8.251 – Homework 3

B. Zwiebach Spring 2007

Due Thursday, March 1.

1. (5 points) Problem 4.1

2. (10 points) Problem 4.2

3. (15 points) Problem 4.3

4. (10 points) Closed string motion.

   We can describe a nonrelativistic closed string fairly accurately by having the string wrapped around a cylinder of large circumference $2\pi R$ on which it is kept taut by the string tension T_0. We assume that the string can move on the surface of the cylinder without experiencing any friction. Let x be a coordinate along the circumference of the cylinder: $x \sim x + 2\pi R$ and let y be a coordinate perpendicular to x, thus running parallel to the axis of the cylinder. As expected, the general solution for transverse motion is given by

   $$y(x,t) = h_+(x - v_0 t) + h_-(x + v_0 t)\,,$$

   where $h_+(u)$ and $h_-(v)$ are arbitrary functions of single variables u and v with $-\infty < u, v < \infty$. The string has mass per unit length μ_0, and $v_0 = \sqrt{T_0/\mu_0}$.

   (a) State the periodicity condition that must be satisfied by $y(x,t)$ on account of the identification that applies to the x coordinate. Show that the *derivatives* $h'_+(u)$ and $h'_-(v)$ are, respectively, periodic functions of u and v.

   (b) Show that one can write

   $$h_+(u) = \alpha u + f(u)\,, \quad h_-(v) = \beta\, v + g(v)\,,$$

   where f and g are periodic functions and α and β are constants. Give the relation between α and β that follows from (a).

   (c) Calculate the total momentum carried by the string in the y direction. Is it conserved ?

5. (10 points) Problem 5.3

6. (10 points) Problem 5.4

7. (10 points) Problem 5.5

While not assigned, I think all students should know how to solve problem 4.6 and problem 5.1.