

PROFESSOR: So let's attempt to solve something. And that first step is going to be nice and simple and very illuminating, as well. So we're going to do a three step procedure. Three step procedure. Three step procedure. And basically, we're going to use this equation, the ordered lambda equation, to find the energy corrections. One, ordered lambda equation for energy corrections. Two, ordered lambda equation to find the component of n_1k in v hat. And three, we're going to look at the ordered lambda 2 equation to find the component of that same state in the degenerate subspace.

So let's do the first step. And for that, we're going to overlap that equation, this ordered lambda equation. So for step one, we're going to hit the equation with n_0l from the left.

OK. n_0l from the left. Here it comes. Boom. It's here. What happens now? Well, h_0 on that state, n_0l , that is part of the degenerate subspace. So it has energy n_0 with respect to the original unperturbed Hamiltonian. So you get n_0 minus n_0 , it's 0. So the left hand side vanishes. Kills left hand side.

So we just look at the right hand side. And we got $n_0l \text{Enk1} - \delta H n_0k = 0$. That was the right hand side. So far, so good. Nice and simple.

From here, what do we get? Let's write it just by moving one term to the left and one term to the right. $n_0k \text{Enk1} - \delta H l$. And this should be true for all k and l in 1 up to n . Because those equations are for all k and we could have hit with all l .

And now this is an amazingly nice equation. It tells you a nice story. What is this story? The story is that look at this. This equation says that if you want this to hold it better be that δH is diagonal on the basis that you're using. Because for k and l running in n by n . So this says that δH is diagonal in the chosen basis of the subspace v_n . δH is not diagonal elsewhere. On the big space, it may not be diagonal. But on the little degenerate space, it better be diagonal.

So what it says is that, if you want to start your perturbation theory, you cannot use an arbitrary basis of states. Maybe you chose it wrong. You have to start in the degenerate subspace with a basis that makes the perturbation diagonal.

And that's what our little example had. It says, even to begin with, you should have started with

the basis that makes the perturbation diagonal. In that case, you will find the formulas giving you the right answers. And then, once you have the matrix being diagonal, you can take I equal to k . And then you'll find on the right hand sides $E_{nk}^{(1)}$ is equal to $n \cdot I$ is equal to k , so $n_0 k \Delta H_{n0k}$.

So the energy corrections are the diagonal elements of this matrix. What you would expect. So we will write this as ΔH . We could put k , but to remind you, it's degenerate. You put n_k .

So we've done the first step and we've realized that this perturbation is a little funny. You really have to get started with the right basis. And what is the right basis? The basis that makes ΔH diagonal. We'll see that everything is much simpler if the eigenvalues of this matrix are all different. So the degeneracy is broken. Then things are easier. If the degeneracy is not broken, then the degeneracy may be broken to order λ^2 , which is more interesting and it will be something we'll study next time. So that's all.