squeezing factor.

AUDIENCE: [INAUDIBLE] only that should be the [INAUDIBLE].

PROFESSOR: Yes, actually, homodyne means we use the same frequency. Heterodyning would mean we use two different frequencies. But I'm not talking about that. So we have to use exactly the same frequency here.

AUDIENCE: So instead, this [INAUDIBLE] oscillation also needs a laser to [INAUDIBLE]?

PROFESSOR: Yes. So, to address your question, Angie, what usually happens is you start with one laser in those experiments. You frequence the top of the laser. If you wanted to do some squeezing, you remember that we need a parametric oscillator, where one energetic photon gives us two photons.

So what you do is you start with a laser, [INAUDIBLE]. A 1064 nanometer. You frequency double it to green laser. The green laser pumps your parametric oscillator. And then you get, for down conversion, you can squeeze light at 1064.

But this is because you first stopper the laser. And then you break the photon into new pieces. It has exactly the same frequency as your laser you started with. And this laser is in the local oscillator, or the reference clock, for your whole experiment.

So everything in your experiment, the double laser, the parametrically down converted beams. Everything is related to the single laser you started with. And everything is phase coherent. So that's how, usually, the experiment is done.

Before I tell you what we're doing quantum mechanically, let me just also get another question out of the system, which I've been asked several times. People ask me, well, the problem is that the ellipse rotates like this. Isn't there a way-- now I need my hand-- that we can have an ellipse rotating like this? And that would be great. But this is sort of unnatural because the harmonic oscillator does that.

So if you wanted to do that, you need an operator which is really, at every cycle of the electromagnetic field, is when the light want to sort of do this. No. Always push it back. And this is impractical. You need, really, an oscillator which would completely change the quadrature components of your harmonic oscillator in every single cycle of the electromagnetic field.

But sort of what homodyne detection is, instead of now forcing the light to stay aligned to sort of do this, which is very unnatural, we allow the light to freely evolve. But we have now an observer, our local oscillator, which is rotating synchronously with the ellipse. So we have a local oscillator which is cosine omega t.

It does, so to speak, exactly what the ellipse is doing. So in that sense, the local oscillator allows us, now, to observe the ellipse always from its narrow side. Because the local oscillator is [? cooperating ?]. But the mathematics is pretty much the [? Fourier ?] transform. The mathematics is a [? Fourier ?] transform. The physics is the physics of a [INAUDIBLE] detector.

OK, now the only question remaining is, how do we mix? How do we get a product of our signal and the strong local oscillator? In an old radio, it's done by an element. Maybe a diode, which is a nonlinear circuit. If you drive a nonlinear element with two input sources, you get something which involves the product of the two.

OK, so the principal of homodyne detection is now that we want to mix light at the beam splitter. So the device which does all that for us is, after using so many words, I would say it's disgustingly simple. It's really a half [INAUDIBLE].

We talked about beam splitters a lot here because beam splitters perform wonderful unitary transformations. And we'll exploit them for many purposes. For the purpose of this lecture, I simply assume that we have a 50-50 beam splitter. So, what I'm telling you now about the beam splitter will be generalized. Either later today, or in the lecture on Wednesday.

So a beam splitter has two input ports and output ports. We have light impinging on the beam splitter. We call those modes a and b. And after the beam splitter, we have two output modes. And let me call them a0 and b0.

And what we measure is the output of the beam splitter. So we have two photo detectors. And we measure the output. OK. The output modes, a naught and b naught, are simply obtained by taking the input modes and propagating them.

We have two modes, a, b. You can say, what goes vertically is a. Vertical is a. We call it a naught. Here, it's b. It becomes b naught. And what we have to do is we have two transform the operator a naught.

Let me just make a comment. Sometimes in the beam splitter, you want to think a quantum state comes and is transformed. But instead of transforming the state of a photon, I can also transform the operator, which creates a photon. One is the Heisenberg picture. One is a Schrodinger picture. So right now, I'll use the Heisenberg picture.

I have two operators, a, b. Before the beam splitter is set, I have two operators, a naught, b naught. Afterwards, I have two other operators. I have operators a, b. Afterwards, i have operators a naught, b naught. And, if you do the time evolution, it's a unitary transformation.

And for operators, we have to multiply the operators from the left and right hand side, with unitary operator, u and u dagger. And we really talk about the beam splitter in its full beauty in a short while. It will be a special case of what we discuss later.

But I think it's pretty obvious that a 50-50 beam splitters is simply creating two modes. One of the symmetric combination. And one of the anti symmetric combination.

OK. So in our homodyne detector, we are now measuring the number of photons in the mode a naught. We measure with the upper detector, the number of photons in the mode b naught. But what we are then doing is we run it through a different [INAUDIBLE]. We want to cancel certain noises, as you will see in a moment.

This is why we obtain the difference signal, which we call I minus. So let's now calculate what I minus is. Well, the number of photons in the mode a naught is a naught dagger a naught.

That's the operator for the number of photons. We subtract b naught dagger b naught. And, as you can immediately see, is that it because of the beam splitter, this is now involving a product of a and b. ab dagger plus a dagger b.

So in other words, when you ask yourself, how can we-- remember, I said we peak out the cosine omega t component by multiplying our signal with the local oscillator. And you would say, how do you multiply two modes of the electromagnetic field? Well, just send it to a beam splitter.

In a beam splitter, you create the sum of them. But then your photo detector is square [INAUDIBLE] detector. You measure the electric fields squared or you have to measure the operator, a dagger a.

And now, you get the [INAUDIBLE] between ab dagger and a dagger b. So this is how we multiply two operators. How we get a signal, which is a proportional to a or a dagger times b or b dagger.

By the way, if it wouldn't take the difference, we get terms of aa dagger bb dagger. Just the mode a or the mode b by themselves. And we want to get rid of them. And by taking the difference between the two photo detectors, those parts of the signal are becoming mode. And are subtracted out.

So therefore, this is called balanced homodyne because we have the balanced beam splitter. We measure to two signals. And then we take the difference of the two signals. OK.

There's one more thing we have to add. And then we find we understand the balanced homodyne detection. We want to explore it. That our mode b, remember we want to measure mode a. a squeezed. a has quantum properties. a has only single photons. And b is just a trick to project out the cosine omega t.

And we do that by choosing for b a strong coherent state, described by a coherent state parameter beta. Of course, whenever we have a strong coherent state, that means that we can replace b by beta, and b dagger by beta star. This is sort of the classical field limit of a strong coherent state.

And now we can ask, what this our different signal I minus? Well, the coherent state depends on the phase angle theta. So it will be phase sensitive. It will depend on the angle theta.

But now there is one more thing. And that is, if you use a stronger coherent state, of course both of our outputs go up in proportion to beta. So therefore, we want to go to a normalization by dividing by beta. So this is now our normalized output.

I'm looking at this. So this is our operator for the signal I minus. And that is a. b has been replaced by, b dagger has been replaced by b star. That gives us a times e to the-- because of the complex conjugation-- minus i theta plus a dagger times e to the i theta divided by 2.

So therefore, if we choose the phase to be 0, we measure this symmetric combination, a plus a dagger over 2. And this is-- just comparing notes-- simply the a1 quadrature operator divided by square root 2. And if you put a phase shifter, just a dispersive element, into the strong local oscillator and shift the phase by pi over 2, by delaying the light pulse by a quarter wavelength, what we are now projecting out is the other quadrature component. So therefore, using the beam splitter and the local oscillator, we can now measure the expectation value for a1 and for a2.

I've posted a few papers on the website which show some of the pioneering experiments where people did exactly that. So then, when they observed the quadrature comportment, which was squeezed, they usually did it for a squeezed vacuum, so a1 and a2 had 0 expectation value. But the interesting part is, how much noise was there?

If you don't squeeze, you find that your normalized noise simply corresponds to the classical photon shot noise. But if you are squeezed, you find that one quadrature comportment has a smaller noise than shot noise. So [INAUDIBLE] photons in your different signal. And your noise is less than square root n. Where as in the other quadrature, the component is larger.

So you can look up on the website papers where people slowly varied the phase of the local oscillator. And you see, this is shot noise, how the noise is below shot noise. Exceeds shot noise. It's below shot noise. And exceeds shot noise. And this was the first evidence for the generation of squeezed light. Questions? OK.

So we have discussed the detection of squeezed light using balanced homodyne detector. And balanced means that the beam splitter was 50-50. So now we are ready to discuss the unbalanced case.

So let's get our unbalanced beam splitter and, just for the sake of the argument, let's say it has a really good transmission of tt. So transmission coefficient t squared tells you what fraction of the power is transmitted. And let's just assume for the sake of the argument, 99% are transmitted.

So therefore, if we start with our signal a, which may be [INAUDIBLE] squeezed quantum state, number state. You name it. a naught is pretty much the same as a.

We haven't lost so much. But now we use our local oscillator, which is very strong. Only a very small fraction of it will be reflected. But we can always compensate for the small reflection by making b even stronger.

Let's see what we get. So the mode, a naught, the output mode, is now the transmission coefficient times a. a and a. The operators are amplitudes. So we have to use, if you have a 99% beam splitter, we have to take the square root of 0.99. This is the transmission coefficient.

And now we have the transmission coefficient for the strong local oscillator, which we will again approximate by its eigenvalue. Coherent state eigenvalue. And the reflection coefficient is 1 minus t squared.

So if we make the approximation that t is approximately 1, we can take it out off the parenthesis. And we obtain that. If t is close to 1, we can neglect it.

And what we see now is what we have obtained is actually the original quantum state a. Or I should say. The operator. I'm in the Heisenberg picture. a is the mode operator for the input of the unbalanced homodyne detector. And what we have simply done is we have displaced the operator.

We have displaced the mode operator a. And the displacement operator has an [? argument ?] of 1 minus t squared beta. And this is a Hermitian conjugate. 1 minus t squared beta. So what I've shown you here is that the local oscillator and the beam splitter is the realization, or implementation, of the displacement operator.

So in the limit that t goes to unity, we are not losing anything of out quantum state. But by reflecting in the amplitude of a strong coherent state, we simply take our quantum state, and we displace it in this two dimensional plane. So that's what this beam splitter does for us. Any questions?

In the next few classes, we really take advantage of those elements. We know now that the displacement operator is just a beam splitter. We know when we have squeezed some light by using balanced homodyne detection, we can just do measurement of one quadrature component or the other.

So what I hope for those of you who haven't heard about it, what you take away from that is that these are extremely simple elements. But by combining them, we can really realize very sophisticated schemes of quantum optics. To some extent, when I heard about it for the first time, I think, the mathematics look so fancy. I couldn't believe that such simple elements can actually realize what those operators describe. So I gave you the example for the displacement operator.

When I learned about the beam splitter, and its underlying physics, there was one thing which really fascinated me. And this is the most simple element you can think of. I mean, what is simpler than a beam splitter?

A beam splitter has two inputs, two outputs. The simplest optical element is just an attenuator. Put in a window in your laser beam and you lose 4% of your power preserves. Or put in just a little bit of dirty optics and you lose a few percent.

So what I want to discuss with you now is, what really is an attenuator quantum mechanically? Well if, classically, the attenuator would do the following. An attenuator is a device which has a transmission, which is a transmission coefficient squared, which is smaller than unity.

And, in a classical system, if you have a coherent state, you would simply assume that the coherent state gets multiplied by the transmission coefficient. In other words, that you have your original state, described by this phase, or alpha. And then, the action of the attenuator would simply be to scale everything down by a transmission effect of alpha.

So the picture you should have is the following. You have a coherent state. It gets attenuated by the transmission coefficient. But if there are fluctuations about the coherent state, also the fluctuations get attenuated. Because everything gets attenuated by this attenuator.

So if you look at that, you should immediately say, no. This is quantum mechanically forbidden. Because a coherent state with a minimum uncertainty state, this shaded area cannot be smaller than 1/2.

But here it has become smaller. So what I've shown to you here is it's a violation of quantum mechanics. It would actually mean, let me just give you the example. It would mean that if-- yeah.

The coherent state is quantum limited. And if you calculate what is the fluctuations in the photo number, it's a shot noise. So just to give you sort of simple, intuitive example, if you have 10,000 photons plus minus 100, it's a shot noise. Square root n.

If you could now attenuate it by a factor of 100, and you would go from 10,000 plus minus 100, to 100 plus minus 1. That's much better than the shot noise. I mean, this is what I'm just telling you what a simpleminded attenuator would do. And you would immediately say, that's too good to be true. I cannot get sub shot noise light.

So what is wrong? Impossible. Not allowed. Well, we've just tried to formulate something intuitively. And we have to be careful. Well, we know already one way how we can attenuate an input beam.

And maybe we should go back to the situation and analyze it. We know what if we can attenuate it with a beam splitter. And this beam splitter has a transmission coefficient of t.

And then we get our transmitted coherent light. There is something getting reflected. But now we realize that this beam splitter is not just a device which has one input. It has another input. And you may say, well, I don't care about the other input. I don't want to use it.

Well, if you don't want to use the input, it has the vacuum state. So therefore, if you would realize the attenuator with the beam splitter, it would mean that, in addition, and this is what the math really shows, in addition to the attenuated coherent state, which mathematically is also attenuating the fluctuations, you have to add something.

Which is the reflection coefficient times the vacuum state. Which is this. And if you now correctly do the math, if you add the two together, you find you get a coherent state which has an amplitude of t alpha. But has the correct [INAUDIBLE] fluctuation is again a minimum uncertainty state. So the disk of your attenuated state hasn't exactly the same area as the unattenuated state.

So I've shown you the physical part. I have shown you the graphical solution. The math is very simple. But I really want you to do the math yourself. This is a new homework problem we designed to illustrate the physics. But what it tells you is the following.

If you take a neutral density filter out of the lab [INAUDIBLE] and say, this is not a beam splitter. This is an attenuator. Sorry. You cannot simply attenuate a quantum mechanical mode. This is not a unitary time evolution.

What you attenuate [INAUDIBLE] is, without you knowing it, it couples the electromagnetic wave to whatever. To the heat path of the [INAUDIBLE], which is in your attenuator. I don't even want to describe it.

But you're not circumventing the limitation of the beam splitter. Whenever you attenuate, whenever you have a laser beam, and it undergoes losses, when you send a laser beam through the atmosphere, and it undergoes some losses by, who knows, [INAUDIBLE] scattering through the air or something like this.

That means you get an attenuated coherent state. But you couple in the fluctuations off the vacuum. And this establishes shot noise now, at the lower level of intensity.

So your attenuator is not a single state device. Dissipation. Attenuation really means you connect with other parts of [INAUDIBLE] space. You cannot attenuate in a small part [INAUDIBLE] space. This is impossible. This is not unitary time evolution.

So what I've just told you has dramatic consequences for any form of non classical, or squeezed, light. And that's the following. Most of you are experimentalists. And you know that when you run a laser at one or two watts, you send it through optics, and shudders, and optical fibers. How much do you get at the end of your experiment? Not even 50%.

So, whenever you create light, and then you want to do something, you lose some of the light. And let's now assume that you have done what was really a breakthrough in scientific headlines within your lifetime. You have generated squeezed light.

And now we want to use the squeezed light. Shine it on atoms and do a precision measurement, which is better than the standard quantum limit because you have squeezed the ellipse. And you want to now exploit the sharpness of the ellipse. What happens to your aspect ratio of the ellipse when your beam is attenuated?

So let me just discuss it with you graphically. So let's assume we have a squeezed state. And we send it through an optical fiber. The result is you will never send a squeezed state through an optical fiber. But I want you to realize that.

So let's assume we have our squeezed state, symbolized by that. Let's use some red color for the state. And now, in the time evolution, we have some [INAUDIBLE] scattering. Some fiber absorption. But you know already, the absorption is in reality a beam splitter, which happens in the vacuum. So

We have now some finite transmission coefficients. And that would mean, which is the bad news, that your ellipse gets shrunk. It shrinks. It shrinks by the transmission factor t.

But that's what I did bad. You lose some of your power. But the really bad thing is that if I multiply it with the reflection coefficient, you couple in the vacuum. Oops. I'm going to change to red.

And, as a result, since the noise in the vacuum is equal in both quadrature components, you've worked so hard to squeeze it, to make it asymmetric. But what you get now is from your ellipse, you get something which is much more egg shaped now.

So in other words, you can write it down with operators. But once you understand what is going on, you immediately realize that losses reduce the squeezing. And this is a challenge to all the experiments using squeezed or non classical light.

And you see how the scaling works. If you have squeezed your ellipse by a factor of 100, even 1% of the vacuum will spoil your squeezing. So the more you have squeezed, the more non-classical the light is-- the more valuable it is. The more sensitive it is, to even very small losses.

Questions?

OK, good.

Now we have 10 more minutes. What I want to do now is, I want to show you that the language which we have used, the methods we have introduced, can now be used for something which is really cool-- teleportation.

I want to show you how balanced homodyne detection, quadrature measurement, and displacement operator can be put together to generalize is scheme which is a scheme for teleporting quantum states.

This is an application of squeezing, homodyning, and all that. Teleportation of light.

Let me just illustrate what the problem is. I know we have only 10 minutes, and I've decided not to write down all the math for you. I went, actually, to the Wikipedia page, and corrected a few mistakes in the equations and edited a few explanations. I think you can just sit down and read it by yourself.

What I want to explain to you, here, is the physical concepts behind teleportation, and you the crucial steps to realize teleportation. So first, what is teleportation?

Well, teleportation has a sender and a received, which in quantum information science are called Alice and Bob. Teleportation means Alice has a quantum state, psi. And she wants to send this quantum state, psi, to Bob.

In other words, you have, maybe, some squeezed light. You have something-- a quantum state, psi-- which is interesting. And the question is, how can Bob create an identical copy of this quantum state, psi, that he can sort of [INAUDIBLE] with the same quantum state.

Before you appreciate teleportation, you have to realize what the problem is. The problem is, really, in fundamental properties of quantum systems, fundamental limitations-- what you can do with a quantum system.

First, I have to tell you what is allowed, and what not. In this game, teleportation means, of course, you will not take your atom and your photons in the state psi, just propagate them to Bob, and Bob has them. That's trivial.

I can send any quantum state to you by transmitting an atom or a laser beam to you, and you have the same quantum state I had earlier. This is not teleportation. This is trivial propagation. What is meant is that we don't have a quantum channel-- I'm not allowed to send my quantum state to you. But in teleportation, I can do a measurement on the quantum state, call you up through a classical communication channel, and tell you, the result of my measurement is such and such.

And then you would say, well, if I have a spin system, and I measure spin up-- I call you and say, my measurement was spin up, and you create a spin up system. Isn't that teleportation?

The answer is no, because maybe the state I had was a superposition of spin up and down. And what I measured is only spin up. And by telling you it's spin up, you would never create a superposition state, you would just create a spin up state.

So the effect is, that if I do a measurement on my quantum state, and report my results to you, you have insufficient information. Because a projective measurement on a quantum state inevitably leads to loss of information.

A measurement on a single quantum state does not create enough information to recreate the quantum state. Of course, there is an obvious solution. When we want to obtain information about a quantum state, we often, in quantum mechanics, have to do many measurements. And we can take a spin state, we can measure what is its x, what its y, what is the c component of the spin. We can completely characterize the spin state, and then we know everything about it.

But the problem is, we have only one quantum state, and we can't do repeated measurements. Then you would say, well, the next thing is, why don't you just take your quantum state and Xerox it, make many, many copies. And then you have an ensemble, and you can take as many measurements you want. You can do an x measurement, p measurement.

I mean, you can reconstruct the complete wave function. You can measure with an x basis. You can measure with a momentum basis. You can collect all the information. But the problem is-- otherwise the whole teleportation would not be an issue at all-- there is the no-cloning theorem. You cannot duplicate a quantum state.

If you have an atom in a certain quantum state and another atom, it is quantum-mechanically forbidden-- there is no unitary transformation, no way of creating a situation that you have one unknown quantum state, plus another atom, or another light beam, and after some interaction you have two times the same quantum state. You cannot clone.

Therefore, based on all that, we cannot clone the quantum state. We are only left with the one copy of the quantum state, which Alice has. Alice is not allowed to send it to Bob. Maybe Bob is on the other side of the ocean, or on another planet. All what Alice can do is, she can do one measurement and tell Bob, this is my measurement. So this is the problem of teleportation. How is that possible?

Well, the way how I put it, it seems impossible. But there's a way out of it.

Let me just write down what I said. So the goal is now-- Alice performs measurement, reports result to Bob. And now, Bob will recreate the quantum state.

What I've just said is, to perform a single measurement is not enough information. So this is not enough. We need one more resource. And the resource, which is now used to use quantum teleportation, is that you take some entangled system, or-- and this why I talk about it today-- or squeezed light. That's the same thing. It's a form of entanglement.

I told you that when we generate squeezed light with a parametric down conversion, the parametric down conversion takes a green photon and creates two identical infrared photons. Until now, we have discussed that those identical infrared photons go into the same mode, which is squeezed. But now, a slight extension of this concept would mean, in parametric down conversion, you squeeze something, but one photon goes to Alice, one photon goes to Bob.

So now, Alice and Bob have an additional resource. They sort of own-- each of them-- half of the two twin brothers, which are the photons created in the parametric down conversion process. And that will work.

So the idea is the following. To create those twin beams of photons is simply done with an optical parametric oscillator. There is one extension, which is described on the Wiki. I won't have time to explain it to you. But it's a two-mode OPO. It puts the two identical photons-- not in the same beam, as we did before, we had the a dagger squared, a squared operator-- they go into two different modes.

You know the magic of beam splitters by now. Now, two beams come out. One goes to Alice, one goes to Bob. If Alice would take her input state-- this unknown state which is handed to her by Victor, somebody else who participates in the game-- we know already, if Alice would perform a measurement, the quantum state would be destroyed.

The result of the measurement-- let's talk about spin-1/2-- would only be one spin projection. It's not enough to reproduce a state.

But what she is doing is, she uses, again, the magic of the beam splitter. So one of those twin brother photons is now mixed with the unknown quantum state at the beam splitter. And the output of the beam splitter is now entering the balanced homodyne detection, which we just discussed.

The output of this beam splitter-- and there are two outputs-- both of them is now becoming part of a balanced homodyne measurement. And you see the ingredients. So this, and this, will be measured. The other input for the balanced homodyne is a strong local oscillator.

The phase of the local oscillator is chosen, in one case, that you measure the x or a 1 quadrature component, or the p or the a 2 component.

So now what Alice has done is, by using this balanced homodyning, she has-- with this local oscillator-- has actually now performed two measurements. The quantum state is destroyed, but here she gets an x value, and here she gets a p value.

So how the magic works out. It's really just a few lines of mathematics, now. These were sort of, you know, twin brothers. But it wasn't clear in which quantum state the twin brothers are.

If you write it down, and I can show you the formula in a moment, these are twin brothers. But those twin brothers-- this is Brother 1, this is Brother 2-- are in sort of a continuum of states. And when Alice this would measure that this twin brother is in state big X, that would be a projective measurement. And Bob's twin brother would now, with certainty, also be in the state, X.

So therefore, what happens is, the measurement of x and p is now producing-- is now, through the measurement process-- putting the other photon, or the other beam, the other twin brother, into a specific quantum state.

And if you look for few lines of math, the magic is that the quantum state-- which is now here, with Bob-- turns out to be a displaced copy of the original state. And the displacement depends on x and p. So if Alice now tells Bob, hey, I measured x and p, and Bob is now [INAUDIBLE] his displacement operator-- remember, a displacement operator is nothing else than an unbalanced beam splitter, with a huge local oscillator as an input.

So if Bob is now setting up his displacement operator, it makes a displacement which depends on x and p. He can take this other twin brother, shift it back, and he will exactly regenerate the quantum state which Alice had.

So this is, now, how a quantum state can be transmitted without having any quantum channel for transmission. You're not propagating the quantum state. You use classical communication, but the resource you use is some EPR pairs, or squeezed light.

It's a few lines of equations, but I don't have time to go through it. They're really annotated in a way that I think it will be an enjoyful reading for you.

Any questions?

Time is over.

OK. A reminder for those who came late-- this week we have three classes, Monday, Wednesday, Friday.

Have a good afternoon.