PROFESSOR: All right. We have this, so let's complete the Einstein discussion. And that's practical formulas that are relevant to some of the exercises you will find in homework.

So when we set up the rates in Einstein's argument, we said that for spontaneous emission, Einstein's spontaneous emission, he put a rate that was-- he said the number of transitions that occur due to spontaneous emission is proportional to the number of atoms, Nb, the number of photons present, times some coefficient that I don't know, Bab.

That's how the rate of atoms changing and going from the top level to the low level was. But this is the one per atom. This is the transition rate per atom, multiplied by the number of atoms present in the top level gave you the rate for spontaneous emission. This quantity is the one that is equal to that omega ab. So this is equal to omega ba, like this.

Therefore, if you compare the Bab of Einstein, it's all except the U. So Bab is 4 pi squared over 3h squared dab vector squared. So Einstein could not do that because he didn't have quantum theory. But we do, and we now can calculate from first principles this quantity in an atomic transition.

And the other good thing was that even though there was a rate for spontaneous emission, that rate was fixed by Einstein's argument to be h bar omega ba cubed over pi c cubed Bab.

So if you knew the b coefficient, you knew the a coefficient for spontaneous emission-- many times were interested in spontaneous emission. So we calculate the stimulated emission coefficient, here, and that's given in terms of this matrix element. We might as well give the general formula.

So you just substitute the Bab, and now we have that A is equal to 4/3 omega ba cubed over h bar c cubed Bab squared 4/3 omega ba cubed. There's one h bar in the bottom. There's the c cubed. Yep.

So that completes that argument. We now have combined our development in time dependent perturbation theory with a statistical physics argument. [INAUDIBLE] to determine those rates.

A little more is needed if you have to compute a practical rate, as you will have to do for the homework. It's just the concept of a lifetime. And a couple of selection rules.

So let me make a few comments about it. We get, for example, a decay rate, A. It's a rate for spontaneous emission. You have an atom. There's a decay rate, A. So if you have a population of atoms, N atoms, at t equals 0, and A being the decay rate per atom, it means that the number of states that are changing in a little time, dt, is minus A times dt-- the rate multiplied by the time per atom-- times the number of atoms.

So dN dt is minus AN. And the number of atoms as a function of t go N at t equals 0 times e to the minus At. So since we always call processes that go e to the minus t over tau-- tau is the lifetime. That is nothing else than 1 over A.

So you get the decay rate. It's synonymous to a lifetime. You just invert it. No big problem. Nevertheless, if you have an object, sometimes can decay in different ways. It can decay maybe to one type of state, another type of state, a third type of state. So the total A might be equal to A1 plus A2 when there are various decay channels.

You can decay-- a system can decay from an excited state. It can go to one state to another state to a third state. Each one contributes a total decay rate because a total disappearance of those atoms is because they either go into one or the other or the third. So this is true that the total decay rate is that.

So this gives you a relation. The total lifetime, which is 1 over this, is the sum of the inverses of the partial lifetimes. So if you have a process that can go in various ways, you may want to combine them to get a total lifetime.

So the last thing we should say about this-- the last thing we're going to do is mention what happens when you really try to calculate these things for hydrogen levels. You're going to have the ability to calculate the lifetime of an excited state of hydrogen now. That's quite the thing, to be able to get a number that is correct up to factors of 2 and is measurable.

So what do you have to consider? Well, the only thing we haven't quite done is computed this matrix element of the dipole operator. You see, everything else that could be done was done. We integrated over the incoherent radiation, we found the Fermi golden rule, we have everything simplified. But still, the matrix element is something that you have to calculate.

So there's some selection rules. Rules. And this will be discussed in recitation. It's pretty important stuff. I'll just mention the results. It's very readable in Griffiths. And it has to do with the matrix elements.

So d is the dipole operator, so it boils down to r. So if you really knew n l prime m prime, the matrix elements of nlm, if you knew all of this, you would be done. You could calculate anything.

So the question is, when do these elements exist, and when are they nonzero? You need them to be nonzero to get a nonzero transition rate. So when do they not vanish?

So the only cases these-- I'll summarize the answer-- are nonzero, these can be nonzero-- there can still be accidents and they could cancel-- only if delta m, the change in m from one side to another, which is m prime minus m, is either equal to 0 or plus or minus 1.

You cannot change that quantum state of a system with a dipole interaction unless m changes at most by 1. And moreover, the one that is quite interesting as well is that delta l, which is l prime minus l, must only be plus or minus 1.

So those changes have to happen. So now you can at least get a qualitative understanding of the hydrogen atom stability. You have the ground state here, 1S. Here is 2S, 2P, 3S, 3P, and 3D. Let's see who can go to who.

Well, the ground state cannot go anywhere. That's at the bottom. So OK. 2S going to 1S. Is it possible?

AUDIENCE: No.

PROFESSOR: No. Very good. It violates delta l. So this doesn't happen. So if this doesn't happen, this 2S can almost go nowhere because it doesn't have-- it has to go by spontaneous emission. Something has to go to something lower, so it almost doesn't go anywhere.

So actually, it has a long lifetime, and it can decay via two photons or more complicated decays. And we will not consider it. But this is fairly stable.

Now, this one, 2P, can go here because it can go from l equal 1 to l equal 0, and it can change m by some value. The 3S you would say, oh, it cannot go here, cannot go here, but it can go laterally in that direction because it can go change l from 1 to 0. So it's a change of l, and that is possible.

The 3P can go here. And my diagram starting to get cluttered. And it could go here as well, all the way down. And the 3D can only go here. Cannot go up to here because that would be l

equal 2. So that is your diagram of possible decays of hydrogen. And you're going to be computing a couple of those.