8.08 Problem Set # 9

April 6, 2005
Due April 13, 2005

Problems:

1. Consider an interacting bosonic gas in 1D. The Ginzburg-Landau free energy is given by

$$A = \int_{-\infty}^{+\infty} dx \ \left(\frac{1}{2m} |\partial_x \psi|^2 + \left(\frac{a(T)}{2} + U(x) \right) |\psi|^2 + \frac{b}{4} |\psi|^4 \right)$$

Where ψ is the amplitude of condensed bosons (the order parameter) and $a(T) = a_0(\frac{T}{T_c} - 1)$ for T near T_c. Here a_0, b and m are constants. The external potential $U(x)$ has the following form

$$U(x)|_{x<0} = +\infty, \qquad U(x)|_{x>0} = 0$$

(a) Show that there is a boson condensation for $T < T_c$ and find the amplitude of condensed bosons $\psi(x)$ for $x \to +\infty$.

(b) Near $x = 0$, the amplitude of condensed bosons is suppressed by the potential $U(x)$. To gain a more quantitative understanding of the suppression, we assume $\psi(x)$ to have a form

$$\psi(x)|_{x<0} = 0, \qquad \psi(x)|_{0<x<\xi} = \frac{x}{\xi}\psi(+\infty), \qquad \psi(x)|_{\xi<x} = \psi(+\infty)$$

We want to adjust ξ to minimize the total free energy for the above form of boson condensation. Calculate the ξ dependence of the free energy. Find the value of ξ that minimizes the free energy.

(c) Show that near T_c, ξ diverges as $\xi \propto |T_c - T|^\nu$. Find the critical exponent ν.

(The length scale ξ is called the coherent length. It is a very important length scale in superfluid. For example, the size of the vortex core is given by ξ.)

2. Problem 8.2 in K. Huang's book.

3. Problem 8.3 in K. Huang's book.

4. Problem 9.4 in K. Huang's book.