PROFESSOR: OK. In That case, let's get started. First with our review of what we talked about last time. We began by talking about the dynamics of a flat radiation-dominated universe, which is very straightforward. We start with Friedmann's equation for a flat universe. For radiation, rho is proportional to one over a to the fourth. So that gets us a dot over a squared is equal to a constant over a to the fourth.

Rearranging that we can write it as a times da is equal to the square root of that constant times dt. And then we can just integrate both sides. So you get one half a squared is equal to the square root of the constant times t, plus a new constant, constant prime.

OK, then we say that we can adjust this constant prime by resetting our clocks. And we haven't said anything yet that determines how our clocks are set. So the standard convention is to set your clocks so that t equals 0 is the time when the scale factor is zero. And that corresponds to constant prime equals zero, as one can see by just looking at that equation. So that's what we do when the constant prime term disappears.

And then we don't care about the constant of proportionality anyway. For a flat universe the constant of proportionality is completely meaningless. It just tells you how many meters per notch you're dealing with, so it defines your notch, but otherwise has no physical meaning whatever. So the important bottom line is that a of t is proportional to the square root of t for a flat [INAUDIBLE] dominated universe.

And once one knows that, one can easily get quite a few other things. OK. As I was just saying, once you now how a of t behaves you can immediately calculate lots of other things. And in particular h is a dot over a and that is just 1 over 2t. So we know what h is, a function of time, even without putting any more details about what kind of radiation we have. It doesn't matter. You still get h is equal to 1 over 2t.

The horizon distance is given by the formula a of t times the integral from 0 to t of c over a of t prime, dt prime, the interval represents the total co-moving distance that light could travel between time 0 at time t. And then one multiplies that by the scale factor time t to turn it into a physical distance, and that then becomes the horizon distance of time t. And that is two times ct. You might remember it was three times ct for the matter-dominated case.

And then finally you even know exactly what the mass density is as a function of time. Because 8 squared is 8 pi over 3 rho a pi g over 3 times rho. And we know what h is. So that tells me what row is also. We actually know the mass density as a function of time, independent of anything else.

OK. Then we began talking about black-body radiation, which is basically a gas of massless particles at a temperature t that is a gas of massless particles in thermal equilibrium. And it turns out that the temperature t determines almost everything. So the energy density of black-body radiation, u, which is the same as rho mass density times c squared, is equal to a kind of a fudge factor g times pi squared over 30 times kt to the fourth, over h bar c cubed.

And this fudge factor g is equal to 2 for photons. And the reason we gave it a letter instead of just writing a two in the formula in the first place is that we'll soon be talking about black-body radiation of other kinds of particles, and g will be different for different kinds of particles.

The two for photons is given the number two because there are two spin states of photons. Photons that are massless spin one particle and massless particles, all we have maximum spins, they don't have spins in the middle, so they spin along one axis for a photon is either plus or minus 1, which is two spins. That's the quantum mechanical description. It corresponds completely to the classical description that you can separate any light wave into left-circularly polarized and a right-circularly polarized part. Or equivalently, you could separate it into an x-polarized and and y-polarized part.

And you can get x-polarization by superimposing left and right, so these are not alternative polarizations. They're just two different ways of describing the general polarization. The general causation is a linear combination of either left and right or linear combination of x and y. In any case, the basis has two basis elements, and that's where the two comes from.

Next item on our list is the pressure which could be calculated from the statistical mechanics, and we've already calculated by other means. But the answer, not matter how you calculate is is p equals 1/3 u.

Next we talked about the number density. And these are all calculations that we didn't really do. I'm just quoting standard calculations from statistical mechanics. And you can learn how to do them presumably by some [INAUDIBLE] class.

The number density is a different constant g star, in general it's different. For photons it ends up being the same. But in general a different constant, g star times zeta of 3 where zeta of 3 is the Riemann zeta function evaluated at argument three. And that's equal to 1 over 1 cubed plus 1 over 2 cubed, plus 1 over 3 cubed dot dot dot. And it adds up to 1.202 to three decimal places. And then the rest divided by pi squared then times kt cubed, over h bar c cubed. So the number [INAUDIBLE] like the cube of the temperature, while the energy density went like the fourth power of the temperature.

And the g star that appears here as I mentioned already I think is 2 for photons. And we'll learn more later about how to apply this to other kinds of particles.

And finally the entropy density is the same g as we had for energy densities times 2 pi squared over 45 k to the fourth t cubed, again it goes like t cubed, divided by h bar c cubed. And entropy is a somewhat subtle concept. Fortunately for our purposes we will not need to know much at all about what entropy actually is. I might just say for some sense of completeness that entropy is a measure of quote the disorder of a state. And this "disorder" means some measure of the number of different microscopic quantum states that contribute to a given macroscopic classical description.

The important thing for us is, well first of all, that the second law of thermodynamics tells us that entropy never decreases, but we're going to make use of a much stronger statement which holds very well for the early universe, which is that if the system stays close to family equilibrium, then the entropy doesn't change. And in the early universe I think for every process that we're going to discuss that condition holds. The system stays close to thermal equilibrium and entropy is conserved. The one exception will be inflation, which we'll learn about near the end of the course. At the end of inflation there's a humongously entropy producing phase transition. And if inflation is right essentially all of the entropy that we have int he universe today was produced during that phase transition. Before that there was only negligible entropy.

OK. That's it for my summary. Any questions? Yes?

AUDIENCE: I know it's a law that entropy only increases. Is there anything behind that or is it just it's a law of physics?

PROFESSOR: Yeah, that's a subtle question, which I think if you ask 10 different physicists you'll likely get 10 different answers. But one thing's for sure is that it does not follow as a consequence from the other laws of physics that we know. The other laws of physics that we know are essentially time, reversal, and variant. So why entropy always increases is something of a mystery. The usual story is something like the early universe, for reasons unknown, started in the state of peculiarly low entropy. And because it started low it's approaching the equilibrium value, which is much larger. So that's a possible explanation. I actually am in the process of writing a paper about the growth of entropy in the hour of time, and maybe I'll get a chance to tell you about that sometime before the end of the course, but I won't try to explain it now. But it's a slightly different idea about what could explain it. But it's something of a mystery. Nobody really knows what determines the arrow of time, why entropy only goes one way. Any other questions? OK.

What I want to do now is to continue talking about black-body radiation. I think we've said everything that we wanted to say about photons, but now we want to apply to other kinds of particles. And we're going to begin with neutrinos, because neutrinos certainly do account for a significant fraction of the entropy in the universe today even.

Neutrinos were, until around 1990 or so, thought to be massless. Now we know that they in fact have a small mass. I think I mentioned last time that we have never measured the mass of a neutrino and largely for that reason, we don't actually know what the masses are. But what we have measured is something that's quantum mechanical and rather peculiar, which is that one type of neutrino, and neutrinos come in three types, or flavors.

And that is nu sub e, nu sub mu, and nu sub tau. And what we've seen is that neutrinos of one flavor can turn into, just by traveling through space, a neutrino of the other flavor. And that can only happen if there's a non-zero mass. And what it particular measures is delta m squared between the neutrinos. And I think I even wrote on the formulas on the blackboard for the known limits of that. So I won't write it again.

But neutrino oscillation, which is what we call this, the conversion of one kind of neutrino to another, implies that delta m squared is not equal to zero. So it's still, in principle, possible that one of these neutrinos could be massless, but they can't all be massless. If one is massless, the other two have mass. And of delta m squared is very small. So these would be smaller masses than any other particles that we know of in the universe.

Now for purposes of cosmology it turns out that you can get by by thinking of the neutrinos as being massless. And then we'll start by discussing it that way. It's not quite as trivial as just saying that the mass is a small number. So if you have a formula that involves the mass, you can usually neglect it. It's a little bit more subtle because whether the neutrinos are massless or massive affects the number of spin states. And we're going to even treat the number of spin states as if the neutrino is massless, which obviously is a bit of a cheat and needs further explanation. But I'm going to do it first for massless neutrinos and we'll come back later to discuss why you can get the same answers if the neutrino has a mass.

So we're going to start out imagining that the nu's are massless. And in that approximation there is only one spin state for a neutrino and it's left-handed. And that means that the spin points the opposite direction from the momentum. So if the neutrino is spinning like this, which my right hand will correspond to a spin in that direction, the momentum will always be in that direction. And I can do the same thing this way. If it's spinning like this, the spin is that way, the momentum would be that way. The momentum's always the opposite direction of the spin.

Now you might realize and that that leads to a question about Lorentz invariance. If the spin and the momentum point in opposite directions in some frame, it's not obvious if they would also point in opposite directions in some other frame. But it turns out to be true. One can verify that, which we're not going to do, but it is a Lorentz invariance statement, as long as the particle's massless. If the particle's not massless it is clearly not a Lorentz invariance statement. And that's easy to say. If the particle were not massless, and say it was spinning this way, so the spin is that way, and the momentum is that way, that way the situation and the particle had a mass, then I could change Lorentz frames by jumping into a rocket ship, and shooting off in this direction, the same direction the particle is moving, and since the particle is moving with some finite velocity, if it has a mass, there has to be a finite velocity, in principle the spaceship can overtake it.

And when the spaceship overtakes it, from the point of view of the spaceship, the momentum will now be pointing that way, but the spin doesn't change when the spaceship overtakes it. So this relationship between spin and momentum is manifestly not Lorentz invariant when the particle has a mass, which is why we are cheating in a somewhat big way here, by saying that we're going to treat the neutrinos is massless. We're going to even be ignoring this fact that what we're saying about the spin and momentum is not even Lorentz invariant if the particle has a mass. And we'll make better excuses for that later, but for now we'll see simple picture first, and then talk about the more complicated picture later.

Anti neutrinos, which I'll write with a bar over it, also have one spin state. And as you might guess since anti particles are kind of the opposite of particles, that is right-handed. Now in addition neutrinos differ from photons in that neutrinos belong to a class of particles called fermions. Photons are bosons. How many of you already know about [INAUDIBLE] fermion or boson? OK. Most, but not quite all. Fair enough.

So fermions are particles that obey the Fermi exclusion principle that you're very likely learned about in a chemistry class someplace. It says that no two particles can be in exactly the same quantum state. Bosons do not obey such a principle. In fact, bosons are even more likely to be found in the same quantum state. And that's the important difference.

In quantum field theory, and only in quantum field theory, you can't do this in just non relativistic quantum mechanics, but in quantum field theory you can prove something called the spin statistics theorem, which says that particles with half integer spins, that is 1/2, 3/2, et cetera, are necessarily fermions. And particles with integer spins are necessarily bosons. So we know whether a particle is a fermion or a Boson as soon as we know its spin. And in the case of neutrino, it's spin 1/2. So nu's have spin one half. And that makes them fermions.

Now the statistical mechanics formulas that we just went over come about from counting states. Basically the underlying principle is that the system is likely to be in any state that you could imagine with the probability of e to the minus the energy of that state, divided by kt all in the exponent. So it's state counting that determines these formulas. And that means it's going to be different for fermions and bosons because for bosons you're going to count situations where there are many particles in the same state. And for fermions you're not.

And in particular that means you're going to be counting more states for the bosons and to be counting fewer states for the fermions. So you would expect these constants g and g star to be smaller for fermions than they are for those bosons.

And that indeed is true.

So for fermions g gets an extra factor that is multiplied by 7/8s. And g star is multiplied by 3/4. So I think we could have predicted that these g's would get factors that are less than 1, simply because we're counting fewer states. I think we can also predict without doing any calculations that g should have a bigger factor than g star. Remember g star is the factor that appears in the number density. So if we want to calculate the average energy per particle at a given temperature, we would take a formula which has a g in it for the energy density, and divide it by a formula that has a g star in it for the number density. The energy density divided by the number density is just the energy per particle. And this tells us that we'll get a number that's bigger than 1 for the fermions.

So for fermions, there's slightly more energy per particle than there is for bosons. And I think that's easily believable because fermions obey this exclusion principle. It means once you put one particle in the lowest energy state, you can't put any more there. You have to put them in higher energy states. So the expectation would be that you have more energy per particle with the fermions and that is indeed what the [INAUDIBLE] calculation indicates.

So now we're ready to write down a formula for g, for the neutrinos. And this will be the overall g that occurs in the formula for the energy density and the entropy density, and the pressure for neutrinos. And it will, we'll write it out as a string of factors. We'll first of all have this factor 7/8 coming from the fact that they're fermions. Then we'll have a factor of three because there are three flavors. And we're just going to add them together here. So we get a factor of three.

Then we get a factor of two because there's particle anti particle pairs. That is, we have to count both neutrinos and anti neutrinos, and the total energy density. For photons, a photon is the same as an anti photon, so you don't get an extra particle connected with the anti particle. But for neutrinos you do, so that gives you a factor of two here. And then you always have the number of spin states, but in this case, the of number spin state is one, but I'll write it just so we can keep track of the general pattern. And when you do that multiplication, you get 21/4. So g sub nu for all three neutrinos put together is 21/4. And you could put this into all of our formulas and get the energy density pressure, and entropy density for neutrinos.

Then we're also interested in g star, which will give us the number density. So g star sub nu. And it differs only by the first factor. 3/4 for the fermion nature of the particle. And the rest is just dittos. And what you get in the end is 9/2.

OK, now this does not end our discussion of black-body radiation in the early universe

Because as we go back in time to earlier times we can come to a times when KT, the main thermal energy, is large compared to the rest energy of an electron, m sub e c squared.

So at these very high temperatures even e plus, e minus pairs contribute to the black-body radiation. So we'd like to write down a g four e plus, e minus pairs. Actually, maybe I shouldn't call them pairs because they really are contributing individually, but g for e plus, e minus. Yes?

AUDIENCE: So I know said that we're assuming, even though it's a spin 1/2 particle that's just one spin state. I was wondering is it possible for a spin 1/2 particle to be massless? I can't think of any offhand, because don't they have to have m equals negative 1/2 state? [INAUDIBLE] in general.

PROFESSOR: Right. No, it is perfectly consistent, would be perfectly consistent for neutrinos to be massless. And then they would only have and m equals 1/2 state in spite of the fact that their spin 1/2. And the m equals minus 1/2 state would just be missing, but that's OK. It's similar to what happens with photons. Photons are spin one particles. And the m equals 0 state is missing. And that can only happen with massless particles. For massive particles, by basically the argument that I gave about catching up to the neutrino, it's not possible to have one helicity and not have the other. But for massless particles it is possible. The photon does it. We used to think the neutrino did it. Turned up the neutrino doesn't do it, but we can still describe the old theory, which is simpler than the new theory. And that's what I'm doing.

AUDIENCE: Is there any reason they're missing those?

PROFESSOR: OK. The question is is there any reason why they're missing those spin states. The answer basically is that there's no solid reason, which is why we didn't know if it was going to be missing the states or not. But for the case of massless particles only the m states are all completely disconnected from each other under Lorentz transformations. For any non-zero mass, all the spin states mix. And because they all mix under Lorentz transformations you can't have any one without having all of them. But that statement disappears when the particle becomes massless. So when the particle becomes massless, essentially, each spin state is its own kind of particle. And it might be part of a spectrum of real particles, or might not. And we don't know any fundamental principle that answers that question, other than experiment.

OK. So I was just going to write down a g for e plus, e minus pairs. Remember this g determines the energy density, the pressure, and the entropy density. And it again has a factor of 7/8 because these are fermions. It has a factor of one to sort of follow the general pattern because there's only one species of electrons. For neutrinos we had three, but for electrons we just have one. We do get a factor of two for particles, anti particles because an e plus is different from an e minus. And there are two spin states for the electron. If could be spin up, or spin down, or spin equals 1/2, and there's spin equals minus 1/2. So there are two spin states. And that gives us a factor of 7/2 for g.

And for g star e plus, e minus, the only difference again, is in the first factor, which is 3/4 for fermions for g star. And then times dot, dot, dot. And that then is equal to just a nice round factor of three.

And then if we go back to this earlier time, we have not only the e plus, e minus pairs, which started to exist when we crossed this threshold, we also have photons and neutrinos. So for kt, large compared to m sub ec squared, and I should say small compared to the next threshold, m sub nu c squared, mass of a muon. The mass of the muon as 106 MeV Mass of electron is a half of an MeV. So there's are good long range here where this is the right number. So g total is equal to 2 plus 21/4 plus 7/2 equals 10 and 3/4. So this number 10 and 3/4 plays an important role in a long segment of the history of our universe.

Another number that you might be interested in is the energy density of radiation in the universe today. And the e plus, e minus pairs are of course long since frozen out because kt is a lot less than half of an MeV. So we just have neutrinos and photons. You might think that we could just add the formulas for photons to the formula for neutrinos, but that turns out not to be right. And the reason it's not right now is that we believe that the temperature of the neutrinos is not the same as the temperature of the photons. And this is a problem that you'll be working out on the next homework set, so I won't give all the details here because I want you to have the fun of working out those details. But I'll tell you the outline of what it is.

The transition occurs when kt crosses this magic threshold of m sub e c squared, which means that the electron positron pairs are going to start to disappear from the thermal equilibrium mix. Now those electron positron pairs have both energy and entropy. It turns out that the energy is very hard to keep track of. And the reason why that is true is that we know that du dt is equal to minus 3h, times rho plus p, I think i have this right. times u. No, not times u. And this might be a u. Yeah, I think this is right.

But what is definitely true is that it involves the pressure as well as the energy density. And keeping track of what the pressure's doing as a function of time is complicated because depending on exactly what the mix of particles are and the pressure is even given by a more complicated formula when you're very near the threshold, that is when kt is near the mass of a particle, there's a more complicated formula for the pressure. So the pressure is hard to keep track of.

But the entropy turns out to be easy to keep track of because entropy is simply conserved during this process. So on the problem that you'll be doing for the next problem set you'll be looking at the entropy contained in these e plus e minus pairs.

And then there's an important assumption which is valid. We will not really try to justify it, but at the time when the e plus, e minus pairs go out of equilibrium, when they disappear from the thermal equilibrium mix as kt falls below m sub ec squared, at that point the photons are still interacting strongly with everything else, and with each other. But the neutrinos have pretty much decoupled. They're not really interacting with anything anymore.

So when the e plus, e minus pairs disappear they give essentially all of their entropy to the photons, and essentially not of their entropy to the neutrinos. And that means that you can calculate the entropy density of the photons, and the entropy density of the neutrinos and you can calculate from that the relative temperatures between the two. And the net effect is that the photons end up with a higher temperature than the neutrinos.

So cosmology makes a clear prediction here. The universe today should be bathed in a thermal distribution of neutrinos. The temperature I think ends up being about 2.4 degrees Kelvin, a little colder than the photons. And it's really the great challenge of observational cosmology to try to measure those thermal neutrinos. Because neutrinos interact so weakly, nobody has come close to measuring the existence of those neutrinos. Everybody thinks they must be there, if anybody's discovers they're not there, it'll be a major shift in our understanding of cosmology. But nobody really knows how to measure them. So you guys could try to do that sometime during your career as physicists.

So putting these things together what you'll show is that the temperature of neutrinos should be equal to 4/11 to the 1/3 power times the temperature of the photons. And given that, we can write down a formula for the energy density in radiation today. So u [? rad ?] 0 for today is equal to g for photons, plus 21/4, which is g for the neutrinos, but then we have to correct for the fact that the neutrinos are at the lower temperature, and remember energy density's go like the fourth power of the temperature. So there's a correction here, which is the fourth power of this 4/11. So this multiplies 4/11 to the 4/3 power, the fourth power of the temperature.

And then the rest of the formula for energy density. Pi squared over 30 times k, times the temperature of the photons to the fourth power, divided by h bar c cubed. And if you plug numbers into this, you get a number which I wrote on the blackboard when we started talking about radiation for the radiation density of today's universe. It's 7.01 times 10 to the minus 14 jewels per meter cubed.

So now we know how to derive this formula that I pulled out out of hat when we first started talking about energy density radiation. You might remember we used this to calculate when radiation matter equality took place. OK. Any questions about this?

OK. Well in that case now I'm ready to come back and talk about neutrino masses more realistically. OK. [INAUDIBLE] the mass we now know. Or at least two out of the three do. And still I argued, or told you, that the mass didn't matter for the calculations we just did. And that seems a little strange because the calculation we just did depended on counting the numbers of spin states, and the number of spin states, of course, changes if the particles have a mass. Neutrinos could not have just one spin state if they had a mass. So the question then is what happens to these right-handed neutrinos. Remember the neutrinos we know and love are left-handed.

OK. Nobody has ever seen a right-handed neutrino. But if the neutrino has a mass, they must exist. And one way to see is that argument I gave you about catching up to the neutrino and going faster than it. And then a left-hand neutrino would turn into a right-handed neutrino, which is, this is all you did, was change frames, and the world is Lorentz invariant. It means that right-handed neutrinos must in principle exist in any frame.

OK. It turns out that there are two theories of how the neutrino mass works. And we don't know yet which of these theories is correct. And they are called the possibility of a Dirac mass or the possibility of a Majorana mass. And these are both named after people, so don't try to parse anything about the physical meaning of those words from the name.

The Dirac mass is the easier one for us to understand because it's the same kind of mass that an electron has. So if the neutrinos have a Dirac mass it really would mean that there are right-handed neutrinos, which are a new species a particle that we haven't seen yet, but which would be implied by the existence of this mass.

The catch is that because of the very peculiar way that the neutrinos interact it is possible for the right-handed neutrinos to interact vastly more weakly than the left handed neutrinos. And that would be the explanation for why we've never seen them because they interact so extraordinarily weakly. And even in the early universe where everything almost reaches thermal equilibrium, the cross-section for producing right-handed neutrinos would be so small that they would essentially never be produced.

So right-handed nu's interact so weakly that they're not seen in lab, or in early universe.

OK. The other type is more peculiar. And there are no other particles in nature that are known to have this type of mass, but it's a theoretical possibility. Majorana masses can only be possessed by particles which are neutral, but neutrinos are neutral. And if neutrinos have a Majorana mass it would mean that the right-handed partner of the left handed neutrinos that we see would in fact be a particle that we already know about, but it will be the particle that we have previously identified as the right-handed anti neutrino. So if the Majorana hypothesis is true, the anti neutrino and the neutrino would be the same particle. And one would be the right-handed version and the other would be the left-handed version. And that is also a possibility, we just know know.

And in this case, clearly what we said about the early universe still works because we did the counting right. If it's the direct possibility we would have to do a further argument, we justify the fact that these right-handed partners interact so weakly that we would never see them even in the early universe, but that is the case. And for the Majorana case there clearly is no real difference from the calculations that we did. Any questions about that? Yes?

AUDIENCE: What exactly do you mean when you say the direct mass is like the electrons?

PROFESSOR: Right. What I mean is that it appears in the equations of motion in the same way that the that the electron mass appears, and therefore like the electron there's a right-handed electron and there's also a left-handed handed electron, which are just related to each other by a parity transformation. Yes?

AUDIENCE: Is it not a problem in the Majorana case [INAUDIBLE] particle? Doesn't there have to be an [INAUDIBLE]?

PROFESSOR: OK. The question is doesn't there have to be an anti particle, how can the neutrino and the anti neutrino be the same particle. The answer is no. The photon does not have an anti particle. So particles that have a charge of any kind must have an anti particle, but if the particle is really neutral then it can just be it's own anti particle, and that would be the case here. Yes?

AUDIENCE: Sorry, I have two questions. One is what is the difference between a neutrino and a anti neutrino besides? I mean, how do we know they're the same, that they are the same particle [INAUDIBLE]. It's not just a matter of semantics. If the only difference that we can see is [INAUDIBLE]. Whether we call it the anti neutrino or say that it's the same particle, just [INAUDIBLE].

PROFESSOR: Right. OK. You're saying how do we know whether the anti neutrino was the same particle as the neutrino, and what do we mean by that statement anyway. I guess the answer is really something that comes out of the fundamental equations in the context of the quantum field theory. But maybe I can say something more concrete, however. There's a quantum number called Lepton number, and actually divides into three kinds of quantum numbers, so there's an electron number, muon number, and a tau number. And for all the interactions that we've seen that number is conserved, that is the sum of the number of electrons minus the number of anti electrons, plus the number of electron neutrinos, minus the number of anti electron neutrinos is conserved.

And if that were a rigorous conservation law, then it would really mean that the neutrino was not neutral. It would have this nonzero charge called electron charge for the electron neutrino. So the Majorana possibility requires that that quantity is not really conserved. So that's an important fact about nature that we haven't really learned yet, whether or not it's exactly conserved or only highly approximately conserved. The belief is that it's only highly approximately conserved, but we don't know. Yes?

AUDIENCE: Does a neutrino [INAUDIBLE]?

PROFESSOR: Yeah. You are right. You're very quick. You are right that neutrino oscillations are certainly enough to prove that the individual Lepton numbers are not conserved, but they still have the possibility that total Lepton number can be conserved, which we don't think is true, but I don't think we've ruled it out yet either. Any other questions? Yes?

AUDIENCE: In the Majorana case, if they are the same particle, do they still come in left and right-handed forms so as to keep the [INAUDIBLE] the same value, or do they only come in the left-handed phase, in which case the [INAUDIBLE] would be decreased [INAUDIBLE]?

PROFESSOR: Well, the counting that we had before would be correct. And the only question is whether the factor of two that we put here for particle, anti particle is here where maybe the particle's really neutral and the factor of two is the number of spin states, but it's the same result either way. It's all in the question of whether the left-handed neutrino is the anti particle or the right-handed neutrino, or another spin state of the right-handed. I'm sorry. I'm saying this wrong. The question is whether the right-handed neutrino is the anti particle of the left-handed neutrino or whether it's another spin states of the left-handed neutrino. But either way the number particles that we think exists would be the same in the mayorana description. OK?

OK. Now we're ready to actually write down for example a formula for kt. So sticking to this time range of kt is much, much bigger than m sub eb squared, but kt is much, much less than m sub nu c squared. We can write down a formula-- I'm sorry. Actually the formula's more general. I didn't really look carefully what I wrote. Forget this. As long as we know what little g is, and this formula's going to have a little g in it, and then you can fill in the right value for any time period you want. We can write down a formula for kt as a function of time. And that's because we've learned how to write energy density as a function of time, and we've learned how to express energy density in terms of the temperature. And putting those two together gives us a formula for the temperature as a function of time. And that formula has the form of 45 h bar cubed c to the fifth, divided by 16 pi cubed little g times capital G, Newton's constant, whole thing to the 1/4 power, times 1 over the square root of t.

So if we had nothing but radiation, and if little g were constant, this would be the formula for the temperature as a function of time. Now what actually happens is that little g changes as we go through these different thresholds for which particles are contributing to the black-body radiation. And that means that the exact formula is a little bit more complicated than this. But as long as you're well into any one of these periods, as long as you're not near any of these border lines, this formula is in fact a very accurate approximation to the temperature as a function of time.

And an interesting time period is about one second after the Big Bang. And that corresponds to this g equals 10 and 3/4 that we talked about earlier, where we have neutrinos e plus, e minus pairs, and photons. And since this is before the e plus, e minus pairs have disappeared, the temperature of the neutrinos at this stage is still the same as the temperature of the photon. So everything is at the same temperature here. We just add up the g's. And the end result for that is that kt is equal to 0.860 MeV million electron volts, divided by the square root of t in seconds.

There are the units that you could express it in in the notes. Other sets of units are given. So if 0.86 is about 1, which it is for many purposes, then roughly speaking we're saying at 1 second after the Big Bang, kt was about one MeV, which means it's higher than the rest mass of the electrons, which is 1/2 MeV. So the electron positron pairs were still pretty much present at one second after the Big Bang, but they start to disappear pretty much immediately after that.

One can also write down what the temperature itself is. The temperature is 9.98 times 10 to the ninth k, divided by the square root of t in seconds.

OK. The next item we want to talk about following the history of the early universe is recombination and decoupling. Until the temperature fell to about 4,000 Kelvin, the hydrogen in the universe, and the universe was mostly hydrogen, about 20% helium and the rest hydrogen, and until the temperature fell to 4,000 Kelvin, the hydrogen would be ionized. This is another [? stat net ?] calculation that we're not going to be doing. 4,000 degrees is not some magic temperature associated with hydrogen. The point at which the hydrogen ionizers depends on its density. But for the densities in the early universe, the ionization point is about 4,000 Kelvin. So at t equals 4,000 Kelvin the hydrogen recombines.

Now the word recombine has somehow historically taken hold. So everybody calls it recombination in spite of the fact that according to our theory it was never combined previously at any time. So the prefix re there has absolutely no meaning whatever, but nonetheless it is completely conventional.

OK. To estimate when this happens we can use an important fact, which we haven't said yet. Because entropy is conserved and entropy density is equal to some constant times t cubed, if entropy is conserved then like mass density in a matter-dominated universe, as the universe expands, the entropy thins out. And therefore just like the mass density in a matter-dominated universe, the density should go down like 1 over a cubed, 1 over the volume.

Now strictly speaking this is only true if g is fixed. Well, actually that's not right. It's always true. What I'm about to write next is only true if g is fixed. If g is fixed then entropy density is proportional to the temperature cubed. And if you put these together you could see that t is just proportional to 1 over the scale factor. So as long as little g, as long as the number of degrees of freedom contributing to this black-body radiation is fixed, the temperature just falls like 1 over the scale factor. OK.

OK. So we said that recombination occurs at 4,000 degrees. There's actually another number that's more interesting, which is decoupling, which is slightly colder. Now the definitions are that recombination is usually defined as the temperature at which half of the hydrogen has recombined, but some significant fraction of it is still not yet recombined. It doesn't happen suddenly. It happens gradually. So you have to pick some point along the way to say this is the temperature that defines the recombination. Usually take us to the halfway point, which I think is the number used to calculate that. But perhaps of more interest is the temperature at which the photons decouple in the sense that as the photons scatter between then and now, a typical photon has not scattered at all. And that's colder because when you cross 4,000 K you still have half of the hydrogen ionized, which means there's still plenty of electrons around for these photons to scatter off of. So you have to cool to a colder temperature until the photons cease to interact with the electron positrons in any significant way. And that's why decoupling temperature is somewhat lower than recombination temperature.

We can estimate at what time decoupling happened. Now this is only a crude estimate, but will in fact be pretty accurate. Since we don't yet even know about dark energy, we're going to estimate the evolution of the universe between the time of decoupling and now as being entirely matter-dominated. The time of decoupling is long after the time of matter radiation equality. So the universe is matter dominated at the time of decoupling and we're going to assume it's matter dominated up to the present day. And that means that we know that the scale factor will evolve like t to the 2/3, and therefore the temperature will evolve, like 1 over t to the 2/3. And that will be enough to tell us how much time is needed to go from 3,000 Kelvin to the present temperature 2.7 Kelvin.

So the time of decoupling making this approximation is just the ratio of the temperatures t zero over t decoupling, to the 3/2 power, times the time today. [INAUDIBLE] this fraction in the past. And plugging in numbers that's 2.7 over 3,000 to the 3/2 times 13.8 trillion years. And that's about 380,000 years.

And this in fact this is pretty much exactly the number that people quote when they calculate this more accurately. So we really hit it just about on the nose. So the time of decoupling was about 400,000 years after the Big Bang. And I should add, and then we'll stop, that when we look at the cosmic background radiation, what we are really seeing is an image of the universe at this time, at the time of decoupling, because from this time onward photons have pretty much just travelled in straight lines. And that means that when we look at the cosmic background radiation we're really seeing an image of the early universe in exactly the same way as you're seeing an image of me when you observe the photons there traveling from my face to your eyes along straight lines. It's the same principle. So this determines what we're actually seeing in the cosmic background radiation. And therefore it's a very, very important number. And we'll stop there now and continue next time.