Problem Set 1 Solutions

1. The Lagrangian in question is

$$\mathcal{L} = -i\bar{\psi}(\partial\!\!\!/ - m)\psi \tag{1}$$

where ψ is a spinor doublet.

(a) Since the generators T_a are Hermitian, the transformation laws are

$$\delta\psi = i\epsilon_a T_a \psi \tag{2}$$

$$\delta\bar{\psi} = -i\bar{\psi}\epsilon_a T_a \tag{3}$$

The Lagrangian is quite trivially seen to be invariant

$$\delta\mathcal{L} = -i\delta\bar{\psi}(\partial\!\!\!/ - m)\psi - i\bar{\psi}(\partial\!\!\!/ - m)\delta\psi = -i\bar{\psi}(-i\epsilon_a T_a)(\partial\!\!\!/ - m)\psi - i\bar{\psi}(\partial\!\!\!/ - m)(i\epsilon_a T_a)\psi = 0 \ , \tag{4}$$

where we used the trivial relation $[T_a, \gamma^\mu] = 0$.

(b) We use the so called Noether method (described in e.g. Di Francesco et al.: CFT Section 2.4.2) to find the conserved current, i.e., we pretend that the transformation parameter ϵ is spacetime dependent. Then

$$\delta\mathcal{L} = (\partial_\mu \epsilon_a)\bar{\psi}\gamma^\mu T_a \psi \tag{5}$$

which implies

$$J_a^\mu = -\bar{\psi}\gamma^\mu T_a \psi \tag{6}$$

(Note, that Lie algebra indices are up and down without any regard for their placement). Using the EoMs the current can be shown to be conserved.

(c) The conserved charges are thus

$$Q_a = -\int d^3x\, \bar{\psi}(x)\gamma^0 T_a \psi(x) = \int d^3x\, \psi^\dagger(x) T_a \psi(x) \tag{7}$$

In the following we will repeatedly use the relations

$$[T_a, T_b] = i f^{abc} T_c \tag{8}$$

$$\{\psi_i(x), \psi_j^\dagger(y)\}|_{x^0=y^0} = \delta(\vec{x} - \vec{y})\delta_{ij} \tag{9}$$

where i, j are indices in the Lie algebra representation space. We write:

$$i\left[\epsilon_a Q_a, \psi_k(x)\right] = i\epsilon_a T^a_{ij} \int_{x^0=y^0 \text{choice made}} d^3x\ \left[\psi_i^\dagger(y)\psi_j(y), \psi_k(x)\right] \tag{10}$$

Now we use the relation:

$$[AB, C] = A\{B, C\} - \{A, C\}B \ , \tag{11}$$

which for our case gives ($A = \psi_i^\dagger(y),\ B = \psi_j(y),\ C = \psi_k(x)$):

$$i\left[\epsilon_a Q_a, \psi_k(x)\right] = -i\epsilon_a T^a_{ij} \int_{x^0=y^0} d^3x\ \delta(\vec{x} - \vec{y})\delta_{ik}\psi_j(y) = -i\epsilon_a T^a_{kj}\psi_j(x) = -\delta_\epsilon \psi_k(x) \tag{12}$$

(d) We wish to compute $[Q_a, Q_b]$. We write down the commutator explicitly

$$[Q_a, Q_b] = T^a_{ij} T^b_{kl} \int_{x^0=y^0 \text{choice made}} d^3x\, d^3y \, \left[\psi_i^\dagger(y)\psi_j(y), \psi_k^\dagger(x)\psi_l(x)\right] \tag{13}$$

We use the relation

$$[AB, CD] = A\{B, C\} D - AC\{B, D\} + \{A, C\} DB - C\{A, D\} B \ , \tag{14}$$

which for our case gives ($A = \psi_i^\dagger(y),\ B = \psi_j(y),\ C = \psi_k^\dagger(x),\ D = \psi_l(x)$):

$$[Q_a, Q_b] = T^a_{ij} T^b_{kl} \int_{x^0=y^0} d^3x\, d^3y \, \delta(\vec{x}-\vec{y}) \left(\psi_i^\dagger(x)\psi_l(x)\delta_{jk} - \psi_k^\dagger(x)\psi_j(x)\delta_{il}\right) = \int d^3x \, \psi^\dagger(x) \left[T_a, T_b\right] \psi(x) = i\epsilon_{abc} Q_c \tag{15}$$

as expected.

(e) We will use the Hadamard lemma that can be found e.g. under the wikipedia article Baker-Campbell-Hausdorff formula, i.e.

$$e^X Y e^{-X} = Y + [X, Y] + \frac{1}{2}[X, [X, Y]] + \dots \tag{16}$$

We use the property of Qs derived in this problem

$$[i\Lambda_a Q_a, \psi(x)] = -i\Lambda_a T_a \psi(x) \tag{17}$$

$$[i\Lambda_b Q_b, [i\Lambda_a Q_a, \psi(x)]] = \Lambda_a \Lambda_b T^a_{ij} [Q_b, \psi_j(x)] = -\Lambda_a \Lambda_b T^a_{ij} T^b_{jk} \psi_k(x) = -(\Lambda_a T_a)^2 \psi(x) \tag{18}$$

$$\dots \tag{19}$$

and plugging into (16) we get

$$\hat{U}^\dagger \psi \hat{U} = \psi + i\Lambda_a T_a \psi(x) + \frac{1}{2}(\Lambda_a T_a)^2 \psi(x) + \dots = \exp\left(i\Lambda_a T_a\right)\psi = U\psi \ . \tag{20}$$

2. (a) Take the parallelogram to have it's left corner at x, an upper edge given by the vector a^μ and a lower edge given by b^μ. The quantity we are interested in is

$$U(x, x) = exp\left[ie \oint A_\mu dx^\mu\right] \tag{21}$$

infintesimally. Let's circle around the parallelogram, counterclockwise, starting at x. Since we are interested in the loop infintesimally, the integral along each path just becomes A multiplied by an infintesimal parameter. We evaluate A at the midpoint of each segment of the parallelogram. We take the vectors to be

$$a^\mu = \epsilon_a e_a^\mu \tag{22}$$

$$b^\mu = \epsilon_b e_b^\mu \tag{23}$$

with infintesimal ϵ_a, ϵ_b and unit basis vectors e_a, e_b. For convenience we take $x = 0$, and any time A is written without an argument, i.e. A^μ, we mean $A^\mu(0)$. Taylor expanding to $O(\epsilon^2)$, the contribution from the first segment to the integral is

$$\epsilon_b e_b^\mu A^\mu(\frac{\epsilon_b}{2}\vec{e}_b) = \epsilon_b e_b^\mu (A^\mu + e_b^\nu \partial_\nu A^\mu \frac{\epsilon_b}{2}) \tag{24}$$

The contribution from the second segment is

$$\epsilon_a e_a^\mu A^\mu(\epsilon_b \vec{e}_b + \frac{\epsilon_a}{2}\vec{e}_a) = \epsilon_a e_a^\mu (A^\mu + e_b^\nu \partial_\nu A^\mu \epsilon_b + e_a^\nu \partial_\nu A^\mu \frac{\epsilon_a}{2}) \tag{25}$$

From the third,

$$-\epsilon_b e_b^\mu A^\mu(\epsilon_a \vec{e}_a + \frac{\epsilon_b}{2}\vec{e}_b) = -\epsilon_b e_b^\mu (A^\mu + e_a^\nu \partial_\nu A^\mu \epsilon_a + e_b^\nu \partial_\nu A^\mu \frac{\epsilon_b}{2}) \tag{26}$$

and from the fourth

$$-\epsilon_a e_a^\mu A^\mu(\frac{\epsilon_a}{2}\vec{e}_a) = -\epsilon_a e_a^\mu (A^\mu + e_a^\nu \partial_\nu A^\mu \frac{\epsilon_a}{2}) \tag{27}$$

Adding everything together, we get

$$\oint A_\mu dx^\mu = \epsilon_a \epsilon_b e_a^\mu \epsilon_b^\nu F_{\mu\nu} + O(\epsilon^3) = a^\mu F_{\mu\nu} b^\nu + O(\epsilon^3) \tag{28}$$

for the abelian field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{29}$$

And so

$$U(x,x) = 1 + iea^\mu F_{\mu\nu} b^\nu + O(\epsilon^3) \tag{30}$$

Note, that you might think there's an $O(\epsilon^2)$ coming from the next term in the exponential expansion of (21), but we will verify in the next part of the problem that this term vanishes in the abelian case.

(b) In the non-abelian case, we are interested in

$$U(x,x) = \mathcal{P}exp\left[ig \oint A_\mu dx^\mu\right] \tag{31}$$

$$= \left[1 + ig \oint A_\mu dx^\mu + \frac{(ig)^2}{2} \oint dx^\mu \oint dy^\mu \mathcal{P}(A_\mu(x) A_\nu(y)) + O(\epsilon^3)\right] \tag{32}$$

where $\mathcal{P}$ is the path ordering symbol, putting higher values along the path to the left of the expression. The calculation of $ig \oint A_\mu dx^\mu$ to $O(\epsilon^2)$ is exactly the same as in the abelian case.

$$ig \oint A_\mu dx^\mu = iga^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) b^\nu + O(\epsilon^3) \tag{33}$$

The new complication is the calculation of

$$\frac{(ig)^2}{2} \oint dx^\mu \oint dy^\mu \mathcal{P}(A_\mu(x) A_\nu(y)) \tag{34}$$

by choosing pairs of sides on the parallelogram. For example, integrating over the pair of the first side and the second side gives

$$e_b^\mu \epsilon_b e_a^\nu \epsilon_a \mathcal{P}\left[A^\mu(\frac{\epsilon_b}{2}\vec{e}_b) A^\nu(\epsilon_b \vec{e}_b + \frac{\epsilon_a}{2}\vec{e}_a)\right] = e_b^\mu \epsilon_b e_a^\nu \epsilon_a A_\nu A_\mu + O(\epsilon^3) \tag{35}$$

because the second side is further along the path than the first side. Doing this for all pairs of sides gives

$$\frac{(ig)^2}{2} \oint dx^\mu \oint dy^\mu \mathcal{P}(A_\mu(x) A_\nu(y)) = g^2 a^\mu b^\nu \left[A_\mu, A_\nu\right] \tag{36}$$

Combining this with (33) gives

$$U(x,x) = 1 + ig \oint A_\mu dx^\mu + \frac{(ig)^2}{2} \oint dx^\mu \oint dy^\mu \mathcal{P}(A_\mu(x) A_\nu(y)) + O(\epsilon^3) = 1 + iga^\mu F_{\mu\nu} b^\nu + O(\epsilon^3) \tag{37}$$

now with F as the non-abelian field strength. For an arbitrary infintesimal loop, this will generalize to

$$U(x,x) = 1 + \frac{1}{2} ig\sigma^{\mu\nu} F_{\mu\nu} + O(\epsilon^3) \tag{38}$$

where $\sigma^{\mu\nu}$ is the surface area element.

3. We want to show that

$$0 = D_\mu F_{\nu\lambda} + D_\lambda F_{\mu\nu} + D_\nu F_{\lambda\mu} = \epsilon^{\alpha\mu\nu\lambda} D_\mu F_{\nu\lambda} \tag{39}$$

where α is different from μ, ν, λ. We will show that the RHS can be written as

$$\epsilon^{\alpha\mu\nu\lambda} \left[D_\mu, [D_\nu, D_\lambda]\right] = 0 \ , \tag{40}$$

which vanishes by the Jacobi identity. We now act with (40) on an arbitrary vector in the fundamental representation $\phi(x)$ and use $[D_\nu, D_\lambda] = -igF_{\nu\lambda}$:

$$0 = \epsilon^{\alpha\mu\nu\lambda} \left[D_\mu, [D_\nu, D_\lambda]\right] \phi = -ig\epsilon^{\alpha\mu\nu\lambda} \left[D_\mu, F_{\nu\lambda}\right] \phi \ . \tag{41}$$

We use the defining property of the covariant derivative $D(F\phi) = (DF)\phi + F(D\phi)$, which can be checked to be true in this special case by calculation to simplify our formula to

$$0 = -ig\epsilon^{\alpha\mu\nu\lambda} \left[D_\mu, F_{\nu\lambda}\right] \phi = -ig\epsilon^{\alpha\mu\nu\lambda} \left(D_\mu \left(F_{\nu\lambda}\phi\right) - F_{\nu\lambda} D_\mu \phi\right) = -ig\epsilon^{\alpha\mu\nu\lambda} \left(D_\mu F_{\nu\lambda}\right) \phi \ . \tag{42}$$

Because ϕ is arbitrary we proved that (39) holds.

One could of course explicitly calculate out all terms and see that (39) is true, but this is a shorter way. You can test your understanding of covariant derivatives by going through the calculation. You should pay attention to the way the covariant derivative acts on a vector in the adjoint representation:

$$(D_\mu M)^a = \partial_\mu M^a - igA^b_\mu (T^{(adj)}_b M)^a = \partial_\mu M^a + if^a_{bc} A^b_\mu M^c = \left(\partial_\mu M - ig\left[A_\mu, M\right]\right)^a \ . \tag{43}$$

4. We will stick with the conventions used in class opposite to what Peskin uses. Here we first demonstrate that the Feynman propagator can be written as an integral of the propagator for the non-relativistic Schrödinger equation given by Peskin. From the definition of the Feynman propagator

$$\left(-\partial^2 + m^2 - i\epsilon\right) D_F(x, y) = -i\delta(x - y) \tag{44}$$

we have

$$D_F(x, y) = -i\left\langle x \middle| \frac{1}{\hat{p}^2 + m^2 - i\epsilon} \middle| y \right\rangle = \int_0^\infty dT \left\langle x | e^{-i\hat{H}T} | y \right\rangle, \qquad \hat{p}_\mu = -i\partial_\mu, \qquad \hat{H} = \hat{p}^2 + m^2 - i\epsilon \tag{45}$$

Note that $D(x, y, T) = \left\langle x | e^{-i\hat{H}T} | y \right\rangle$ is precisely the propagator for the non-relativistic Schrodinger equation given by Peskin. We thus obtain

$$D_F(x, y) = \int_0^\infty dT \, D(x, y, T) \ . \tag{46}$$

If we want to represent $D(x, y, T)$ as a path integral we can easily do that by determining the Lagrangian corresponding to $\hat{H}$ by Legendre transformation.

$$L = \frac{1}{4}\left(\frac{dz}{dt}\right)^2 - m^2 \tag{47}$$

$$D(x, y, T) = \left\langle x | e^{-i\hat{H}T} | y \right\rangle = \int_{z(0)=y,\ z(T)=x} Dz(t) \ \exp\left[i \int_0^T dt \ \left(\frac{1}{4}\left(\frac{dz}{dt}\right)^2 - m^2\right)\right] \tag{48}$$

Here the particle with the non-relativistic mass given by $\frac{1}{2}$ moves in four dimensions (3 spatial and 1 time of the Minkowski spacetime) with an additional constant potential m^2.

(a) The propagator for a one-dimensional non-relativisitic Schrodinger equation is given by (with the non-relativistic mass given by $\frac{1}{2}$)

$$K(x,y,T) = \frac{1}{\sqrt{4\pi i T}} e^{\frac{i(x-y)^2}{4T}} \tag{49}$$

Here the particle moves in four dimensions (3 spatial and 1 time of the Minkowski spacetime) with an additional constant potential m^2, thus

$$D(x,y,T) = \frac{1}{(4\pi i T)^{\frac{3}{2}} \sqrt{-4\pi i T}} e^{\frac{i(x-y)^2}{4T} - i m^2 T} \tag{50}$$

Note that the time direction contributes to an additional $-$ sign in the prefactor. We thus find for spacelike separation (time-like separation can be obtained by analytic continuation)

$$D_F(x,y) = \frac{1}{(4\pi)^2 i} \int_0^\infty dT \, T^{-2} \, e^{\frac{i(x-y)^2}{4T} - i m^2 T} = \frac{1}{(4\pi)^2} \left(\frac{m}{|x-y|} \right) K_1(m|x-y|) \tag{51}$$

which is precisely the expression for the Feynman propagator in coordinate space.

(b) The propagator in a background Maxwell field satisfies the equation:

$$\left(-D^2 + m^2\right) D_F^{(A)}(x,y) = -i\delta(x-y), \qquad D_\mu = \partial_\mu - ieA_\mu = i(\hat{p}_\mu - eA_\mu) \tag{52}$$

From the discussion of (72)–(46) we then conclude that

$$D_F^{(A)}(x,y) = \int_0^\infty dT \, D^{(A)}(x,y,T), \qquad D^{(A)}(x,y,T) = \left\langle x | e^{-iTH^{(A)}} | y \right\rangle \tag{53}$$

with

$$H^{(A)} = (\hat{p} - eA)^2 + m^2 \tag{54}$$

Since the corresponding Lagrangian is given by

$$L = \frac{1}{4} \left(\frac{dz}{dt} \right)^2 + eA_\mu \frac{dz}{dt} - m^2 \tag{55}$$

$D^{(A)}(x,y,T)$ has the standard path integral representation given by

$$\begin{aligned} D^{(A)}(x,y,T) &= \int_{z(0)=y,\ z(T)=x} Dz(t) \ \exp\left[i \int_0^T dt \ \left(\frac{1}{4}\left(\frac{dz}{dt}\right)^2 + eA^\mu \frac{dz_\mu}{dt} - m^2 \right) \right] \\ &= \int_{z(0)=y,\ z(T)=x} Dz(t) \ \exp\left[i \int_0^T dt \ \left(\frac{1}{4}\left(\frac{dz}{dt}\right)^2 - m^2 \right) + ie \int_0^T dt \ A^\mu \frac{dz_\mu}{dt} \right] , \end{aligned} \tag{56}$$

where in the las step we separated the A_μ dependence into a Wilson line.
As asked by the problem we now verify the above path integral representation does satisfy the Schrodinger equation (for $T \neq 0$ and $x \neq y$)

$$\left(i\partial_T - H^{(A)} \right) D^{(A)}(x,y,T) = 0 \tag{57}$$

Note that H only acts on x. This is easily carried out by using

$$D^{(A)}(x,y,T+\epsilon) = \frac{1}{i(4\pi\epsilon)^2} \int d^4z \ \exp\left(i\epsilon \left[\frac{1}{4}\left(\frac{x-z}{\epsilon}\right)^2 - m^2 \right] + ie(x^\mu - z^\mu) A_\mu \left(\frac{x+z}{2} \right) \right) D^{(A)}(z,y,T) \tag{58}$$

and expanding the right hand side of the equation to $\mathcal{O}(\epsilon)$. Note that as first pointed out by Feynman himself it is important to evaluate A_μ at middle point $(x+z)/2$. We use our knowledge about the path integral in general (or the Brownian motion) to note that the important domain of z will be in a $\sqrt{\epsilon}$

neighborhood of x. This can also be directly seen from the Gaussian integral itself, because it has a variance $\sigma^2 = \mathcal{O}(\epsilon)$. So we relable $z = x - \sqrt{\epsilon}u$ and expand in ϵ to get:

$$\begin{aligned}
D^{(A)}(x,y,T+\epsilon) &= \frac{1}{i(4\pi)^2}\int d^4u\,\exp\left(i\left[\frac{1}{4}u^2-\epsilon m^2\right]+ie\sqrt{\epsilon}u^\mu A_\mu\left(x-\sqrt{\epsilon}\frac{u}{2}\right)\right)D^{(A)}(x-\sqrt{\epsilon}u,y,T)\\
&= \frac{1}{i(4\pi)^2}\int d^4u\,\exp\left(\frac{i}{4}u^2\right)\left\{1+\sqrt{\epsilon}\,iu_\mu\left[eA^\mu(x)+i\partial^\mu\right]\right.\\
&\quad\left.-i\epsilon m^2+\epsilon\frac{u_\mu u_\nu}{2}\left[e^2A^\mu(x)A^\nu(x)+ie(\partial^\mu A^\nu(x))+2ieA^\mu\partial^\nu-\partial^\mu\partial^\nu\right]\right\}D^{(A)}(x,y,T)+\mathcal{O}(\epsilon^{3/2})
\end{aligned} \tag{59}$$

Now we make use of the fact that the remaining Gaussian integral has zero expectation value and hence terms with odd powers of $\sqrt{\epsilon}$ vanish:

$$\frac{1}{i(4\pi)^2}\int d^4u\,\exp\left(\frac{i}{4}u^2\right) = 1 \tag{60}$$

$$\frac{1}{i(4\pi)^2}\int d^4u\,u^\mu\exp\left(\frac{i}{4}u^2\right) = 0 \tag{61}$$

$$\frac{1}{i(4\pi)^2}\int d^4u\,u^\mu u^\nu\exp\left(\frac{i}{4}u^2\right) = -2i\eta^{\mu\nu} \tag{62}$$

$$\frac{1}{i(4\pi)^2}\int d^4u\,u^\mu u^\nu u^\lambda\exp\left(\frac{i}{4}u^2\right) = 0 \tag{63}$$

Thus we get:

$$D^{(A)}(x,y,T+\epsilon) = \left\{1-i\epsilon\left[m^2+e^2A^2(x)+ie(\partial_\mu A^\mu(x))+2ieA^\mu\partial_\mu-\partial^2\right]\right\}D^{(A)}(x,y,T)+\mathcal{O}(\epsilon^2) \tag{64}$$

$$0 = \left[i\partial_T-\left(m^2+e^2A^2(x)+ie(\partial_\mu A^\mu(x))+2ieA^\mu\partial_\mu-\partial^2\right)\right]D^{(A)}(x,y,T)\ , \tag{65}$$

which is the Schrödinger equation.

There is a technically different derivation of the same result that we are going to demonstrate in part (d).

(c) The gauge invariant Lagarangian is:

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right]-(D_\mu\Phi)^*\,D_\mu\Phi-m^2\Phi^*\Phi \tag{66}$$

$$D_\mu\Phi = \partial_\mu\Phi-igA_\mu\Phi \tag{67}$$

(d) We write the non-Abelian generalization of the result of (b) with an arbitrary ordering O:

$$D^{(A)}(x,y,T) = \int_{z(0)=y,\ z(T)=x} Dz(t)\ O\exp\left[i\int_0^T dt\ \left(\frac{1}{4}\left(\frac{dz}{dt}\right)^2-m^2\right)+ig\int_0^T dt\ \frac{dz_\mu}{dt}A^\mu(z)\right] \tag{68}$$

which is a matrix in group space. Then applying (58) to this case, it can be immediately seen that the correct Schrodinger equation is recovered only if one uses path ordering (i.e. A_μ with larger t stands to the left).

However, we will give an alternative derivation of the Schrödinger equation:

$$\begin{aligned}
i\partial_T D^{(A)}(x,y,T) &= \int_{z(0)=y,\ z(T)=x} Dz(t)\ O\left[\exp\left[iS[z]\right]\ \left(-\frac{\partial S[z]}{\partial T}\right)\right]\\
&= \int_{z(0)=y,\ z(T)=x} Dz(t)\ O\left[\exp\left[iS[z]\right]\ \left[\frac{1}{4}\left(\frac{dx}{dt}\right)^2+m^2\right]\right]
\end{aligned} \tag{69}$$

where we used the formula well known from classical mechanics, $-\partial S[z]/\partial T = H$ and plugged in the boundary conditions $z(0) = y,\ z(T) = x$. Secondly we recall the role of time-slices played in the definition of the path integral:

$$D^{(A)}(x,y,T) = \int dz\ O\left[D^{(A)}(x,z,\epsilon)D^{(A)}(z,y,T-\epsilon)\right] \quad (70)$$
$$= \int dz\ O\left[\exp\left[i\left(\frac{(x-z)^2}{4\epsilon} - m^2\epsilon\right) + ig(x-z)A^\mu(x)\right]\ D^{(A)}(z,y,T-\epsilon)\right] + \ldots$$

Thirdly we collect the terms from the Schrödinger equation:

$$0 = \left[i\partial_T - \left(m^2 + g^2A^2(x) + ig(\partial_\mu A^\mu(x)) + 2igA^\mu\partial_\mu - \partial^2\right)\right] D^{(A)}(x,y,T) \quad (71)$$
$$\partial^2 D^{(A)}(x,y,T) = \partial_x^2 \int dz\ O\left[\exp\left[i\left(\frac{(x-z)^2}{4\epsilon} - m^2\epsilon\right) + ie(x-z)A^\mu(x)\right]\ D^{(A)}(z,y,T-\epsilon)\right] + \ldots$$
$$= \int_{z(0)=y,\ z(T)=x} Dz(t)\ O\left[\exp\left[iS[z]\right]\ \left[-\frac{1}{4}\left(\frac{dx}{dt}\right)^2 - g\frac{dx_\mu}{dt}A^\mu(x) - g^2A^2(x)\right]\right] \quad (72)$$
$$-\left(ig(\partial_\mu A^\mu(x)) + 2igA^\mu(x)\right)\partial_\mu D^{(A)}(x,y,T) = \int_{z(0)=y,\ z(T)=x} Dz(t)\ A^\mu(x) O\left[\exp\left[iS[z]\right]\ \left[g\frac{dx_\mu}{dt} + 2g^2A_\mu(x)\right]\right]$$
$$-g^2A^2(x)D^{(A)}(x,y,T) = \int_{z(0)=y,\ z(T)=x} Dz(t)\ (-g^2A^2(x))O\exp\left[iS[z]\right] \quad (73)$$

To have (69), (72)-(73) add up to zero we need the following relation to be true (schematically):

$$A(x)O\left[\exp[iS]\ldots A(x)\ldots\right] = O\left[\exp[iS]\ldots A(x)\ldots A(x)\ldots\right] \quad (74)$$

This is true only if the operation O is the path ordering, P. Then the Schrödinger equation is indeed satisfied.