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8.21 The Physics of Energy
Fall 2009

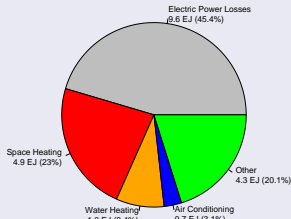
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8.21 Lecture 4

Heat and Thermal Energy

September 16, 2009

2001 U.S. Residential Energy Use
Total 21 EJ



Heating/cooling:

> 50% of U.S. household energy use

~ 40% of U.S. total energy use

Thermal energy conversion:

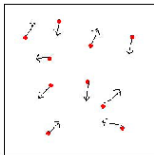
> 97% of U.S. energy direct/converted TE
(exceptions: hydro, solar PV, wind)

Basic energy applications of thermal physics:

- Heat capacity [Heat a teapot]
- Heat flow [Building insulation]
- Thermal energy conversion [Power plants, cars]

Last lecture we reviewed mechanical energy: **Kinetic/Potential**

Many particle system: micro kinetic/potential $E \rightarrow$ **Thermal Energy**



For a closed, isolated system:

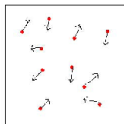
Thermal Energy: Total energy from relative motion, excitations of particles above absolute 0.

Internal Energy: *All* contributions to energy, including binding, rest mass energies– Needed for chemical, nuclear Rx

Temperature: Measure of internal/thermal energy (Def. L8)

Heat: Transferred thermal energy (high $T \rightarrow$ low T)

Example: Monatomic ideal gas



N molecules

- Ideal gas: pointlike particles, elastic collisions
- Monatomic: no internal excitations [e.g. Argon]
- Thermal equilibrium (“settled down”)

$$E_i = \frac{1}{2}m(\mathbf{v}^{(i)})^2 \text{ for molecule } i = 1, \dots, N$$

Total thermal energy:

$$(k_B = 1.38 \times 10^{-23} \text{ J/K})$$

$$U = \sum_{i=1}^N E_i = N \left\langle \frac{1}{2}mv^2 \right\rangle = N \frac{1}{2}m (\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle) \equiv \frac{3}{2}Nk_B T$$

Equipartition of energy: At high T , each DOF has energy $\frac{1}{2}k_B T$

Thermal energy of a fixed system: $U(T)$

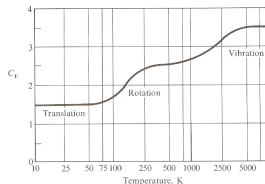
Example: monatomic ideal gas $U = \frac{3}{2}Nk_B T$

Heat capacity: $C_V = \left(\frac{\partial U}{\partial T} \right)_V \quad dU = C_V dT \quad (\text{constant volume})$

Ideal gas: $C_V = \hat{c}_v N k_B$ (monatomic: $\hat{c}_v = 3/2$)

Real gas:

- DOF increase at higher T
- Complicated by interactions, finite volume effects



\hat{c}_V of H_2 [Kittel/Kroemer]

Specific heat capacity

(J/g · K):

Examples

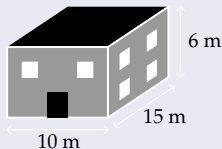
| substance | c_p | c_v |
|-----------------------|-------|-------|
| water (liquid, 25° C) | 4.18 | 4.14 |
| air (room temp.) | 1.01 | 0.72 |
| concrete | 0.88 | — |
| wood | 0.42 | — |

Example: water for a shower, 5 gal./min., 8 minutes, 50°F → 104°F

$$Q = c_p m \Delta T$$

$$\cong (4.18 \text{ J/g} \cdot \text{K})(40 \text{ gallons})(3.79 \text{ L/gal})(1 \text{ kg/L})(30 \text{ K}) \cong 19 \text{ MJ}$$

Example: air in a house



900 m³ from 32°F → 67°F

$$(1.01 \text{ J/g} \cdot \text{K})(900 \text{ m}^3)(1.2 \text{ kg/m}^3)(20 \text{ K})$$

$$\cong 22 \text{ MJ}$$

Notes: only air, neglects heat loss

Phase Transitions



Photo courtesy of Jon Wiley on Flickr



Photo courtesy of totalpodcatastrophe on Flickr



Photo courtesy of xtrarant on Flickr

H₂O latent heat
of melting:

334 J/g

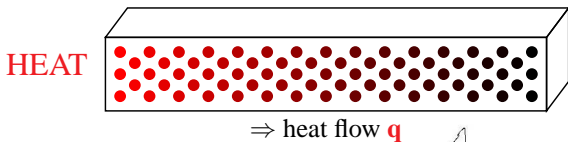
H₂O latent heat
of vaporization:

2260 J/g

- Extra E needed to break intermolecular bonds in solid/liquid
- Constant p enthalpy of transformation
- Water steam stores lots of E (steam engine/turbines)

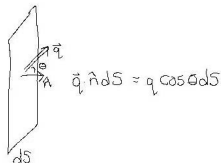
Example: evaporate 1 oz. sweat: $28 \text{ g} \times 2260 \text{ J/g} = 63 \text{ kJ}$
cool by $63 \text{ kJ}/(4.18 \text{ J/gK} \times 70 \text{ kg}) = 0.22 \text{ K} = 0.39 \text{ F}$

Heat Conduction



\mathbf{q} = heat current density [$\text{J}/\text{m}^2\text{s}$]

heat flowing per second through \perp surface



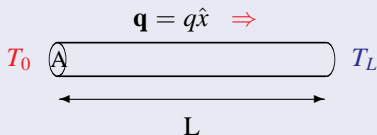
Fourier law: $\mathbf{q} = -k \nabla T$

- k : thermal conductivity [W/mK]

- Different for different materials
- Can be temperature dependent

- Intuitive: heat from hotter \rightarrow colder

- Depends only on local properties (gradient)

Heat conduction from Fourier law $\mathbf{q} = -k\nabla T$ —Analogy with electric conductivity [$\mathbf{j} = -\sigma\nabla V$]Example: 1D problem: thin bar, length L , area $A \rightarrow \mathbf{q} = q\hat{x}$ Solve for $T(x)$, $q(x)$ constant in time with fixed boundary conditions

- No change in time $\Rightarrow q$ constant in x (or T changes locally in time)

$$\Rightarrow q = -k \frac{dT}{dx} = \text{constant} \Rightarrow T(x) = T_0 + (T_L - T_0) \frac{x}{L}, \quad q = -k \frac{T_L - T_0}{L}$$

Rate of heat transfer:

$$qA = \frac{-k(T_L - T_0)A}{L} \Rightarrow T_L - T_0 = (qA) \left(\frac{L}{Ak} \right) \sim V = IR$$

“thermal resistance”

Heat conduction: example

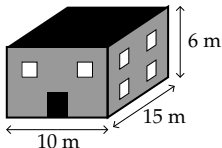
Heat xfer through 0.2m concrete



| material | k [W/mK] |
|-----------------------|----------|
| air | 0.026 |
| fiberglass insulation | 0.043 |
| hard wood | 0.16 |
| concrete | 1.4 |
| steel | 52 |

$$q = -k \frac{\Delta T}{L} = (1.4 \text{ W/mK})(20 \text{ K}/0.2 \text{ m}) = 140 \text{ W/m}^2$$

Heat loss from our building if concrete?

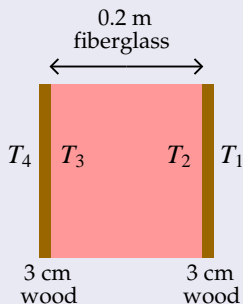


$$\begin{aligned} \text{area} &= 300 \text{ m}^2 + 150 \text{ m}^2 = 450 \text{ m}^2 \\ &\Rightarrow 60 \text{ kW} \times 24 \text{ hours} \cong 1500 \text{ kWh} \end{aligned}$$

Roughly 5 GJ/day!

$$\text{Wood: } \times \frac{0.16}{1.4} \Rightarrow \sim 500 \text{ MJ/day}$$

Concrete: lose 5 GJ/day. Wood: 500 MJ/day. **What if we insulate?**



$$q = \Delta T \frac{k}{L} \text{ same for each material}$$

$$\begin{aligned} T_4 - T_1 &= (T_4 - T_3) + (T_3 - T_2) + (T_2 - T_1) \\ &= q \left(\frac{L_w}{k_w} + \frac{L_f}{k_f} + \frac{L_w}{k_w} \right) \\ &\cong q \left(2 \frac{0.03 \text{ m}}{0.16 \text{ W/mK}} + \frac{0.2 \text{ m}}{0.043 \text{ W/mK}} \right) \\ &\cong q (5.1 \text{ Km}^2/\text{W}) \end{aligned}$$

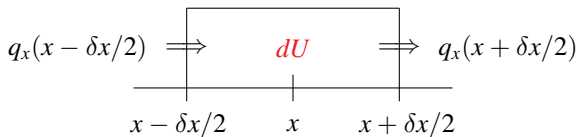
$$\text{So } qA = 900 \text{ m}^2 \frac{20\text{K}}{5.1 \text{ Km}^2/\text{W}} \cong 1900 \text{ W} \cong 160 \text{ MJ/day}$$

- Note analogy to serial resistors $R_{\text{total}} = R_1 + R_2 + R_3$
- Heat loss proportional to ΔT
- In construction: use “R-value” $R = \Delta T/q$ (L/k for single material)

⚠ Heat equation: Include time dependence in heat flow

- Begin with Fourier equation $\mathbf{q}(\mathbf{x}, t) = -k\nabla T(\mathbf{x}, t)$

For simplicity, work in one space dimension, $q_x(x, t) = -k \frac{\partial T(x, t)}{\partial x}$



$$dU = -\frac{\partial q_x}{\partial x} dx A dt = c_p \rho (dx) A (dT)$$

so in 1D $\frac{\partial T(x, t)}{\partial t} = -\frac{1}{\rho c_p} \frac{\partial q_x(x, t)}{\partial x} = a \frac{\partial T(x, t)}{\partial x^2}$

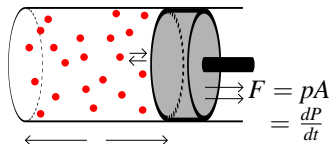
$$3D : \frac{\partial T(\mathbf{x}, t)}{\partial t} = a \nabla^2 T(\mathbf{x}, t) = a \left[\frac{\partial^2 T(\mathbf{x}, t)}{\partial x^2} + \frac{\partial^2 T(\mathbf{x}, t)}{\partial y^2} + \frac{\partial^2 T(\mathbf{x}, t)}{\partial z^2} \right]$$

where $a = k/(\rho c_p)$ is Fourier coefficient of material.

Pressure: Bouncing molecules \rightarrow force on piston

$$\text{Each impact: } \Delta P_{\text{piston}} = 2mv_x$$

$$\text{Time between impacts: } 2L/v_x$$

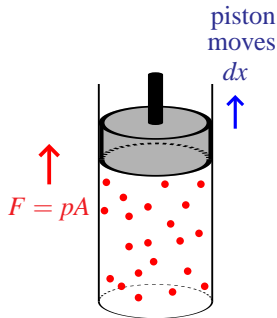


$$F = \frac{dP}{dt} = \sum (2mv_x) \left(\frac{v_x}{2L} \right) = Nm \frac{\langle v_x^2 \rangle}{L}$$

$$\text{Pressure : } p = \frac{F}{A} = \frac{N}{A \cdot L} m \langle v_x^2 \rangle = \rho \langle v_x^2 \rangle$$

For ideal gas: $pV = Nk_B T$ ($= nRT$)

Expansion



Now, **heat up** air in cylinder

p increases \Rightarrow piston moves

Work done by gas:

$$dW = Fdx = pAdx = pdV$$

So $dU = -pdV$

First Law of Thermodynamics

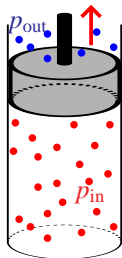
$$\text{heat input } dQ \Rightarrow dQ = dU + p dV$$

Heat engine: Raise $T \Rightarrow$ Raise $p \rightarrow$ expand + do work \Rightarrow cycle

Question: how much thermal energy can be used?

How much useful work can we get from thermal energy?

$$F_{\text{net}} = A(p_{\text{in}} - p_{\text{out}})$$



Work done when piston moves dx :

$$dW = p_{\text{in}}dV.$$

Some is **work on outside gas**

$$dW_{\text{lost}} = p_{\text{out}}dV.$$

Usable work is $dW_{\text{useful}} = (p_{\text{in}} - p_{\text{out}})dV$.

For ideal gas: $p = Nk_{\text{B}}T/V$.

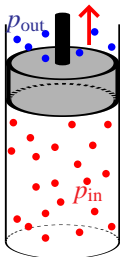
$$dW = \frac{Nk_{\text{B}}}{V}T_{\text{in}}dV, \quad dW_{\text{lost}} = \frac{Nk_{\text{B}}}{V}T_{\text{out}}dV,$$

$$\text{efficiency} = \frac{dW_{\text{useful}}}{dW_{\text{tot}}} = \frac{dW - dW_{\text{lost}}}{dW} = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}}$$

Remarkably, this is **maximum efficiency possible!** [L8].

Enthalpy

$$F_{\text{net}} = A(p_{\text{in}} - p_{\text{out}})$$



Recall specific heat at constant volume
 $dU = dQ = C_V dT \Rightarrow C_V = \left(\frac{\partial U}{\partial T}\right)_V$

At constant p , some energy $\rightarrow pdV$ work

$$dU + pdV = dQ = C_p dT$$

Define **Enthalpy**: $H = U + pV$ at constant pressure

$$\text{so } dH = d(U + pV)_p = dU + pdV = C_p dT \Rightarrow C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

Ideal gas:

$$U = \hat{c}_v N k_B T, \quad \hat{c}_v = \frac{1}{2} (\# \text{ DOF})$$

$$H = (\hat{c}_v + 1) N k_B T = \hat{c}_p N k_B T \Rightarrow \hat{c}_p = \hat{c}_v + 1$$

SUMMARY

- Thermal Energy: internal energy above $T = 0$,
 $U = \frac{3}{2}Nk_B T$ for monatomic ideal gas
Internal Energy: all energy in isolated system (w/ bond, rest E)
- Heat capacity $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ (constant volume),
 $C_p = \left(\frac{\partial H}{\partial T}\right)_p$ (constant pressure, *enthalpy* $H = U + pV$)
- Rate of heat flow in material: $\mathbf{q}(\mathbf{x}) = -k\nabla T(\mathbf{x})$
- Heat equation: $\frac{\partial T(x,t)}{\partial t} = a\nabla^2 T(\mathbf{x}, t)$
- Pressure = Force/Unit Area, $pV = Nk_B T$ ideal gas law
- Not all heat E \rightarrow useful work, $\eta = \frac{T_H - T_C}{T_H}$ will be max.