

PROFESSOR: OK. So time to complicate the model a little bit to get more interesting physics from it. So what am I going to do? I'm going to add an extra term. So the system goes now to an H of t that is going to have the αt over 2. And now it's gonna have a little term of diagonal.

So there's several ways of thinking about this. H_{12} is going to be constant in time. So it's a number. H_{12} star is its complex conjugate. If you're looking at t equals 0, H becomes just H_{12} , H_{12} star. And the energy eigenstate or the energy eigenvalues of this matrix are plus or minus the norm of H_{12} , the absolute value of H_{12} . Just put a couple of lambdas, calculate the eigenvalues. It involves H_{12} times H_{12} star, which is the square of the norm of H_{12} . And the energies are those.

So look at what's happening in your energy diagram as a function of t . At t equals 0, there are two energies, H_{12} and minus H_{12} . Those are the energies at time equals 0. We're trying to get the analog of what was going on there. And now you could say, OK, at time equals 0, that's what I get. What do I get at large times?

Well, at large times, these are dominant. And these are very small. So you must get something similar to these arrows here. So what I'll draw here is this and this. And I cannot trust this here for small time. But presumably, this is about right here and about right here and about right here and about right here.

And the states must be the same one, 0, 1 here; 0, 1; 1, 0; 1, 0. So that's what you know just without doing any calculation. That's what your system does. And now, a Hamiltonian in general doesn't get levels crossing. That requires a coincidence like having no off-diagonal element. So actually, what this will give you is this.

So that's how the system will look as a function of time. Those are the energy levels as a function of time, the instantaneous energy levels. At every instant of time, you now have the energies. These are the energies of this matrix, the energies, the eigenvalues. One line computation, our E plus minus equal plus or minus square root of H_{12} squared plus α squared t squared over 4, so plus or minus. So this is E plus and E minus of t .

All right. So here we go. We have a real system. And we have something quite interesting actually. When you let H_{12} go to 0, you're back to that place. And you just zoom through like here you did. You just go through and go through. So when the levels do that, you just

continue through any particular state you were in. You don't start here and then go here. You just go through. That's what you prove here.

And therefore, when you take the limit of H_{12} going to 0, if H_{12} goes to 0, these things collapse. And you're back there. And you zoom by through. So when H_{12} will be very, very small, you will be likely, in fact, more likely to make the transition than not to make it. So if you have a system for-- this is just a micron separated here, a milli-electron volt-- let's be more precise here-- you're more likely to zoom through and make the nonadiabatic transition because this will not be an adiabatic process. They're getting to close to each other.

On the other hand, if you are far away, you're going to be very unlikely to make the nonadiabatic transition because you are very separated. The nice thing about this problem is that it can be solved analytically. Not terribly easy. It involves a little bit of hypergeometric functions and some differential equations. But it can be solved.

You will solve it numerically in the homework. I will say a few words about it. And we'll discuss the transition in this case. And the answer is known analytically. It's a very famous result. In fact, many people have written papers trying to give simple derivations of this answer.

So in order to just write the answer and to see how it looks-- so this is the answer for a transition-- we try to discuss the notion of adiabatic process here. When is this transition adiabatic?

And what we do is this. You can imagine taking a tangent here and another tangent here. And when it hits the lines, the linear lines that we plotted here, bring them down. So you get a rectangle here. So you hit the αt over 2 line. And you call this $2\tau d$ or 2τ , no, τd .

OK. So let's figure out what that is because I say that this time is the time that going up on the line, αt over 2, this time $2\tau d$, you get the height, H_{12} . That means that H_{12} is α times $2\tau d$ divided by 2. So τd is H_{12} over α .

So τd is usually thought as the timescale associated to the change in the Hamiltonian. That is you have an original Hamiltonian, that's where this linear [INAUDIBLE]. And over a time $2d$, the shape is changed into the final Hamiltonian. This is the process in which the original system is changed into the new system within this timescale. The timescale τd .

But we have another time that is interesting here. You see, while this change is happening, the

separation of the energy levels is by this distance H_{12} . So the Hamiltonian at t equals 0, the Hamiltonian looks like $0, H_{12}, H_{12}$, here.

And if you have two states governed by such Hamiltonian that is valid near time equals 0, there's going to be oscillations between states here. You can go from the first state to the second state with some frequency governed by this number. That is called the Rabi frequency, Rabi oscillation. It's something you've done in 805. In two-state systems, you oscillate.

And the frequency of oscillation between the states 1 and 2 is H_{12} divided by \hbar or 2π divided by the period T_{12} . That's the definition of the frequency.

So what do we have here? The process is adiabatic if the time τ_d for the change to happen is much larger than the period T_{12} of the oscillation. So you can think of this system during the time it spends in this box like a two-level system separated by H_{12} that oscillates between the two states. And in that time, $2\tau_d$, all the change is happening.

So you should have that τ_d is much bigger than the period of oscillation. And this corresponds too because T_{12} is the inverse of ω_{12} to $\omega_{12}\tau_d$ much greater than 1, so either one. And that would be adiabatic condition.

Let's put the numbers here. ω_{12} is H_{12} norm over \hbar . And τ_d -- we often found it there-- is H_{12} over α . So over α . So this is much greater than 1. I don't know. This 2 here has nothing to do with that formula.

So this is the adiabatic condition. And this is reasonable. You see, what is going to make it adiabatic? The more these branches are separated, the more difficult the transition, the better the adiabatic approximation. And it's this thing here.

The bigger this number, the better you are. The slower α is, the lower the slope is, the more time it's going to take, the better the adiabatic approximation. So this is really the thing. And the final formula that I will write here is the probability for a nonadiabatic transition is exponential of minus $2\pi\omega_{12}\tau_d$. So the probability that you cross the thing and jump from this down to here is this. It's suppressed by the adiabatic factor. And that's what you will check in the homework.