(1) Introduction

1 Welcome to 8.223

- Classical Mechanics II
- Matthew Evans lecturer
- Lectures M-F 10-11:30 AM in 4-270
- Recitations M-F 12-1 PM in 4-270

- Recitation TAs:
  - Stephanie O'Neil (weeks 1-2)
  - Yuanhong (Jason) Luo (weeks 3-4)
  - Aravind Devarakonda (general, Psets)
  - *TAs: make suggestions and ask questions!*
  - website: linked from stellar

- Hi folks! Welcome to 8.223...
  - Who am I?
    - Astrophysics Division
    - Experimentalist
    - Gravitational Waves!
  - Who are you?
    - course 8?
    - sophomores? juniors?

Class Objectives:

- Classical Mechanics Power Tools
  - How to solve the really hard problems with relative ease through Lagrangian Mechanics

- Preparation for Statistical Mechanics and Quantum
  - The theoretical foundation for advanced physics lies in Hamiltonian Mechanics
  - Unfortunately this means you will have to believe me when I say ”this will come in handy in 8.04”

- Course Structure
  - Daily Lecture and Recitation - must attend!
  - Psets - 70% - do them daily, hand in Friday 10am
  - “Project” - 30% - one hard problem to do alone

No late psets! No late projects!!

Doing the psets is *critical* to getting the most of this class. Try them alone, then in a group, then ask an upperclassman, then email the TA.

The project problem should look impossible given only 8.01 physics, but will not be so bad by week 2 or 3! Do it alone (without help from the internet) whenever you like. Turn it in with the last pset on Friday February 2 at 10 AM (or earlier for bonus credit).

Rough Course Outline

Topic	LL Chapter	Lecture
Lagrangian Mechancis	1-5	1-4
Conserved Quantities	6-10	5-6
Orbits and Scattering	11, 13-15, 18-19	7-10
Oscillations	21-22, 25-26	11-12
Tricky Potentials	30+	13-14
Hamiltonian Mechanics	40, 42, 45-46	15-19

A few words about reading LL: it is overly dense, which means that the implications and examples are generally left to the reader. Suggestions:

LL Suggestions

1. Read with a notebook and work out the "evident" transitions between equations.
2. Do the exercises - they overlap with the pset, and the solution is given.
3. Do 1 and 2 before lecture, and bring questions!

Also consider consulting

Classical Dynamics by Marion and Thornton
Classical Mechanics by Goldstein

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2 8.01 Work Flow Review

In 8.223, I will assume that you remember 8.01, and I will occasionally use it as a point of reference.

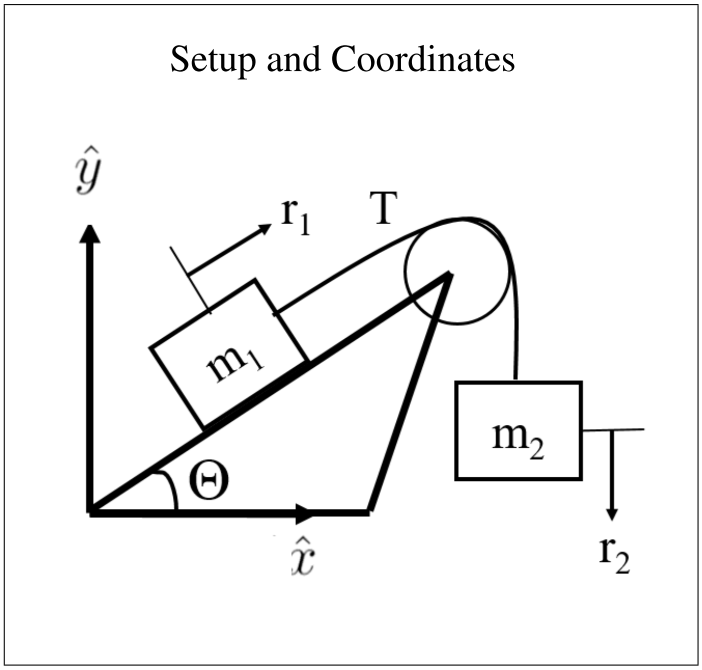
In 8.01, Newton's second law was king. All you needed was $F = ma$ and some coordinates, and you were ready to go. Sure, you added springs and strings and pulleys and went from point masses to rigid bodies; you integrated in space and time to find energy and momentum, and studied motion in various potentials, but in the end it was always just $F = ma$ in some coordinates.

For any given problem you might

8.01 Work Flow

1. set up a coordinate system
2. draw free body diagram
3. write equations of motion
4. eliminate forces of constraint (e.g. tension in a string)
5. solve for final equations of motion (e.g. $\ddot{x} = \frac{-k}{m}x$)

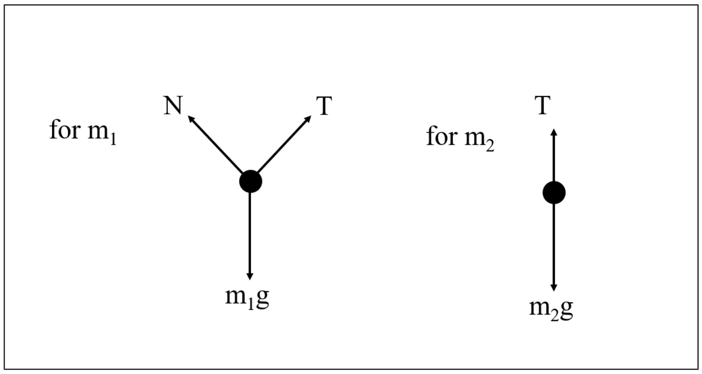
A typical 8.01 problem:



A typical 8.01 problem:

$$\begin{aligned}
r &= r_1 = r_2 \\
x_1 &= r\cos\theta \\
x_2 &= x_0 \\
y_1 &= r\sin\theta \\
y_2 &= y_0 - r \\
\widehat{r_1} &= \cos\theta\,\widehat{x} + \sin\theta\,\widehat{y} \\
\widehat{r_2} &= -\widehat{y} \\
\widehat{\theta} &= -\sin\theta\,\widehat{x} + \cos\theta\,\widehat{y}
\end{aligned}$$

The next step is a free body diagram.



Analyze the free body diagram:

$$\begin{aligned}
F_{x_1} &= T_s \cos\theta - N \sin\theta \\
F_{x_2} &= 0 \\
F_{y_1} &= T_s \sin\theta + N \cos\theta - m_1 g \\
F_{y_2} &= T_s - m_2 g
\end{aligned}$$

Eliminate forces of constraint:

$$\begin{aligned}
\frac{d}{dt}\theta = 0 &\Rightarrow \vec{F}_1 \cdot \widehat{\theta} = 0 \Rightarrow N = m_1 g \cos\theta \\
\ddot{r}_1 = \ddot{r}_2 &\Rightarrow \frac{\vec{F}_1 \cdot \widehat{r_1}}{m_1} = \frac{T_s}{m_1} - g\sin\theta = \frac{\vec{F}_2 \cdot \widehat{r_2}}{m_2} = \frac{-T_s}{m_2} + g \\
&\Rightarrow T_s\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g(1 + \sin\theta) \\
&\Rightarrow T_s = \frac{m_1 m_2}{m_1 + m_2} g(1 + \sin\theta)
\end{aligned}$$

Write equation of motion:

$$\begin{aligned}\ddot{r} = \frac{-F_{y_2}}{m_2} &= g(1 - \frac{m_1}{m_1 + m_2}(1 + \sin\theta)) \\ &= \frac{g}{m_1 + m_2}(m_1 + m_2 - m_1 - m_1 \sin\theta) \\ \ddot{r} &= \frac{g}{m_1 + m_2}(m_2 - m_1 \sin\theta)\end{aligned}$$

Given some initial conditions (r and $\dot{r}$ at $t = 0$) you can solve this ODE.

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3 Generalizing $F = ma$

In the next few lectures we'll explore a different approach. Here's a brief taste:

Generalizing F = ma

$$\text{Generalize } F = ma \text{ to } \frac{-\partial U}{\partial x} = \frac{d}{dt}(m\dot{x}),$$

where U is the potential energy.

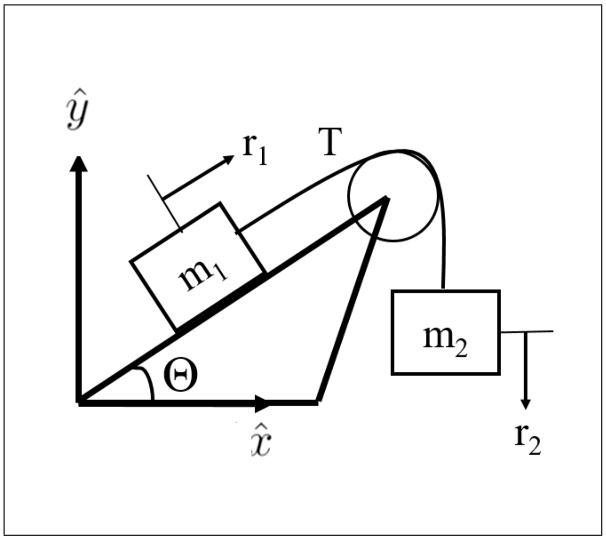
$$\text{note } m\dot{x} = \frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2\right) = \frac{\partial T}{\partial \dot{x}}$$

where T is the kinetic energy

$$\text{thus } F = ma \Rightarrow -\frac{\partial U}{\partial x} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right)$$

Let's try this with our example problem.

Note: "T" is used for kinetic energy here (it was tension in the string!)



To find equations of motion:

$$
\begin{aligned}
U &= g(m_1 y_1 + m_2 y_2) = g(m_1 \sin\theta - m_2) r \\
T &= \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) = \frac{1}{2}(m_1 + m_2)\dot{r}^2 \\
-\frac{\partial U}{\partial r} &= g(m_2 - m_1 \sin\theta), \frac{\partial T}{\dot{r}} = (m_1 + m_2)\dot{r} \\
\Rightarrow \ddot{r} &= \frac{g}{m_1 + m_2}(m_2 - m_1 \sin\theta)
\end{aligned}
$$

DONE!! No vectors, no forces of constraint to eliminate!

Kapitza Pendulum Example, NDSolve demo

For tomorrow

1. read Feynman Lecture (pages 1-7, non-relativistic)
2. do pset problems 1-3

Note about lectures: I aim to finish $\approx$ 15 minutes early, so that there is time for questions (varying from 5-30 minutes early).