PROFESSOR: So I want to demystify a little of this equation. We sometimes use the basis to show everything that is happening. So we have a good basis. So let's look at it. So how does it all look in an orthonormal basis?

So which basis? Well, you have one. H 0 was supposed to be known. So we'll call these states n. These are the eigenstates of H 0. We'll have E n. We could put a zero. We used to have that zero there. But since we're never going to try to solve this equation in a different way, as time goes on, the time-dependent system, I said to you, doesn't have energy eigenstates. So perturbation theory is not going to be about perturbing these energies. So I will erase. E n, and there will be no confusion. That's never going to change.

Now I'm going to write an equation that may seem a little strange. We want to solve for this psi ~. So let's write an ansatz for psi ~ of t. It's going to be the following, sum over n C n n. If C ns are constants, this is definitely not right. This state must have some time-dependence. So at least, I should put a time-dependence here. But even that sounds a little wrong, a priori.

You knew that in time-independent problems, you can always write a state as a superposition of your energy eigenstate. So now, this is the interacting picture we have not any more energy eigenstates in any sense. Can I write this? Does this make sense still?

Well, the answer is yes, you can. And the reason is that whatever you can say about this energy eigenstate, they apply for early times. They apply for late times. They anyway exist. They're a basis. And what I said, they are a basis. So any wave function can be written in terms of them.

So if I look psi ~ at time 1 second, I should be able to find numbers that make this possible. And if I look at it at 2 seconds, I will find another set of numbers and make it possible. So by the fact that I've included here a time-dependence for the coefficients, it is possible to write this.

These coefficients change in time in strange ways, but that's our unknown. If I knew how they change in time, I would have solved the problem. I don't know how the change in time. But in general, we will try to find for this coefficient.

So if you have psi ~, you know at the end of the day, your goal is psi of t. So what is psi of t?

Psi of t, from this blackboard, is the action of this operator on psi ~. That operator moves through the constants and through the sum. And with the end states eigenstates of H 0, this just gives me minus I E n t over H bar n.

And this expansion should make you feel good. You say, OK, here it is. These are my states in my time-dependent we function, let's say. And it's given by this formula. If I didn't have a delta H, we know that without a delta H, psi ~ is constant. So they C ns would be constant if there is no delta H. And if the C ns are constant and there's no delta H, that's exactly how energy eigenstates evolve in time. They are evolving with exponentials of the action of each 0.

So this is consistent with all you know. If delta H turns on, the C ns are going to acquire time-dependence. And we will know what the state is doing. So this is good.

So let's plug, into the new Schrodinger equation, this expansion. And this is our new Schrodinger equation. So what do we get? I get I H bar d/dt of this. I'll write it here with a sum. I'll change to letter m. I'll use dots. C m of t dot. For time derivative, we'll many time use dots. So I'm taking the time derivative of psi ~ as in the Schrodinger equation. I go here, C m m.

And this is supposed to be equal to delta H ~ times this same state. So it's going to be sum over n C n of t delta H ~ n. So this is the Schrodinger equation for this state. We're now looking at the Schrodinger equation in a basis, because that may be a good way to solve it.

Well, one way to solve it is to do, in the right-hand side, a complete set, introduce a complete set of states. So I'll put a sum over m m m and this whole sum over n C n of t delta H n. So this will be equal to sum over m. The m is here. And sum over n, the bra can go all the way in. And you write C n of t m delta H n. So this is our Schrodinger equation still. And now it's kind of in a nice way in which we have similar letters on both sides.

A little bit of notation. I'll call this delta H ~ m n. We've done that in perturbation theory many, many times.

So what is our equation? Well, compare terms with equal value of the function in front of the state m. So we gave I H bar C m dot of t is equal to the sum over n C n or delta H m n C n of t. That's it.

It's a nice looking equation. There's a couple sets of differential equations for an infinite set of functions in which the derivatives are obtained in terms of the Hamiltonian matrix elements for the transition Hamiltonian for the perturbation times these functions there.

A little more notation here in the sense of just understanding the structure of that matrix element. That's useful, because in practice, all the tilde things, at the end of the day, we don't want them. We want the original ones, things without tilde. And we always look for them.

So what is this matrix element delta H m n ~ is m delta H ~ n. But remember what delta H ~ was. You have it here. So we have m E to the I H 0 t over H bar delta H E to the minus I H 0 t over H bar n. That's no problem. Everything is good here, because those are eigenstates.

So we know how much you get by letting this act on that state and that other exponential act on this state on the right. So what do we get? E to the I e m t over H bar. And from the other one, E n t over H bar like this. The two exponential give you that. And then you have just delta H, your original perturbation, between states of the Hamiltonian.

I'll write it this way. Delta H mn, just without the tilde, which is m delta H n. Most people call this the frequency m n. E over H bar is a frequency. So the harmonic oscillator reminds you of that, E equal H bar omega. And for easier writing, you write an omega mn. So this becomes E to the I omega mn t times delta H mn. So this is kind of a nice notation.

So this matrix elements there-- well, I'll write again the equation I H bar C m dot of t would be the sum over n of E to the I omega mn t delta H mn C n of t. That same equation has been rewritten in the viewpoint of this, where we simplified the tilde. And we can refer everything to our original basis.

So in a sense, I think this should demystify things. What's the situation? What has happened is that if you have a basis, you write an ansatz for the wave function of this form. And the coefficients are solutions of those differential equations. So we've translated the problem to something doable.

If you have lots of resources, a computer, and you solve coupled time-dependent differential equations first order. There this are not partricularly-- well, it all depends how difficult is the time-dependence here. The exponential is not bad. This? Well, it all depends. But this can be solved numerically. It can be solved with many methods. This has made the problem concrete. And we're going to try to understand how to solve it in cases of interest.