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**PROFESSOR:** Good afternoon. We are on the finish line. Two more weeks to go. Our last chapter is coherence, and I can promise you this chapter of coherence has some highlights, so we are not going to and more boring subject. Actually, some of the best subjects, some of the most exciting topics are still to come.

So today, we continue our discussion of coherence. As I pointed out last week, we first talk about coherence in single atoms and then coherence between atoms. In the first part on coherence, I want to come back to this topic of spontaneous emission, because many of us have deep rooted misconceptions about what spontaneous emission is. We discussed on Wednesday that spontaneous emission is not so spontaneous as many of us assume because it's a unitary time evolution with an operator with a term in the Hamiltonian. It is exactly this operator which takes the wave function of the total system, the atoms and the light, to whatever it is later on. There is no random phase, there is no random variable in this time evolution, exclamation mark.

But there are certain aspects associated with spontaneous emission, and I want to address them. On the other hand, if you think about spontaneous emission in the most fundamental way, the first thing you should think about it, vacuum Rabi oscillation. Here you see in the simplest possible system what spontaneous emission can do for you.

The way we want to discuss an important aspect of spontaneous emission, we want to go beyond the vacuum Rabi oscillation, is the following. We start with an atom in the ground state and the cavity is in the vacuum state, but now we take a short pulse of a laser and we prepare the atom. And because the laser outputs a coherent state, the coherent state has a well defined phase, and this phase appears

in the superposition between ground and excited state because this superposition is created with the matrix element which has the electric field of the laser.

But then we allow spontaneous emission to happen, and spontaneous emission to happen means we take our operator which I just showed you, we propagate forward in time in such a way that we just go through half a cycle of a vacuum Rabi oscillation, which means everything which was in the excited state is now in the ground state.

And by just exactly propagating this system forward in time, we obtain this state, and that's something I hope very, very insightful which we arrived at the end of the last lecture, that the quantum state of the atom has been perfectly mapped onto the photon field. So all the information which was in the atom before spontaneous emission is now available in the photon field. So the next thing to address is the phase  $\phi$ . What is the phase of the spontaneously emitted photon, and this is what we want to understand now.

So how well can we measure the phase  $\phi$ ? You should first assume the phase  $\phi$  is perfectly determined with extremely high accuracy if you use a laser beam which has a macroscopic electric field. The phase  $\phi$  is a classical variable and can be determined with arbitrary precision. And therefore the phase  $\phi$ , which we have imprinted first into the atomic wave function and then in the photon field, is an exact number. It comes from the laser beam.

What I'm showing here is phase space plots for the photon field. I know we talk about photons and two dimensional phase-space distribution mainly in 8-422, but I think the pictures speak for themselves. A lot of you have seen the harmonic oscillator and harmonic oscillator, if you start to prepare the system here, this is position, this is momentum, the system evolves in a circle.

A lot of you have seen, if we regard photon states as states of a harmonic oscillator, which they are, that you have flux states, which are just circles, or the vacuum state is just a tiny circle at the center. And if you have a coherent state, a coherent state is maybe a little blob out there, and for the coherent state, you can determine the

phase because the angle of this little blob relative to the origin is well determined. I think you all have seen a version of that.

So anyway, what is done here is I show you this phase-space plot for the photon field, and what happens is if initially, the excited state was zero, this is just the ground state of the harmonic oscillator. It's a circle. If the excited state was occupied with unity probability, it's a flux state with  $n$  equals 1, and here you see the phase-space plot of a flux state with  $n$  equals 1. And of course, you realize if you have exactly one photon or one atom in an excited state.

There is no phase information left because the phase is actually the relative phase in the superposition between ground and excited state. If you have an excited state, only an excited state with a phase factor, you know if a phase factor can simply be factored out of the total wave function, it's never measurable. What is measurable are phases which are relative phases between two amplitudes which are populated.

And of course, not surprisingly, if we now vary the excited state fraction of the atom, that's a probability to have a photon in the photon field from zero or one in between, we sort of see that this phase-space distribution, it points along the 45 degree axis and we can measure the phase. And the most accurate phase measurement can be done if the superposition between ground and excited state is 50-50, or, talking about the photon field, we have a 50-50 superposition state between no photon and one photon.

But the phase here is indetermined and the phase here has quite a bit of variance because if you have a single photon, there's only so much accuracy for the phase. It would require more discussion, but sometimes you talk even about an uncertainty relation,  $\Delta n \Delta \phi$  equals 1. So if you have one photon, you can only measure the phase with precision on the order of unity. If you had millions of photons, then you can do very accurate phase measurements.

So what we have is-- let me just summarize the conclusion. So the phase  $\phi$  is best defined in the atom, and therefore also in the photon field, when we have an equal superposition of spin up and spin down, of ground and excited state. And you also

get that from the Bloch vector picture, if you have a Bloch vector which is pointing like that, it doesn't have a phase, it's just pointing up. If it's pointing down, there is no phase. But if it's a 50% superposition state, it points in the xy plane and you have the best definition of the relative phase of the amplitude between ground and excited state.

So I mentioned this Heisenberg uncertainty relation. The fact is just looking at these phase-space plots, you realize the angle which we can determine here for the photon distribution will have quite a variance, but now I want to discuss with you how would we actually go about it, how would we measure the phase of the photon field? And this requires a homodyne experiment, a beat experiment where we interfere the emitted photon with a local oscillator, which is the laser beam which was used in the first place to excite the atom.

And what we will find out, and it's clear that we cannot obtain a sharp value of the phase, but these fluctuations in the phase do not come from any partial trace, do not come from any fluctuations in the Hamiltonian. Just to address that, when we write down the term in the Hamiltonian, the  $e \cdot d$  term, yes, depending on the basis set, depending how you define spin up, spin down, and what phase factors you put into your basis set, you may have a phase popping up in the Hamiltonian, but this phase is purely definitional. The phase I'm talking about is really a relative phase between two amplitudes, and it is independent of a phase which may be your choice by choosing the basis set in which you formulate the Hamiltonian. Therefore, when we measure the phase, and we find that there are fluctuations, they actually come from the quantum nature of the states involved.

Let's talk about the measurement and let me set it up genetically. Here is our atom, here is the laser beam, and we want to create a Mach-Zehnder interferometer. Let me just use another color for the laser beam. Why don't we take sodium today, which has emission in the orange?

The idea is the following. We have a laser beam which is used to excite the atom, and here we have a switch. And what we let through is only a certain pulse. Let's

say if we want to have a coherent superposition between ground and excited state, it would be  $\pi/2$  pulse.

Then, after the atom has absorbed the pulse, we switch off the light pass. So then in the second stage, the atom can emit, and the emitted light interferes with the local oscillator, which is the laser beam, and we can measure the beat note on the detector. This is the scheme how we do a homodyne measurement.

And so we assume we have a very short pulse which excites the atom. Then we switch off the laser in the upper pass and the light which reaches the detector for homodyne is only the light which has been emitted by the atom maybe a nanosecond later. So we do a homodyne measurement of the phase of the wave or the wave train emitted by the atom. And the distribution of measurements for the phase, I don't want to give you mathematical expressions, but it's pretty much what you can read from the drawing I've shown you.

So for a  $\pi/2$  pulse, we retrieve the phase  $\phi$ , but with fluctuations. Let's now come to the interesting case that we have a  $\pi$  pulse. The  $\pi$  pulse prepares the atom in an excited state, and at  $t = 0$  after the excitation, there's absolutely no coherence. The density operator for the atom has just one in the column and row for the excited state. There's no off diagonal matrix element. There is no phase information. So at  $t = 0$ , no coherence, no phase.

So now we have excited the atom with a  $\pi$  pulse, but there is no phase information in the atomic system, and that would also mean that when we now start mapping the quantum state of the atom onto the quantum state of the light, there won't be any specific phase for the light. We could say after the spontaneous emission is over before we do any measurement process, we have mapped a flux state of the atom onto a flux state,  $n = 1$ , of the photon field, and there is no phase associated with a number state.

But let's be a little bit more specific here. Let's assume we can have an ensemble of atoms, we can repeat the measurement many times, and let's ask the question, what happens after the atom which was originally in the excited state has decayed

to 50%? Well, then we have a wave function which is a superposition of ground and excited state, and there is a phase  $\phi$  now, but this phase  $\phi$  is completely random.

So for those of you who are concerned that I call it a wave function, you can be more specific in the sense of quantum Monte Carlo, that at any quantum Monte Carlo wave function, at any given moment you have a wave function, but the ensemble of your atom is now an ensemble of all those wave functions with a random phase  $\phi$ . This is a way how you can decompose the statistical operator of the system, but the result is the phase is random.

If the phase is random, that means no coherence. The statistical operator does not have an off diagonal matrix element. It also means that, if you would ask what is the ensemble average of the dipole moment, the dipole moment is given by the Bloch vector. Well, if all phases are equally populated in your ensemble, the dipole moment average is 2-0. But, of course, you have a  $d^2$  value, a value of the dipole moment, which is not zero.

So here we have now a situation where the photon field has a random phase because we lost the phase information of the laser beam when we put the atom into an excited state, and you may now ask, what is the origin of this phase uncertainty? And at least the qualitative answer is it's vacuum fluctuations. You can take the concept of vacuum fluctuations a little bit further. I'm just mentioning it, but I will not work it out.

The fact that the phase of this photon which is random is somewhat associated with vacuum fluctuations, you can address this question when you talk about two atoms. So we have two atoms. We excite them both with our  $\pi$  pulse into an excited state, and then, as time goes by, we will have atoms which create photons. And at least as long as the atoms are well localized within optical wavelengths, you could play with the idea that if they're vacuum fluctuations, maybe the two atoms will see the same vacuum fluctuations.

And therefore, indeed, you will actually observe correlations in the relative phase.

So if you measure the phase of the light emitted spontaneously by the two atoms, you will find a correlation which is due to the fact that-- I'm waving my hands here, but that the spontaneous emission was triggered by the same random vacuum fluctuations. So the absolute phase will be completely random from time to time, but the relative phase will be correlated.

But what we are talking about here is correlations between two atoms. We will talk later about superradiance, and maybe this will make it much clearer what it means if several atoms emit spontaneously together. Any questions? Yes, Colin?

**AUDIENCE:** Are there any requirements on these two atoms being located within an optical wavelength of each other?

**PROFESSOR:** Yes and no. In the simplest example of superradiance, we want to put them to within one optical wavelength, and then we do not have any phase vectors, but we will talk about it next week, that we also have superradiance in extended samples, and then we only get the superradiance, the coherence between atoms, into a smaller solid angle where the different phases are very well defined. If you would now average the spontaneous emission over different directions, you would get propagation phases and the atoms would only be coherent in one solid angle but not be coherent in another solid angle. Other questions?

That's, to the best of my knowledge at the most fundamental limit, what spontaneous emission is, how accurately a spontaneously emitted photon carries forward the phase of the laser beam which excited the atom, and then eventually when we have completely lost the phase because we excited the atom to an excited state. Everything that we discussed will be actually carried to the next level when we discuss superradiance because then we have  $n$  atoms--  $n$  can be a big number-- which excite together, and if they emit photons, the phase of this  $n$  photon field can be very precisely measured.

So some of the uncertainties we have here simply come from the fact that, if you have only one photon or one atom, there are naturally quantum fluctuations of any phase measurement. But that part will go away when we go to ensembles of atoms

where we have many atoms, and superradiance is then the way how we can revisit the subject, how well can you retrieve the phase of the laser field from the spontaneously emitted photons. Other questions? Nancy?

**AUDIENCE:** [INAUDIBLE] single atom? So  $\pi/2$ , I can see that we do a homodyne measurement and get the phase out. Do we need dipole moments for  $\pi$  [INAUDIBLE], or this is just a science that we're going to use for many other things?

**PROFESSOR:** What's the question? We have  $\pi$  over--

**AUDIENCE:** What measurements do we need if we have just one atom? Do we make any measurements, or no phase information?

**PROFESSOR:** I think the measurement is, in a way, what I indicated here. We excite the atom, then we switch off this pulse, and then we take this short pulse of light. It's a wave train which has a duration on the order of the natural lifetime of the atom, and this wave train is interfered with a local oscillator, and the interference term allows us to retrieve the phase. And if you use a strong local oscillator, then we pretty much retrieve the quantum limit of the measurement, and the quantum limit of the measurement is what I showed you in these cartoon drawings of the phase-space distribution.

**AUDIENCE:** So essentially, we can [? read a ?] flux state like this?

**PROFESSOR:** If you have a flux state and you repeat the measurement many times, we will measure random phase. So what happens here is-- let me put it this way. The homodyne detection is a way how we want to measure the phase, and whenever you want to measure the phase, you get a phase because the number you get from a phase measurement is a phase. But if you have a flux state which has not a specific phase but an equal probability for all phases between zero and  $2\pi$ , then, if you repeat a phase measurement many, many times, you will get a random result for the phase.

**AUDIENCE:** I think that's what my question originally was. What measurement would you perform for this [?  $\pi$  phase? ?] Would you still do a phase measurement?



**PROFESSOR:**

It's your choice. If you want to do a phase measurement, that's a way to do it, and then for flux state, you will get a random phase. But maybe for the flux state, of course, you can say in hindsight, the flux state doesn't have a phase so maybe you shouldn't bother measuring the phase. The special thing about the flux state that it has exactly one photon, and so maybe you want to have a measurement which is measuring the special character of the flux state, namely that you have sub-Poissonian distribution of the photons.

Of course, this aspect of just having one photon gets completely lost when you have a beam splitter and you have zillions of photons in your laser beam with all the Poissonian fluctuations in the coherent state and you superimpose it. But this is nothing else than complementarity. You can either measure the phase or you can measure the photon number, and the question is, what are you interested in?

This is one aspect of coherence in a two level system, namely that we have a phase in the two level system and the question is, how can we measure it? And the answer is we can map it on the photon field and then perform a quantum measurement on the photon field.

I want to continue with some other aspect of coherence in single atom. Let me just point out one important aspect about coherence in a two level system, and this is related to something very mundane, the precession of spin-- when it's a two level system, it's spin  $1/2$ -- in a magnetic field. In other words, I just want to quickly remind you in a few minutes that for any two level system, we can always map it on spin  $1/2$ . I was really emphasizing this message throughout the whole course.

But for spin  $1/2$ , if you think of spin up or down in a magnetic field, there is a very clear visualization of the coherence. If you have a coherence of position of spin up and down, the phase of the superposition decides whether the spin points in x or y. So the precession of a spin in the transverse xy plane is actually the manifestation of coherence, and it's not just the special coherence of spin  $1/2$  in a magnetic field because all two level systems are isomorphic to that. You can always use it as an intuitive visualization of what coherences are. So what I just want to point out is the

relation to the quantum mechanical or classical precession of spin in a magnetic field, that it is simply an effect of coherence within one atom, coherence between two levels in an atom.

So if spin points in the x direction, it is a coherent superposition of plus z and minus z, spin up and spin down, but this is a situation at time  $t$  equals 0. If we let time evolve, spin up and spin down evolve with the Larmor frequency, actually effect of  $1/2$ , but with opposite phases because one has plus the Larmor,  $\hbar \omega$  Larmor over 2. The other has minus  $\hbar \omega$  Larmor over 2 as energy.

In other words, if you look at the relative phase, it's a beat node with the same unsplitting between spin up and spin down. But that means now, due to this coherent time evolution of the two amplitudes, that the spin precesses in the xy plane. The statistical operator for the spin  $1/2$ , we have 50-50 population in spin up and spin down, but now, the phase here precesses as  $e^{-i\omega t}$  and  $e^{i\omega t}$ , which means, if you use the statistical operator and find the expectation value for the x spin. It means we take the statistical operator describing the pure state of a two level system, we multiply with a Pauli matrix in x, and this is a prescription to get the expectation value for  $\sigma_x$ , and we find it's  $\cos(\omega t)$ . So the spin is precessing the x component changes cosinusoidally. That would mean the y component changes sinusoidally.

Let me just contrast it to the case of no coherence, and this would mean off diagonal matrix elements are 0. then if you have a statistical operator where the off diagonal matrix elements are zero, in one minute, you can show that then any expectation value for the x or y component of the spin vanishes. In that case, if you have a statistical mixer between spin up and down, of course there is no phase determined, and it is the phase of the superposition state which tells you where between 0 and  $2\pi$  the spin is pointing in the xy plane.

I want to come back later on when I discuss an example of coherent spectroscopy, that if you excite coherently a superposition of spin up and spin down, you can perform some form of coherent spectroscopy, which I want to explain first in general

and then come back to the spin as an example. When we talk about coherent spectroscopy, I want to just in 10 minutes or 15 minutes show you some spectroscopic techniques which exploit the coherence between several quantum states. I do it for a number of reasons.

One is coherent spectroscopy actually allows us to obtain information about the level structure even if this level structure is much narrower than the Doppler width. So it is a sub-Doppler technique to exploit coherence. And before people had lasers, before people invented sub-Doppler laser spectroscopy, often, coherent spectroscopy was the only way how you could obtain detailed structure of the atom.

The reason why I explain coherent spectroscopy is to just give you a little bit idea about that you appreciate how smart people were before lasers were developed, but also, it illustrates what coherence can do for us. It's a nice example for the concept of coherence. When I was a graduate student, textbooks had dozens of pages, 50 pages on coherent spectroscopy, the Hanle effect, quantum beat measurements. It's all old fashioned because with a laser, and especially cold atoms and the laser, we have such wonderful tools to go to the ultimate fundamental precision of quantum measurements. But still, coherence is important.

Let me talk about one method, which is called quantum beat spectroscopy. The selling point about quantum beat spectroscopy is that it allows the measurement of narrow level spacings-- just think about Zeeman splitting in a magnetic field-- without any form of narrow band excitation. You can also put it like this. If you don't have any way to selectively excite levels, but you're interested what is the level spacing, but you cannot have a narrow band laser, have atoms which stand still and scan and get peak, peak, peak, what you can still do is you can just excite all of the levels at once. In other words, you hit the atom with a broad laser like with a sledgehammer, and then you see a beat node, you see some blinking, a quantum beat between the excitation of the levels. That's the idea.

We assume we have a ground state and then we have an excited state manifold, and in this excited state manifold, we have several levels distributed over an energy

interval  $\Delta$ . Yes, we don't have a narrow band source. We may just have a classic light source, but if we use a short pulse that the pulse duration is much smaller than the splitting between energy levels, then we create a coherent superposition of those levels.

So therefore, what we create at time  $t = 0$  is a coherent superposition of energy eigenlevels. And the important thing is that this is at time  $t = 0$ , but now, when time goes on, each amplitude, each part of the wave function, evolves with its frequency  $\omega_i$ , and if we would then look at, let's say, the emission spectrum as a function of time, we will find that-- I will give you a little bit more details later-- that yes, there is a decay approximately with the natural spontaneous emission time with the inverse of the natural line widths. But we observe some oscillations which is the interference term of the different terms in the wave function.

So therefore, if we would take this spectrum and perform a Fourier transform, we will actually observe different peaks. This is frequency, and the frequency peaks are at discrete frequencies corresponding to frequency differences between the excited state. And ideally, the widths of these is determined by the natural line widths.

So in other words, what we have actually done is we have done a version of the double slit experiment. We have ground state, we had our excited states,  $e_{sub\ i}$ , and our broad band source was creating a coherent excitation, and then we were observing the light which came out. We were performing a multi-slit experiment. We had a laser pulse and then we see photons coming out, but it is fundamentally not observable which intermediate state was responsible for the scattering.

So therefore, we have in the Feynman sense several indistinguishable paths going through different internal states. And therefore, we get an interference effect. Some of what I'm saying we will retrieve later on when we talk about three level systems. We will also have situations that sometimes we go through two possibilities for the intermediate state, and if we have no way, even in principle, to figure out which intermediate state was involved, we have to sum up the amplitudes, and that's when we get a beat note.

This technique is a Doppler free technique because, even if you take a single pulse from a light source, you have a Doppler broadening, which is  $k \cdot v$ ,  $v$ , the thermal velocity, and this can be much, much broader. You will still see the quantum beats, maybe I should say in principle, because the beat happens at the much smaller frequency  $\Delta$ . Or maybe I should say that the Doppler shift is reduced by the splitting of the excited states over the frequency of the exciting laser. Of course, if you have your different atoms emitting at different frequencies, you have a Doppler shift, but since you measure the difference frequency, you only get the Doppler shift associated with the difference frequency.

Now let me come back to the previous example I had about the spin 1/2 system. If you assume you have a spin 1/2 system, spin up and spin down, which is excited with a laser, which is linear polarization, you would then create a superposition of up and down which, let's say, is now a dipole moment which points in the x direction. A dipole moment which points in the x direction will not emit light along the axis of the dipole moment because of the dipolar emission pattern. It will only emit to the side.

But I mentioned to you that the dipole moment or the spin, which is originally in x, will now oscillate with a Larmor frequency in the xy plane. So the picture you can actually have of such a quantum beat and quantum superposition is like the lighthouse. You have a searchlight at the lighthouse, and the searchlight is just rotating at the Larmor frequency. For instance, you wouldn't see light right now, now you don't see light, now you don't see light. It's really like a classical lighthouse which is emitting light at the Larmor frequency.

So if you have a fluorescence detector which looks at the atoms from a certain direction, you will pretty much see the lighthouse effect that the fluorescence of this coherent superposition of atoms goes on and off, on and off, on and off. This is sort of a very nice visualization how you obtain what I showed here, a beat node in your detected signal.

Let me talk about another aspect of coherence. And of course, they are all related. Coherence is always related to the phase, to beat nodes, to superposition. Let me

now talk about one aspect which is related to delayed detection.

When I was a graduate student and I learned about spectroscopic techniques, somehow I was so fascinated by techniques which could measure spectroscopically transitions better than the natural line widths. I don't know. Maybe from what I had read before, what I learned as an undergraduate, the natural line widths appeared to be the natural fundamental limit. So the topic I'm teaching right now has always had a certain fascination for me, but you will, of course, also realize that in the end, the answer is rather simple. Once you know the answer, most answers are very simple.

So we want to talk about delayed detection. Let's say we excite the system. You can think about a quantum beat experiment. You have a short pulse and then your quantum beats happen. And now the question is, normally, when you do a measurement on a decaying system, you're always limited by the natural line widths, by the inverse of the lifetime.

But now, maybe you want to be smart and you say, well, I start the detection, I only detect atoms after a time  $t_0$ , which is much, much larger than the natural line widths. And the question I have is, can you obtain, with such a measurement, a spectral resolution which is narrower than the natural line widths? Well, we can give two possible answers.

One is yes, because you're looking at atoms which have survived for a long time, so to speak. These atoms are longer lived. We have just selected atoms which happened to survive for several lifetimes. But then there should some lingering doubts.

If you have a sample which undergoes radioactive decay, and you would go to your favorite supplier and buy uranium, which has already decayed for a billion years, it's the same uranium which existed a billion years ago. You will not be able to perform any measurement on your well aged uranium, which has a higher resolution than if you had lived a billion years ago and had done your measurement with younger uranium. So in other words, the exponential decay is self similar. It starts at any

moment and it looks exponential no matter where you start.

I didn't bring clickers today, but with which answer would you side? Is it possible or it's not possible? Maybe just hints of who thinks it is possible by taking advantage of the longtime survivors? OK. Who thinks it's impossible? A few. Good.

The answer is actually depends. If you would go and just look at the longtime survivors, you would not be able to do a more precise measurement. You need a little bit of information from the earlier time. So in that sense, the question is a little bit deliberately confusing, and I want to show you how the mathematics work.

It's just five minutes to show the mathematics of a Fourier transform, and the result will be if you have information about something at  $t$  equals 0, and then you look at the long time survivors, you in essence have a longer integration period for your measurement, and then the Fourier transform of that measurement can be very narrow. But if you do the dumb thing, you just go to the store and buy very well aged uranium or very well aged atoms, and you then start your measurement, you have no chance. You are always back to the spontaneous decay to the natural line widths.

All I have to do is actually just a few lines of mathematics and Fourier transform. Let us assume we have the situation we discussed earlier. We have a quantum beat where we have a beat frequency  $\omega_0$ . Just think about the searchlight, the atoms which oscillate with a Larmor frequency, and you have some cosine  $\omega_0 t$  factor in the intensity of the light you observe.

But now, because the atom in the excited state is decaying, everything will decay with the natural line width. This is sort of what we observe, and the question is, if you observe that in real time, can we then retrieve spectral information from it which is more accurate than  $\gamma$ ? All I want to do is I want to discuss the Fourier transform of this function,  $s$  of  $t$ .

Let me use dimensionless variables. We measure frequencies in units of  $\gamma$ . We use a Lorentzian, which is just  $1 + x^2$ . We just said we want to start

the measurement at time  $t_0$ .

So the question is, if you start later and later and later, do we get higher accuracy because we're talking to the survivors? So let's perform the Fourier transform, and let's use complex notation,  $e$  to the  $i$   $\omega$   $t$ . I will measure times in units of the inverse line widths. So we performed the Fourier transform, and by doing  $e$  to the  $i$   $\omega$   $t$ , I actually performed the Fourier transform for the cosine and for the sine by using the real part and the imaginary part of the complex number. So we will actually be able to look at the real and imaginary part.

The Fourier transform has a real and imaginary part, so let me call the real part  $F$  of  $x$  and the imaginary part  $G$  of  $x$ . You can do the math. It's a straightforward integral. In both cases, will we find that if you do our measurement, well, the longer we wait, the more signal we lose. This is common to all delayed measurements. You're really now talking to an exponentially smaller and smaller signal. Also, because of the exponential decay, we get an envelope which is a Lorentzian.

But then, and this is the interesting part, we have  $\cosine xT$  minus  $x$   $\sin xT$ . So now we have factors which depend on capital  $T$ , and  $t$  is larger the longer we wait. It is actually those parts with sine and cosine which determine whether we can get resolution below the natural line widths.

What is important is-- and this should be sort of an eye opener for you-- if you simply measure intensity, if you look at the power spectrum, you take the real part plus imaginary part and just look at the absolute value, then, because of cosine squared plus sine squared equals 1, all the cosine and sine part, the last part of the expression above cancels out and you find that you have an exponential loss of signal but your spectral distribution is always a Lorentzian. So you always have a Lorentzian line shape completely independent of the delay time, capital  $T$ . And this is what some of you maybe thought. If I start the measurement later and all I can do is look at the power of the emitted light, I have no advantage. I cannot go sub-natural.

However, if you look at the function  $F$  of  $x$  for large values of  $x$ , you find oscillations.



So if you look at the sine or cosine Fourier transform, the real or imaginary part separately, you find oscillations, and those oscillations, similar to Ramsey fringes, have a central peak. The central peak is narrow and the width is now given by not the inverse of the natural lifetime but the inverse of the delay time you wait for your measurement.

So the fact is now we had a signal  $s$  of  $t$  which we assumed was a quantum beat with a well defined phase, and then it was exponentially decaying. If we would now perform a Fourier transform where we do a Fourier transform with cosine  $\omega t + \phi$ , we can get a narrow signal, but we have no idea what  $\phi$  is or if, in repetitions of the experiment,  $\phi$  would be random, then this is sort of what the math does. If  $\phi$  is random, it is the same as if we simply measure the power spectrum because we cannot distinguish between the cosine and sine Fourier transform. So if the phase  $\phi$  is random, that means you only measure what was  $F$  plus  $IG$  before.

In other words, the situation is extremely in the end very simple. If you have quantum beats which start with a well defined phase, and you know the phase was, let's say, zero here, and now you have the decaying function, and now you look at the quantum beat over there, well, in a way, you had  $n$  beats between  $t$  equals 0 and your measurement, and then your resolution goes with  $1$  over  $n$ , but you have to know what the phase was at  $t$  equals 0, and then, by looking at delayed detection, you can do spectroscopy below the natural line widths. So therefore, what is crucial here is knowledge, or at least reproducibility, of the phase  $\phi$ , and then you can get narrow lines.

Examples for techniques where you excite the system at  $t$  equals 0 and then you can do delayed detection of quantum beats. I mentioned earlier in the course the Ramsey spectroscopy where you have one Ramsey zone where you prepare your Bloch vector. Then the Bloch vector oscillates, and if you simply look at the phase angle of the Bloch vector after a very long time, you have very high precision but you're dealing with an exponentially small signal. Another example are heterodyne or homodyne techniques, but you need something which is phase sensitive in order

to obtain sub-natural line widths.

A final comment is if you want to get higher resolution with delayed detection, yes, you can get it, but you was exponentially in signal. And what does it mean in practice? Well, if you know your line shape, you know it's well described by a Lorentzian, it is better to take your full signal and use then an excellent signal to noise to find the line center and split the line. However, if there's any ambiguity, there may be different, not fully resourced lines under the Lorentzian and you don't know how to split the line, then it may be better to do delayed detection and clearly see the structure of the lines with sub-natural resolution. Any questions?

This was coherence with two levels, coherent excitation, coherent observation, some spectroscopic techniques. Now we are ready to do the next step, namely, to talk about coherence in three level systems. If we have three levels, we could think about it, we have terms which connect level one to two, level two to three, and level three to one in all possible ways, but that's not what we want to assume here. The situation where we can discuss some fundamentally new effects is when we have two states connected through a third state.

In other words, if we have two levels, we are not allowing any transition matrix element connecting the two. They are only connected through a third state. This is for obvious reasons called the lambda type system. You can turn it upside down and you have the V type system or, if the intermediate state is between the first and the second state, you have a ladder type system.

But once you start driving it, it may not really matter. There may be a dressed atom description where, if you drive two states coherently in the dressed atom picture, you have degeneracy between this level and one more photon and this level. And then in the dressed atom picture, which includes a number of photons, the two levels have become degenerate. So therefore, it's very important for practical applications or how to implement it in an atom what kind of system you have, but for the description of those systems, some of the differences may simply disappear if you formulate it in the dressed atom basis.

Of course, there's an important practical reason. Usually the lower states are ground states, the upper states are excited states. And here you have the opportunity, and that's why the lambda type system is the most important one, to have some coherent superposition mediated by the third state, and the coherent superposition is stable because it's a coherent superposition of ground states. If any form of coherent superposition involves an excited state, then you have short lived states, and they are often not so useful for certain phenomena.

So if you think you know already everything about how atoms interact with light from two level atoms, I have to tell you that's not the case because a three level system has many new effects. One, of course, is that atoms can now interact with two electromagnetic fields, and those two electromagnetic fields can affect each other, and this can happen through coherent or incoherent mechanisms.

In other words, you can say it simply. If you hit an atom with light and you have a two level system, there is no way how the atom can hide. It's always excited by the laser. But if you have a three level system, you may have a situation where you have destructive interference between what the two lasers can do to the atom, and suddenly, there may be a state where the atom is in the dark where the atom can hide from the laser beam. This is something which is fundamentally new and has no counterpart in a two level system.

I've already pointed out that the lambda system is the most important one because it has two ground states which can be in a long lived superposition state. What we want to discuss as possible consequences is that in a three level system, you can realize a lasing operation without having inversion of the population of the ground and excited state. So if you always thought, if I want to build a laser, the first thing I have to do is make sure I have more atoms in the excited than in the ground state, yes, this is valid for two level system, but it is no longer valid for a three level system.

The reason why you want to invert a two level system is you want to have stimulated emission from the excited state which is stronger than absorption from

the ground state. But if you take advantage of quantum coherence, you may have a situation where two possibilities for stimulated emission add up coherently, but the two possibilities for absorption add up destructively. And therefore, you can avoid destruction. You have only stimulated emission, but you have not achieved that through inversion. You have achieved that through quantum coherence, a fundamentally new effect.

So we have lasing without inversion, we have the phenomenon I mentioned already that atoms can hide in the dark if the two laser beams in the excitation mechanism destructively interfere. This is called electromagnetically induced transparency. Systems which have sharp resonances in three level systems are used for reducing the group velocity of light, which goes under the name slowing light, or even bringing light to a standstill, stopping light. And three level systems are also used for quantum mechanical memories for quantum computation. Any questions? This is an introduction.

Let me connect special effects in a three level system to something which is very basic and you've heard about it, and this is optical pumping. If we set up a system which has two ground states,  $g$  and  $f$ , you may just think about two hyperfine states in your favorite atom, and then they are only coupled through an excited state, you can now drive the system with laser fields  $\omega_1$  and  $\omega_2$ .

Let me also use that example of optical pumping to introduce some notation which I will need to describe the system with a few equations. We will use energy level diagrams, and the energy is referred to the lowest ground state. So here, we have an energy splitting which is by  $\omega_{gf}$ , and the excited state has a splitting of  $\omega_{eg}$ . We will call the photons in one laser the photons created and annihilated with the operator  $a$  and  $a^\dagger$ , and for the photons for the other laser beam, we use  $c$  or  $c^\dagger$ .

Now, there is a very simple solution for this situation, a very simple equilibrium situation, if you have only one laser beam. If you have only one laser beam,  $\omega_1$  or  $\omega_2$ , it's clear what happens. If you have only one laser beam, let's say

$\omega_1$ , it doesn't talk at all to the atoms in the state  $f$ . They are left alone.

But the atoms in the state  $g$  are excited to the excited state, and then there may be a certain branching ratio for spontaneous emission, but let's rather call it fluorescence, two photon scattering. So there be a branching ratio to go back to that state or to go to the state  $f$ . If the latter happens, the atom doesn't interact with the laser light anymore. If it goes back to the original state, the atom will try again and again until after a while, all the atoms have been optically pumped into the state  $f$ . And the same would happen if you have a laser,  $\omega_2$ , then you would pump all the atoms into the state  $g$ . So if you have only one laser beam,  $\omega_1$  and  $\omega_2$ , then in equilibrium, the equilibrium population is 100% of the atoms are in state  $f$  or  $g$  respectively, and this is nothing else than the phenomenon of optical pumping.

We have a very simple solution. We pump all the atoms into one quantum state if we have only one of the laser beams. But the question is now, can we have a similar situation when both laser beams are on? And what I mean by that is, is it possible now to pump all the atoms into a state which does not scatter any light, which does not react with the light, which is never excited to the excited state?

The answer is yes, and this is what we want to derive right now. Before I go into any equation, the result is pretty clear. If you have, say,  $g$  and  $f$  and they are both excited, if the amplitude which you put into the excited state is the same but has opposite sign, the two amplitudes which are added in a time  $\Delta t$  destructively interfere and you have not put any amplitude into the excited state. And that means, if you have this initial superposition state where this complete destructive interference happened, this state will be dark all the time.

But we want to assume-- also it doesn't matter-- that the two states, the two lasers have two different frequencies. So you cannot say, this is just the two lasers have a certain phase and then the laser field interferes and they reach a space which is dark. We assume that the atom is sitting at the origin. Again, we're not putting in motion effects, so it's an atom with infinite mass.

Then we shine two laser lights on it and the atom is not in the dark. It's not at a dark

fringe of the interference of the two laser beams because if you have two laser beams with two different frequencies, there will not be any place in space which is dark all the time. You create maybe interference fringes, but the interference fringes are rapidly running with a beat node,  $\omega_1 - \omega_2$ . So the atom is not in the dark, but nevertheless, it will not scatter light if it is prepared in a suitable coherent superposition state.

We describe this situation with a dipole Hamiltonian, and we make the rotating wave approximation. The Hamiltonian has three lines, three parts. One is we describe each of the laser fields as a single mode. I call the frequency now  $\omega_a$  and  $\omega_c$ , just to connect it with the operator  $c^\dagger$ ,  $c$ .

For the atom, we use the matrix, two by two matrices, so this is the matrix if the atom is in the ground state. Coherences are described by that. And of course, without any interaction with the laser field, the atomic Hamiltonian is atoms are in the ground state. the state  $f$  has zero energy. We use that as the origin of the energy. The state  $g$  has an eigenenergy of  $\omega_{gf}$ , and the excited state  $e$  has an eigenenergy of  $\omega_{ef}$ .

But now the important part is that we want to have the coupling. And actually, I realized I was not saying it correctly.  $\Omega_1$  and  $\Omega_2$ , these are the Rabi frequencies of the two fields, and the two fields are at frequency  $\omega_a$  and  $\omega_c$ .

So now we have the coupling between the excited state and the ground state via photons  $a$  and  $a^\dagger$ . And the coupling happens at the Rabi frequency  $\Omega_1$ . And then we have the second laser field, which is at Rabi frequency  $\Omega_2$ . We have the atomic raising and lowering operator, and we have the photon  $c$  and  $c^\dagger$ .

That's a nice Hamiltonian. It has three lines. The important part here, which we have explicitly assumed is that each of the lasers,  $a$  and  $a^\dagger$ ,  $c$  and  $c^\dagger$ , are only driving one transition. One field is responsible for connecting the state  $f$  to the excited state. The other field is responsible for connecting the state  $g$  to the

excited state.

In practice, this can be accomplished by you have maybe polarization. This is a plus one state, this is a minus one state, and the excited state is  $m = 0$ . Then one laser beam is sigma plus, the other one is sigma minus, so it can be polarized and the two laser beams can only talk to one of the ground states. Or you can have a situation that you have a huge energy separation. Let's say you have a large hyperfine splitting and the two lasers are separated by frequency.

I think I've set the stage, but I think I should stop here and on Wednesday, I'll show you in the first few minutes of the class that this Hamiltonian has a simple solution, which is a dark state, which is a superposition state of  $g$  and  $f$ . Any questions? See you on Wednesday. A few people haven't picked up their midterm quizzes. If you want them, I have them here.