PROFESSOR: We have already begun with the degenerate perturbation theory. And we obtained the first result-- perhaps the most important result-- already. It came rather quick. And let me review what we had.

So we were talking about the Hamiltonian H0 that had degeneracies. And we were now interested to see what the perturbation does to those degenerate states. What does it do to its energies? What does it do to the states themselves?

So we called that degenerate subspace the space V capital N. And the rest of the state space of the theory was V hat. For those states in the degenerate space, we use a label n for being an n-th state or having energy En. They all have the same energy, unperturbed energy, En0.

These are the states. And with k running from 1 to n-- these are a basis of linearly independent states that span the degenerate subspace. So that's our degenerate subspace.

And then, the rest of the space is the space V hat. And it's spanned by states that we call p0's, we used to call them. And the notation is good that allows you to distinguish, with two labels, states in the degenerate subspace and, with one label, a state in the rest of the space.

The rest of the space can also have degeneracies that wouldn't matter. You see, when you do non-degenerate perturbations theories, because you are focusing on a single, non-degenerate state, it's not that you're focusing on a theory in which every state is non-degenerate. Non-degenerate perturbation theory means focusing on a non-degenerate state of a general spectrum.

Same thing with degenerate perturbation theory. You focus on a particular non-degenerate space. Whether there are other non-degenerate spaces doesn't matter. They will be treated as the rest. And that makes your life simple.

So we had our perturbative equations assume that we were going to find states nk that depend on lambda. And they would have energies Enk that depend on lambda. And looking at the Schrodinger equation, we found an expansion in which this state is the state n0 plus lambda n1 plus lambda squared n2. And this energy is En0 plus the first corrections plus the second corrections and those things.

So these were our equations that probably by now, they're starting to look pretty familiar. And

as usual, we say that a correction to a state doesn't receive corrections. We can work in a convention where it doesn't receive corrections proportional to the original state. This could be reabsorbed into normalization.

So that was the setup. And we said we would calculate things using three steps. And the first step was to act with those unperturbed states on the order lambda equation, so lambda to the 1 there. The first equation is always trivial.

So we acted in this equation. And these states are killed by this operator because they are states of energy En0. And therefore, you get the right-hand side equal to 0 when you have the n0l.

And we discovered that the consistency of our expansion requires that the chosen basis-- because, after all, you chose that basis. Nobody gave it to you. If you have a degenerate subspace, you can take any set of linearly independent vectors that are orthonormal, and that's a good basis.

It's not a uniquely defined basis. But in the chosen basis that you're working with, it must happen that the perturbation is diagonal-- delta lk. And therefore, the first order perturbations, when you take l equal to k, is given by this matrix element between nk and nk.

So this is a key result, and our first result that in the degenerate subspace, either you choose a basis that diagonalizes the perturbation to begin with, or you compute the matrix delta H in that degenerate subspace and then work and diagonalize it. So either way, you do it.

So I want to make a couple of remarks on this thing. First remark is a terminology. We can say that the degeneracy was lifted by the perturbation. And that's a great thing if it happens. We'll comment more.

It's a very nice thing if it happens, the degeneracy lifted. And you have the intuition what happens. You have these states are all here of the same energy. And then, if the degeneracy is lifted-- these are four states-- then, suddenly, one, two, three, four, as lambda changes, they split.

So their energies start to become different. And that depends on the first energy corrections being different for every single one of the states. So this means that Enk1 is different from Enl1 for the l-th state. And for the k-th state, the first energy corrections are different for all k

different for l.

So for any two states that you pick, from the list of your N-- capital N states there-- for any two states that you pick, their energies must be different. And then, the degeneracy is lifted. And that means that you've found what we will call a good basis, a basis of states that diagonalizes delta H.

And already, the states have split. You can now follow each of them and see what happens. You know what is a preferred basis. So this is nice. And so this is our first remark. Degeneracy is lifted when this happens.

And this is the case, we're going to assume, for the rest of the discussion, for the first part of the lecture, that the degeneracy is lifted. And then, we'll try to find the correction to the state. Let me comment that our first order shifts that we have found-- these eigenvalues-- are valid even if the degeneracy is not lifted.

The second remark has to do with a rule to help you do this stuff. This will be very useful in about a week, when we apply some of these things to the hydrogen atom. So first matter of notation-- some people use this, I kind of like it-- a basis in VN that diagonalizes delta H is called a good basis.

So good, bad-- it's kind of easy to relate to those things. So it's good. It gives you the answer. So we'll call it a good basis. So here comes a question that you want to answer. Typically, you have some basis that you like. And you have a delta H.

Now, you can say, OK, do I know that delta H is diagonal in this basis? Maybe I don't. So what can I do? I can diagonalize it and check. But sometimes, there is a better way, and it helps you explain why the matrix delta H is diagonal.

So this is a shortcut. It's a conceptual idea that you can check that delta H is diagonal without diagonalizing it. So here it is. So here's the rule-- "rule."

If the basis vectors are eigenstates of a Hermitian K with different eigenvalues-- so so far, what am I saying? I'm bringing up a new operator. Somebody gives you another operator K-- Hermitian. And then, you check that, after all, for this Hermitian operator, the basis, or the basis vectors, are eigenstates of that operator and with different eigenvalues. All of them have different eigenvalues.

[MUMBLING]

And K commutes with delta H, then the basis is good. Basis is good. Now, first time I heard this-- I still remember-- it seemed to me like a very complicated rule. Just diagonalize and forget about it. No. But it actually is very helpful. So let's explain this rule.

So K with delta H is equal to 0. So let's assume we have two eigenstates. So "proof" here. Two eigenstates-- n0p and n0q. And the K eigenvalues are lambda p and lambda q that are different from each other.

So we said that the basis states are supposed to be eigenstates of this Hermitian operator K and all of them with different eigenvalues. So here, I wrote two states that are part of our basis. And I say that K eigenvalues are lambda p and lambda q, and they are different from each other. So far, so good.

So then we can form the following-- n0p, the commutator of K with delta H with K or K with delta H n0q. Consider that matrix element. Since we say that K commutes with delta H, this must be 0. That operator that we've put in between the states is 0. So this is 0 because this is 0.

But on the other hand, let's expand this. This is a commutator. So you have K times delta H minus delta H times K. Since it's a Hermitian operator, the first term K is near this state. And being Hermitian, you get the eigenvalue of 8, which is lambda p n0p delta H n0q. That's from the first term.

And the second term is a minus the delta H on the left. So you get the minus n0qp delta H. And the K acting on the right this time gives you lambda q. So this is equal to lambda p minus lambda q times n-- it's the same factor for both terms-- delta H n0q.

OK. So that matrix element has been evaluated. And it's now equal to the product of the difference of eigenvalues times the matrix element of delta H. But here, we know that this product must be 0. But since the two eigenvalues are different, this factor is not 0.

So the only possibility is that the second factor is 0. And you have shown, therefore, at this moment, since this is 0, this implies that the n0p delta H n0q is equal to 0 whenever p is different from q. And that's exactly the statement that all the off-diagonal elements of delta H are 0.

So indeed, it has been diagonalized. Or you can, without diagonalizing the delta H, you can say, oh, look at this basis. This basis is good. Because there is this operator-- many times it will be angular momentum, lz, it may be other thing-- for which all the states have different eigenvalues. And it commutes with a perturbation. So you look for an operator that commutes with a perturbation for which your basis vectors may be eigenstates.