PROFESSOR: OK, let's start. So we were talking about melting in two dimensions, and the picture that you had was something like a triangular lattice, which at zero temperature has particles sitting at precise sites-- let's say, on this triangular lattice-- but then at finite temperature, the particles will to start to deform. And the deformations were indicated by a vector u.

And the idea was that this is like an elastic material, as long as we're thinking about these long wavelength deformations. u and the energy costs can be written for an isotropic material in two dimensions in terms of two invariants. And traditionally, it is written in terms of the so called lame coefficients, mu and lambda.

Where this uij, which is the strain, is obtained by taking derivatives of the deformation, the iuj, and symmetrizing it. This symmetrization essentially eliminates an energy at a cost for rotations.

And then because of this simple quadratic translation of invariant form, we could also express this in terms of fullier mode. And I'm going to write the fullier description slightly differently than last time.

Basically, this whole form can be written as u plus 2 lambda over 2 q dot u tilde of q squared. And the other term-- other than previously I had written things in terms of q dot u and q squared u squared-- we write it in terms of q crossed with u tilde of q squared.

Essentially, you can see that this ratifies that they're going to have modes that are in the direction of q, the longitudinal modes. Cost is nu plus 2 lambda, and those that are transfers or orthogonal to the direction of q, whose cost is just mu.

And clearly if I were to go into real space, this is kind of related to a divergence of u. And the divergence of u corresponds to essentially squeezing or expanding this deformation.

So what these measures is essentially the cost of changing the density. And this combination is related to the bark modulus. You have that even for a liquid. So if you have a liquid, you try to squeeze it. There will be a bulk energy cost.

And this term, which in the real space is kind of related to kern u, you would say is corresponding to making the rotations. So if you try to rotate this material locally, then the corresponding sheer cost of the formation has a cost that is indicated by mu, the sheer modulus.

And basically what really makes a solid is this term. Because as I said, a liquid also has the bark modulus, but lacks the resistance to try to sheer it, which is captured by this, that is unique and characteristic of a solid.

So this is the energy cost. The other part of this whole story is that this structure has order. And we can characterize that order which makes it distinct from a liquid or gas a number of ways.

One was to do an x-ray scattering, and then you would see the back peaks. And really that type of order is translational. And you characterize that by an order parameter. It's kind of like a spin that you have in the case of a magnet being up or down.

In this case, this object was e to the i g dot u-- the deformation that you have that's on location r. And then these g's are chosen to be the inverse lattice vectors.

It doesn't really matter whether I write here u of r or the actual position. Because the actual positions starts at zero temperature, we devalue r 0, such that the dot product of that g is a multiple of 2pi. And so essentially, that's what captures this.

Clearly, if I start with a zero temperature picture and just move this around, the

phase of this order parameter over here will change, but it will be the same across the system. And so this is long range correlation that is present at zero temperature, you can ask what happens to it at finite temperature.

So we can look at the row g at some position, row g star at some other position. And so that was related to exponential of minus g squared over 2-- something like u squared x, or u of x minus 0 squared.

And what we saw was that this thing had a characteristic that it was falling off with distance according to some kind of power law. The exponent of this power law, when calculated, clearly is related to this g squared. Because this is the quantity that goes logarithmically.

And so the answer was g squared over 4 pi. Heat was dependent on these two modes being present. So you have nu 2 nu plus lambda, and then 2 nu plus lambda. You had a form such as this.

Now this result was obtained as long as we were treating this field, u, as just the continuum field that satisfies this. And this result is really different, also, from the expectation that at very high temperature the particle in a liquid should not know anything about the particle further out in the liquid, as long as they're beyond some small correlation links.

So we expect this to actually decay exponentially at high temperatures. And we found that we could account for that by addition of these locations, can cause a transition to a high temperature phase in which row g, row g star, between x and 0, decays exponentially.

As opposed to this algebraic behavior, indicating that these locations-- once you go to sufficiently high temperature, such that the entropy of creating and rearranging these dislocations overcomes the large cost of creating them in the first place, then you'll have this absence of translational order, and some kind of exponential decay of this order parameter.

So at this stage, you may feel comfortable enough to say that addition of these

dislocation causes our solid to melt and become a liquid. Now, I indicated, however, that the sun also has an orientational role.

What I could do is-- at each location in the solid, I can ask how much has the angle been deformed, and look at the bond angle. So maybe this particle moved here, and this particle moved here. Somewhere else, the particles may have moved in a different fashion. And the angle that was originally, say, along the x direction, had rotated somewhere else.

And clearly, again, at zero temperature, I can look at the correlations of this angular order, and they would be the same across the system. I can ask what happens when I include these deformations and then the dislocations.

So in the same way that we defined the translational order parameter, I can define an orientational order parameter. Let's call it sci at some location, r, which is e to the i. Theta at that location r--

Except that when I look at the triangular lattice, it may be that the triangles have actually rotated by 60 degrees or 120 degrees. And I can't really tell whether I clicked once, zero times, twice, et cetera.

So because of this symmetry of the original lattice on their on their theta going to theta plus 2 pi over 6, I have to use something like this that will not be modified if I make this transformation, even at zero temperature. If I miscount some angle by 60 degrees, this will become fine.

Now I want to calculate the correlations of this theta from one part of this system to another part of the system. So for that, what I need to do is to look at the relationship between theta and the distortion field, u, that I told you before.

Now you can see that right on the top right corner I took the distortion field, and I took it's derivative, and then symmetrized the result in pencil. And that symmetrization actually removes any rotation that I would have. So in order to bring back the notation, I just have to put a minus sign.

And indeed, one can show that the distortion or displacement u or r across my system-- let's call it u of x-- leads to a corresponding angular distortion, theta, at x, which is minus one half-- let's call it z hat dotted with curve of u.

So if, rather than doing the i u j plus d j u i, if I put a minus sign, you can see that I have the structure of a curve. In two dimensions, actually curve would be something that would be pointing only along the z direction. And so I just make a scale on my dot, without taking that in the z direction.

And so you can do some distortion, and convince yourself that for each distortion you will get an angle that is this.

AUDIENCE: Do we need some kind of thermalization to fix the dimensions of this? Because that can go u has dimensions of fields, and u--

PROFESSOR: I'm only talking about two dimensions. And in any case, you can see that u is a distortion-- is a displacement-- the gradient is reduced by the displacement, so this thing is dimensionless as long as you have these dimensions.

AUDIENCE: Sorry.

PROFESSOR: Yes?

AUDIENCE: That's a 2, right? Not a c?

PROFESSOR: That's a 2. It's the same 2 that I have for the definition of the strain. Rather than a plus, you put a minus.

AUDIENCE: So can we think of these as two sets of Goldstone modes, or is that not a way to interpret it? Is it like two order parameters? I mean, you have a think that has u dependence, but--

PROFESSOR: OK, so let's look at this picture over here. You do have two sets of Goldstone modes corresponding to longitudinal transfers. You can see that this curve is the thing is that I call the angle. So if you like, you can put the angle over here.

But the difference between putting an angle here, and this term, is that in terms of the angle, there is no q dependence. So it is not a ghost. Because the cost of making a distortion of wave number q does not vanish as q squared works.

All right, so then I can look at the correlation between, say, sci of x, sci star of zero. And what I will be calculating is expectation value of e to the i 6. And then I will have this factor of--

So let me write it in this fashion. Theta of x minus theta of 0. Since u is Gaussian distributed, theta in the Gaussian distributor. So for any Gaussian distributed entity, we can write the exponential of e to the something as its average as exponential of minus 1/2 the average of whatever is in the exponent.

So I will get 36 divided by 2. I do have the expectation value of delta theta squared. But delta theta is related up to this factor of 1/4 to some expectation value of kern u. So I would need to calculate kern mu at x minus kern u at 0, the whole thing squared with the Gaussian average.

Now, this entity-- clearly what I can do is to go back and look at these things in terms of Fourier space, rather than position space. So this becomes an integral d 2 q 2 pi to the d. I will get e to the i q dot x minus 1. And then I have something like q cross u tilde of q. And I have to do that twice.

When I do that twice, I find that the different q's are uncorrelated. So I will get, rather than two of these integrals, one of these integrals. And because the q and q prime are said to be the same, the product of those two factors will be the integral 2 minus 2 cosine of q dot x term that we are used to.

And so that's where the x dependence appears. And then I need the average of q cross mu of q. And that I can read off the beta [INAUDIBLE], root the energy over here. You can see that there is a Gaussian cost for q plus u of q squared, which is simply 1 of a lingering variance.

So basically, this term you'll the sum of 1 over u. Now the difference between all of

the calculations that we were doing previously, as was asked regarding Goldstone modes-- if I was just looking at u squared, which is what I was doing up here, I would need to put another factor of 1 over q squared [INAUDIBLE].

And then I would have the coulomb integral that would grow logarithmically. But here you can see that the whole thing-- the cosine integrated against the constant-- will average out to 0. So I will think you have 2 over u times this integral is a constant.

So the whole thing, at the end of the day, is exponential of-- that becomes a 9 divided by 2. There's a factor of 1 over the mu, and then I have twice the integral of d 2 q over 2 pi squared. Which is-- you can convince yourself simply the density of the system a number of times.

So as opposed to the translational order, which was decaying as above our lot, then we include the phonon modes. When we include these phonon modes, we find that the orientational order decays much more weakly. So that was falling off as I went further and further. This, as I go further and further, eventually reaches a constant that is less than 1, but it is something.

Using conversely proportional to temperature-- so as I go to 0 temperature, these go to 1. And basically because this order parameter, with respect to-- well, this measure of distortion with respect to that measure of distortion has an additional factor of gradient. I will get an additional factor of q squared, and then everything changes accordingly.

So orientational order is much more robust. This phase that we were calling the analogue of a two dimensional solid had only quasi long range order. The long range order was decaying as a power law.

Yes?

AUDIENCE: Is n dependent on the position, or--

PROFESSOR: No. So basically, if you were to remember the number of points it should be the

same as the number of allowed fullier modes. And this goes to an integral-- 2 q over 2 pi squared-- when I put the area in two dimensions.

So the integral over whatever [INAUDIBLE] zone you have over the fullier modes is the same thing as the number of points that you have in the original lattice, divided by area, or 1 over the size of one of those triangles squared.

Yes?

AUDIENCE: Where is the x dependent in that expression?

PROFESSOR: OK, the x dependence basically disappears because you integrate over the cosine of q x. And if x is sufficiently large, those fluctuations disappear.

AUDIENCE: Oh, so we're really looking at the [INAUDIBLE].

PROFESSOR: Yes, that's right. So at short distances, there are going to be some oscillations or whatever. But it gradually-- we are interested in the long distance behavior. At very short distances, I can't even use the continuum description for things that are three or lattice spacings apart.

So maybe I should explicitly say that this is usually called quasi long range order, versus this dependence, which is two long ranges.

So given that this is more robust than these forum-like fluctuations, the next question is, well does it completely disappear when I include these locations.

So again, this calculation, based on Gaussian's, relies on just the fullier modes of that line that I have up there. It does not include the dislocations, which, in order to properly account, you saw that we need to look at a collections of these locations appearing at different positions on the lattice. And they had these vectorial nature of the fullier interactions among them.

So presumably, when I go into the base where these locations unbind-- and by unbinding-- as I said, in the low temperature picture of the dislocations, they should

appear very close to each other because it is costly to separate them by an amount that grows invariably in the separation.

In the unbound phase, you have essentially a gas of dislocations that can be anywhere. So the picture here now is that indeed this is a phase, that if I just focus on the dislocations, there is a whole bunch of them. In a triangular lattice, they could be pointing in any one of three directions, plus or minus.

And then there is certainly an additional contribution to the angle that comes from the presence of these dislocations. So you calculate-- if you have a dislocation that has inverse b, let's say at the origin, what kind of angular distortion does it cause.

And you find that it goes like v dot x, divided by the absolute value of x. This is for one dislocation. This is the theta that you would get for that dislocation at location x.

Essentially, you can see that if I were to replace the u that I have here with the u that was caused by dislocation, you would get something like this formula. Because remember the u that was caused by dislocation was something like the gradient of the log potential.

It's kind of hard to work, but maybe I'll make an attempt to write it. So let's take a gradient of theta. Gradient of theta, if I use that formula,

you would say, OK, I have minus 1/2 z hat dot kern of something. And if I take a gradient of the kern, the answer should be 0. But that's as long as this u is a well defined object.

And our task was to say that this u, then you have these dislocations, is not a well defined object, in the sense that you take the kern, and the gradient then you would get 0. So essentially, I will transport the gradient all the way over here, and the part of u that will survive that is the one that is characterized by this dislocation feat.

Now, you can see that this object kind of looks like a Laplacian of this distortion. It's two the derivatives of this distortion field that had this logarithm in it. And when you

take two derivatives of a logarithm, you get the delta function.

So if you do things correctly, you will find that this answer here becomes a sum over i v i delta function of x minus xi. So basically, each dislocation at location x i-- again, depending on its v being in each direction-- gives a contribution to the gradient of theta.

And if I were to take the gradient of the expression that I have over here, the gradient of this object is also-- this is like the field that you have for the logarithmic potential-- will give you the data function. So that's where the similarity comes.

So the full answer comes out to be-- if you have a sum over the dislocations, the sum over the distortion fields that each one of them is causing-- and you will have a form such as this.

Yes?

AUDIENCE: Should the denominator be squared?

PROFESSOR: Yes, that's right. The potential goes logarithmically. The field, which is the gradient of the potential, falls off as 1 over separation. So since I put the separation out there, I have to put the separation squared.

So you can see that the singular part, the part that arises from dislocations-- if I have a soup of dislocations, I can figure out what theta is. Now what I did look for-- actually, I was kind of hinting at that-- if I take the gradient of theta-- and I forgot to put the factor of 1/2pi here-- does the 4 vertices that had charged to pi-- I had the potential. That was 1/r. So for dislocations it becomes d/2pi.

If I take the gradient, then the gradient translates to sum over pi, the i data function of x minus x i-- the expression that I have written over there. And if I do the fullier transform, you see what I did over here was essentially to look at theta in fullier space. So let's do something similar here.

So when I do the fullier transform of this, I will get pi q-- the fullier transform of this angular feat. And on the right hand side, what I would get is essentially the fullier transform of the field of dislocations. So I have defined my v of q to be sum over i into the i q dot position of the i dislocation, the vector that characterizes the dislocation.

And it would make sense to also tap into the normalization that gives 1 over the square root of area. If you don't do that, then at some other point you have to worry about the normalization.

So if I just multiply both sides by q-- and I think I forgot the minus sign throughout, which is not that important-- but theta tilde of q becomes i q dot b of q, divided-- maybe I should've been calling this b tilde-- divided by q squared.

So this is important. Essentially, you take the collection of dislocations in this picture and you calculate what the fullier transform is, call that the tilde of q. Essentially, you divide by 1 factor of q, and you can get the corresponding angle of feat.

Now what I needed to evaluate for here was the average of theta tilde of q squared. And you can see that if I write this explicitly, let's say, q i for be tilde i 4, then the two of them I will get q beta b tilde of beta. And then I would have a q to the side.

And the average over here becomes the average over all contributions of these dislocations that I can put across my system. Now, explicitly I'm interested in the limit where q goes to 0.

So these things depend on q. What I'm interested in is the limit as q goes to 0, especially what happens to this average. It becomes-- multiplying two of these things together-- actually, in the limit where q goes to 0, what I have is the sum over all of the b's.

So in the limit where q goes to 0, this becomes an integral or sum. It doesn't matter which one of them I write. q has gone to 0, so I basically need to look at the average

of the alpha of x, the beta of x [INAUDIBLE], divided by area.

So what is there in the numerator? We can see that in the numerator, sq goes to 0. What I'm looking at is the sum of all of these dislocations that I have in the system.

Now the average up the sum is 0, because in all of our calculations, we've been restricting the configurations that we moved from. Because if I go beyond that strategy, it's going to cost too much.

But what I'm looking at is not the average of b, which is 0, but the average of b squared, which is the variance. So essentially I have a system that has a large area, a. It is on average neutral. And the question is, what is the variance of the net charge.

And my claim is that the variance of the net charge is, by central limit theorem, proportional to the area-- actually, it is proportional to the units that are independent from each other. So roughly I would expect that in this high temperature phase, I have a correlation that is c.

And within each portion of side c, will be neutral. But when I go within things that are more than c apart, there's no reason to maintain the strategy. So overall I have something like throwing coins, but at each one of them, the average is 0, with probability being up or down.

But when I look at the variance for the entire thing, the average will be proportional to the area in units of these things that are independent of each other. It was from the normalization factor of 1 over area.

And these, really, I should write as a proportionality, because I don't know precisely what the relationship between these independent sides that correlation [INAUDIBLE]. But they have to be roughly proportional.

So what do you compute? You compute that the limit as q goes to 0, of the average of my theta tilde of q squared is a structure such as this.

I forgot to put one more thing here. I don't expect to be any correlations between

the x component and the y component of this answer-- the variance, the covariance of the dislocations in one direction and the other direction-- so I put the delta function there.

If I put this over here, I would get the q squared divided by q to the 4th. So I will get a 1 over4 q squared. And I have the c squared and then some unknown coefficient up here.

So it's interesting, because we started without thinking about dislocations, just in terms of the distortion field. And we said that this object is related to the angle. And indeed, we had this distortion, that energy cost of distortions is proportional to angle squared. And that angle, therefore, is not the Goldstone mode because it doesn't go like q squared.

Now we go to this other phase now, with dislocations all over the place, and we calculate the expectation value of theta squared. And it looks like it came from a theory that was like Goldstone modes.

So you would say that once I am in this phase, where the dislocations are unbound, there is an effective energy cost for these changes in angle that is proportional to the radiant of the angle squared.

So that means fullier space, this would go to k a over 2, integral into q 2 pi squared, q squared theta tilde of q squared. So that if you had this theory, you would definitely say that the expectation value of theta tilde of q squared is 1 over k a q squared.

The variance is k a q squared invers. You compare those two things and you find that once the dislocations have unbound, and there is a correlation lend that essentially tells you how far the dislocations are talking to each other and maintaining neutrality, that there is exactly an effective stiffness, like a Goldstone note, for angular distortions, that is proportional to c squared.

And hence, if I were to look at the orientation of all their correlations, I would

essentially have something like expectation value of theta q squared, which is 1 over q squared. If I fully transform that, I get the log. And so I will get something that falls off in the distance to some other exponent, if I recall [INAUDIBLE].

If I have a true liquid-- in a liquid, again, maybe in a neighborhood of seven or eight particles, neighbors, et cetera, they talk to each other and the orientations are correlated. But then I go from one part of the liquid to another part of the liquid, there is no correlation between bond angles. I expect these things to decay exponentially.

So what we've established is that neither the phonon nor the dislocations are sufficient to give the exponential decay that you expect for the bond. So this object has quasi long range order, versus what I expect to happen in the liquid, which is exponential of minus x over psi.

So the unbinding of dislocations gives rise to the new phase of matter that has this quasi long range order in the orientations. It has no positional order. It's a kind of a liquid crystal that is called a hexatic.

Yes?

AUDIENCE: So your correlation where you got 1 over k q squared, doesn't that assume that you're allowing the angle to vary in minus [INAUDIBLE] when you do your averaging? What about the restriction--

PROFESSOR: OK, so what is the variance of the angle here? There's a variance of the angle that is controlled by this 1 over k a. So if I go back and calculate these in real space, I will find that if I look at theta at location x minus theta at location 0, the answer is going to go like 1 over k logarithm x.

So what it says is that if things are close enough to each other-- and this is in units of 1/a-- up to some factor, let's say log 5, et cetera. So I don't go all the way to infinity. The fluctuations in angle are inversely set by a parameter that we see as I approach right after the transition is very large.

So in the same sense that previously for the positional correlations I had the temperature being small and inverse temperature being large, limiting the size of the translational fluctuations, here the same thing happens for the bond angle fluctuations. Close to the transitions, they are actually small.

So the question that you asked, you could have certainly also asked over here. That is, when I'm thinking about the distortion field, the distortion field is certainly going to be limited. If it becomes as big as this, then it doesn't make sense.

So given that, what sense or what justification do I have in making these Gaussian integrals? And the answer is that while it is true that it is fluctuating, as I go to low temperature, the degree of fluctuations is very small.

So effectively what I have is that I have to integrate over some finite interval a function that kind of looks like this. And the fact that I replaced that with an integration from minus infinity to infinity rather than from minus a to a just doesn't matter.

So we know that ultimately we should get this, but so far we've only got this, so what should we do? Well, we say OK, we encountered this difficulty before in something that looked like an angle in the xy model-- that low temperature had power-law decay, whereas we knew that at high temperatures they would have to have exponential decay. And what we said was that we need these topological defects in angle.

So what you need-- topological defects-- or in our case, theta is a bond angle. And these topological defects in the bond angle have a name. They're called disconnections.

And very roughly they correspond to something like this. Suppose this is the center one of these discriminations, and then maybe next to this, here I have locally at the distance r-- if I look at a point, I would see that the bonds that connect it to its

neighbor have an orientation such as the one that I have indicated over here.

Now what I want to do is, as I go around and make a circuit, that this angle theta that I have here to be 0, rotates and comes back up to 60 degrees. So essentially what I do is I take this line and I gradually shift it around so that by the time I come back, I have rotated by 60 degrees. It's kind of hard for me to draw that, but you can imagine what I have to do.

So what I need to do is to have the integral over a circuit that encloses this discrimination such that when I do a d s dotted with the gradient of the bond orientational angle, I come back to pi over 6 times some integer. And again, I expect the [INAUDIBLE] dislocations that correspond to minus plus 1.

Then the cost of these is obtained by taking this distortion fee, gradient of theta, whose magnitude at a distance r from the center of this object is going to be 1 over 2 pi r times pi over 6 times whatever this integer n is. And then if I substitute this 1 over r behavior in this expression, which is the effective energy of this entity, I will get the logarithmic cost for making a single disclination.

Which means that at low temperature, I have to create disclination pairs. And then there will be an effective interaction between disclination pairs, that is [INAUDIBLE] in exactly the same way that we calculated for the x y model.

So up to just this minor change that the charge of a refect is reduced by a factor of six, this theory is identical the theory of the unbinding of the x y model [INAUDIBLE] defects.

Yes?

AUDIENCE: Why is it pi over 6 and not 2 pi over 6?

PROFESSOR: You are right. It should be 2 pi over 6. Thank you.

Yes?

AUDIENCE: So when you were saying that the-- so the distance of this hexatic phase would require the dislocations to occur to [INAUDIBLE] [INAUDIBLE] orientational defects. Is there an analogous case where-- I guess you can't have dislocations in the orientation without the dislocation--

PROFESSOR: So if you try to make these objects in the original case, in the origin of lattice, you will find that their cost grows actually like l squared log l, as opposed to dislocations, whose cost only grows as log l. So these entities are extremely unlikely to occur in the original system.

If you sort of go back and ask what they actually correspond to, if you have a picture that you have generated on the computer, they're actually reasonably easy to identify. Because the centers of these disclinations correspond to having points, that have, rather than 6 neighbors, 5 or 7 neighbors.

So you generate the picture, and you find mostly you have neighborhoods with 6 neighbors, and then there's a site where there's 5 neighbours, and another site that's 7 neighbors. 5 and 7 come more or less in pairs, and you can identify these disclination pairs reasonably easy.

So at the end of the day, the picture that we have is something like this. We are starting with the triangular lattice that I drew at the beginning, and you're increasing temperature. We're asking what happens.

So this is 0 temperature. Close to 0 temperature, what we have is an entity that has translational quasi long range order. So this quantity goes like 1 over x to this power a to g. Whereas the orientations go to a constant.

Now, this a to g is there because there's a shear modulus. And so throughout this phase, I have a shear modulus. The parameter that I'm calling u, I had scaled inversely with temperature. So I have this shear modulus u that diverges once we scale by temperature as 1 over temperature.

But then as I come down, the reduction is more than one over temperature because I will have this effect of dislocations appearing in pairs, and the system becomes

softer. And eventually you will find that there's a transition temperature at which the shear modulus drops down to 0.

And we said that near this transition, there is this behavior that mu approaches mu c, whatever it is, with something-- let's call this t 1. T 1 minus t to this exponent mu bar which was planned to be 6963.

Now, once we are beyond this temperature t 1, then our positional correlations decay exponentially at some correlation, like c. And this c is something that diverges on approaching this transition.

So basically I have a c that goes up here to infinity. And the fact that if we calculate the c, it diverges according to this strange formula that was 1 over t minus t 1 to this exponent mu bar Very strange type of divergence.

But then, associated with the presence of this c is the fact that when you look at the orientational correlations, they don't decay as an exponential but as a power-law 8 of c.

And this 8 of c is related to this k a, and falls off as 1 over c squared. So here it diverges as you approach this transition.

Now, as we go further and further on, the disclinations will appear-- disclinations with [INAUDIBLE] resolve of the angles to be parallel to each other. And there's another transition that is [INAUDIBLE], at which this is going to suddenly go down to 0. And close to here, we have that a to c reaches the critical value of 1/4 v to square root of-- let's call it t 2-- v the square root singularity.

And then finally we have the ordinary liquid phase, where additionally I will find that psi 6 of x psi star 6 of 0 decays exponentially. Let's call it psi 6. And this psi 6 is something that will diverge of this transition as an exponential of minus 1 over square root of t minus t2.

So this is the current scenario of how melting could occur for a system of particles in

two dimensions. If it is a continuous phase transition, it has to go through these two transitions with the intermediate exotic phase.

Of course, it is also possible-- and typically people were seeing numerically when they were doing hearts, spheres, et cetera, that there is a direct transition from here to here, which is discontinuous, like you have in three dimensions. So that's an area of a discontinuous transition that is not ruled out.

But if you have continuous transitions, it has to have this intermediate phase in [INAUDIBLE].

Any questions?

Yes?

AUDIENCE: [INAUDIBLE] so the red one is mu. The yellow one is theta psi, and the purple one is [INAUDIBLE].

PROFESSOR: The correlation, then, that I would put here. So they are three different entities.

So throughout the course, we have been thinking about systems that are described by some kind of an equilibrium probability distribution. So what we did not discuss is how the system comes to that equilibrium. So we're going to now very briefly talk about dynamics, and the specific type of dynamics that is common to condensed matter systems at finite temperature, which I will call precipative dynamics.

And the prototype of this is a Brownian particle that I will briefly review for you. So what you have is that you have a particle that is within some kind of a solvent, and this particle is moving around. So you would say, let's for simplicity actually focus on the one direction, x.

And you would say that the mass of the particle times its acceleration is equal to the forces that it's experiencing. The forces-- well, if you are moving in a fluid, you are going to be subject to some kind of a dissipative force which is typically portional to

your velocity.

If you, for example, solve for the hydrodynamic of a sphere in a fluid, you find that mu is related to viscosity inversely to the size of the particle, et cetera. But that behavior is generic. You're not going be thinking about that.

Now suppose that additionally I put some kind of an optical trap, or something that tries to localize this potential. So then there would be an additional force v 2, the derivative of the potential with respect to x.

And then we are talking about Brownian particles. Brownian particles are constantly jiggling. So there is also a random force that is a function of time.

Now we are going to be interested in the dynamics that is very much controlled by the dissipation term. And acceleration we can forget. And if we are in that limit, we can write the equation as mu-- I can sort of rearrange it slightly as-- actually, let me change location to this.

So that the eventual velocity x dot is going to be proportional to the external force. mu the coefficient that is the mobility. So mu essentially relates the force to the velocity. Of course, this is the average force.

And there is a fluctuating part, so essentially, I call mu times this to be the a times function of t.

Now, if I didn't have this external force, the fluctuations of the particles would be diffusive. And you can convince yourself that you can get the diffusive result provided that you relate the correlations of this force that fluctuating and have 0 average, the diffusion constant d of the particle in the medium through delta of t minus t.

So if their track was not there, you solve of this equation without the track and find that the prohibitive distribution for x grows as a Gaussian whose width grows with time, as d t. d therefore must be the diffusion constant.

Now, in the presence of the potential, this particle will start to fluctuate. Eventually if you wait long enough, there is a probability that it will be here, a probability to be somewhere else. So at long enough times, there's a probability p of x to find the particle. And you expect that t of x will be proportional to exponential of minus v of x divided by whatever the temperature is.

And you can show that in order to have this occur, you need to relate mu and d through the so called Einstein relation. So this is a brief review of Brownian particles.

Yes?

AUDIENCE: The average and time correlation of eta can be found by saying the potential is 0, right?

PROFESSOR: Mm hmm.

AUDIENCE: Those will still be true even if the potential is not 0, right?

PROFESSOR: Yes. So I just wanted to have an idea of where this d comes from. But more specifically, this is the important thing. That if at very long times you want to have a probability distribution coming from this equation, that has the Boltzmann form with k t, the coefficients of mu and the noise, you have to relate through the so called Einstein relation.

And once you do that, this result is true no matter how complicated this v of x is. So in general, for a complicated v of x, you won't be able to solve this equation analytically. You can only do it numerically. Yet you are guaranteed that this equation with this noise correlator will have asymptotically a probability distribution of [INAUDIBLE].

The problem that we have been looking at all along is something different. Let's say you have, let's say, a piece of magnet or some other system that we characterize, let's say, by something m of x. Again, you can do it for vector, but for simplicity, let's do it for the scalar case.

So we know, or we have stated, that subject to the symmetries of the system, I know the probability. For some configuration of this field is governed by a form, let's say, that has Landau Ginzburg character.

So that has been our starting point. We have said that I have a prohibitive distribution that is of this form. So that statement is kind of like this statement. But the way that I came to that statement was to say that there was a degree of freedom x, the position of the particle, that was fluctuating subject to forces and external variables from the particles of the fluid, that was given by this so called Langevin equation.

So I had a time dependent prescription that eventually went to the Boltzmann way that I wanted. Here I have started with the final Boltzmann weight. And the question is, can I think about a dynamics for a field that will eventually give this state.

So there are lots and lots of different dynamics that I can impose. But I want to look at the dynamics that is closest to the Brownian particle that I wrote, and that's where this word dissipative comes. So among the universe of all possible dynamics, I'm going to look at one that has a linear time derivative for the field n.

So this is the analog of the x dot. And so I write that it is equal to some coefficient, that with their minds, the ease with which that particle-- well the field of that location x changes as a function of the forces that is exerted on it.

I assume that mu is the same across my system. So here I'm already assuming there's no x dependence. This system is uniform. And then there was a d v by d x. So v was ultimately the thing that was appearing in the Boltzmann weight. So clearly the analog of the v that I have is this Landau Ginzburg.

So I will do a function of the derivative of this quantity that I will call beta h, with respect to m of x. Again, over there, I had one variable, x. You can imagine that I could have had a system where two particles, x 1 and x 2, also have an interaction among them.

Then the equation that I would have had over there would be the force that is acting on particle 1, by taking the total potential-- which is the external potential plus the potential that comes from the inter particle interaction. So I would have to take a derivative of the net potential energy v, divided with respect to either x 1 or x 2 to calculate the force on the first one or the second one.

So here for a particular configuration m of the field across the system, if I'm interested in the dynamics of this position x, I have to take this total internal potential energy, and take the derivative with respect to the variable that is sitting on that side. So that's why this is a functional derivative of this end.

And then I will have to put a noise, eta. Well, again, if I had multiple particles, I would subject each one of them to an independent noise. So at each location, I have an independent noise. So the noise is a function of time, but it is also wearing across my system.

So if I take that form and do the functional derivative-- so if I take the derivative with respect to m of x, I have to take the derivative of these objects. So I will have minus derivative of t m squared is t m. The u m to the 4th is 4 u m q, and so forth.

Once I have gotten rid of these terms, then I would have terms that depend on the gradient. So I would have minus the derivative of this object with respect to the gradient. So here I would get k gradient of m. And then the next term would be Laplacian derivative, with respect to Laplacian. So I would put l Laplacian of m, and so forth with the methodologies of taking function derivatives. And then I have the noise.

So this leads to an equation which is called a time dependent, Landau Ginzberg. Because we started with the Landau Ginzberg weight, and this equation, as we see shortly, subject to similar restrictions as we had before, will give us, eventually, this probability distribution.

This is a difficult equation in the same sense that the original Landau Ginzberg is difficult to look at correlations, et cetera. This is a nonlinear equation, causes

various difficulties, and we need approaches to be able to deal with the difficult, non linealities.

So what we did for the Landau Ginzburg was to initially get insights and simplify the system by focusing on the linearized or Gaussian version. So let's look at the version of this equation that is linearized.

And when it is linearized, what I have on the left hand side is d m by d t. What I have on the right hand side is mu. I have t m. I got rid of the non linear term, so the next term that I will have will be k Laplacian of m, and then will be minus l 4th derivative of m, and so forth. And then there will be a noise [INAUDIBLE].

One thing that I can immediately do is to go to fullier transform. So m of x goes to m theta of q. And if I do that, but not fullier transform in time, I will get that the time derivative of m tilde of q is essentially what I have here.

And I forgot the minus that I have here. So this minus is important. And then this becomes negative, this becomes positive.

So that when I go fullier transform, what I will get is minus t plus k q squared plus l q to the 4th, and so forth. And tilde of q with this mu out front. And then the fullier transform of what my noise is.

First thing to note is that even in the absence of noise, there is a set of relaxation times. That is, for eta it was to 0. Or in general, I would have n tilde of q and p.

I can solve this equation kind of simply. It is the m by d t is some constant times n-- let's call it gama of q-- which has dimensions of 1 over time. So I can call that 1 over tau of q. If I didn't have noise, if I started with some original value at time 0, it is going to decay exponentially with this characteristic time.

And once I have noise, it is actually easy to convince yourself that the answer is 0 2 t d t prime e to the minus this gamma of q or inversify. Tau of q times eta of q i t prime.

So you see that you have a hierarchy of relaxation times, ta of q, which are 1 over u t plus k q squared, and so forth, which scale in two limits. Either the wavelength lambda, which is the inverse of q is much larger than the correlation length-- and the correlation length of this model you have seen to be the square root of t over k, square root of k over t-- or the other limit, where lambda is much less than c.

In this limit, where we are looking at modes that are much shorter than the correlation length, this term is dominant. This becomes 1 over mu k q squared. In the other limit, it goes to a constant 1 over u t.

So this linear equation has a whole bunch of modes that can be characterized by their wavelength or their wave number. You find that the short wavelength modes have this characteristic, time, that becomes longer and longer as the wavelength increases. So if you make the wavelength twice as large, and you want to relax a system that is linearly twice as large, this says that it will take 4 times longer. Because the answer goes like lambda squared.

Whereas eventually you reach the size of the correlation length. Once you are beyond the size of the correlation length, it doesn't matter. It's the same time. But the interesting thing, of course, to us is that there are phase transitions that are continuous. And close to that phase transition, the correlation length goes to infinity. Which means that the relaxation time also will go to infinity.

So according to this theory, there's a particular divergence as 1 over t minus t c. But it will be modified, and as I will discuss next time, this is only-- even within to dissipative class-- one type of dynamics that you can have. And there are additional dynamics, and this system characterizes criticality as single universality class in statics. There are many dynamic universality classes that correspond to this same static.