PROFESSOR: So my comments. First one-- if the psi n of t can be chosen to be real, the geometric face vanishes. So why is that?

The geometric face is, remember, the integral of this new factor, which was i psi n of t psi n of t dot. You had to integrate that thing to get the geometric face. We explained that this quantity in general is-- well, in general, it's always imaginary, this quantity. And then with an i, this is a real quantity which we've been using.

But if this is imaginary-- look at this. This is imaginary. How can it be imaginary if psi is real? It can't be. If psi is real, imagine doing that integral. You can kind of imagine. So it has to be 0 if psi is real. Yes?

AUDIENCE: [INAUDIBLE]. Why did we say that was imaginary?

PROFESSOR: We proved this was imaginary. We did calculate the derivative with respect to time of psi with psi, and we proved that this was imaginary. And now I'm saying if this was imaginary but if psi is real, the only possibility is that this is 0. But you can prove, in fact, that this is 0, if this is real. I can do it in a second.

It's i integral psi n of p-- because it's real-- d dt of psi n of t. Output an x here, and this is dx. That's what it is if psi is real. Otherwise there would be a star here. But if it's real, there's no star.

But this is just the integral of 1/2 of d dt of psi of x and t squared dx. The d dt is that. And then the d dt goes out of the integral-- so this is i over 2 d dt of the integral of psi squared of x and t dx. But that integral is 1, so the derivative is 0. So there's no geometric face. So if you have real instantaneous eigenstates, don't even think of Berry's phase.

There's another case where you don't get a Berry's phase. So when I'm speaking of Berry's phase at this moment, I mean the Berry's phase from a closed pathing configuration space. So if the configuration space is one-dimensional, the Berry phase vanishes. Barry's phase vanishes for 1v configuration space.

Why is that? Well, what do we have to do? The Berry's phase will be the integral over the closed path in the configuration space psi of r d, d r-- because there's just one side of r-- d r. You have to integrate that thing. That is the Berry phase-- is the integral of the Berry connection over-- this is capital R-- over the space.

But now, if this configuration space is one-dimensional-- this is r-- a closed path is a path that goes like this and black. It retraces itself. So it integrates this quantity with increasing r, and then it integrates the same quantity with decreasing r, and the two cancel, and this phase is equal to 0.

So you cannot get a Berry's phase if you have one dimension. You cannot get the Berry's phase if you have real instantaneous eigenstates. But you can get a Berry's phase in two dimensions, in three dimensions, and there are several examples. I will mention one example, and and then leave Barry's phase for some exercises.

So in 3D, for example-- 3D's nice. A 3D configuration space is the perfect place to confuse yourself, because you have three dimensions of configuration and three dimensions of space. So nice interplay between them-- r1, r2, r3. And then you have an integral-- the Berry's phase is an integral-- over a closed path here. So let's call gamma and let's call this surface, s, whose boundary is gamma.

So what is the Berry phase? The Berry phase is the integral over gamma of the Berry connection d r. But what is that? That is, by Stokes' theorem, the integral over the surface of the curl of the Berry connection times the area. And so the area on that surface.

Stokes' theorem-- remember in E&M? So here was a vector potential gives you a magnetic field-- so the integral of a along a loop was equal to the flux of the magnetic field through the surface. And now the berry phase along the loop is equal to the integral of the-- we should call it Berry magnetic field? No. When people think berries-- curvature. As in the sense that the magnetic field is the curvature of that connection. So this is called the Berry's curvature, but you think about magnetic field-- Berry's curvature.

So the Berry's curvature-- people go with d is the curl sub r of the Berry connection a of r. It's the magnetic field, so it's the integral of d over that surface. So they're nice analogies.

So one example you will do in recitation-- I hope-- you have Max over there-- is the classic example of a spin in a magnetic field. So I will just say a couple of words as an introduction, but it's a very nice computation. It's a great exercise to figure out if you understand all these things.

It's done in [INAUDIBLE] explicitly in some ways, and it's a great thing to practice. So if you want the little challenge-- I know you're busy with papers and other things. but the idea is that you have a magnetic field-- the cogs of a magnetic field, B0n-- along some direction n, and you have a spin 1/2 particle pointing in that direction n. But then you start letting it vary in time-- that unit vector.

So this unit vector varies in time and traces a path. And if it's idiomatic process, the spin will remain in the instantaneous energy eigenstate, which is spin up in this direction, and it will track the magnetic field, basically. There will be some small probability it flips, but it's small. And if this is an idiomatic process, that error can be small.

So as you move around here, there will be a geometric face, and the geometric face is quite nice. So for the spin up state-- m+ state-- if it follows this thing, the geometric phase for the spin up state in this closed path in configuration space-- this is the configuration space of the magnetic field-- will just be given by minus 1/2. This comes from spin 1/2 times an invariant of this loop, which is the solid angle traced by this loop.

So the geometric phase that's acquired by this motion is proportional just to the solid angle of this loop. It's a very nice result. Shows how geometric everything is.