

**PROFESSOR:** So that's part of this story. Let's try to understand it a little better still. So what does the state look like? Well the state looks like  $e^{ik_x x}$  times a state of the oscillator and  $y$ .

OK. So what is happening here? We could re-solve this problem using a different gauge. And you will. You will solve it at least once or twice. Again, this problem using a different gauge.

And the solutions are going to be looking a little different but, of course, you're going to find the same Landau levels and the same infinite degeneracy. The wave functions will look sometimes a little more intuitive. So in this case, one calculation that is interesting to try to understand the physics of this degeneracy is to work roughly a little heuristically on a finite size sample. Imagine a material but it's now finite size.

So let me remark to you what are the degrees of freedom you have here. Suppose you solve this Schrodinger equation in a different gauge, a symmetric gauge in which there is an  $a_y$  and an  $a_x$ . Solutions then are going to look a little more like circular orbits. There's a little more mathematics involved in solving it but they're going to look a little nicer.

But how are they related to this one? Well anyone with a circular orbit must be related to this solution by first forming a superposition of those solutions, maybe localizing it or doing something and then doing a gauge transformation. So in order to compare your solutions in different gauges, you have to dig into gauge, you have an infinite degeneracy, and you have gauge transformation. So to see what state here corresponds to a particular state here, it may be the gauge transformation of a particular superposition in this side. So it's, in general, not all that easy to do.

OK. So let's take a count state in a finite sample. So same picture, but now the material is here. And we'll put  $L_x$  and  $L_y$  here. So finite size in the  $L_x$ , finite size in  $L_y$ . So given our intuition with quantization, this suggests that we impose periodic boundary conditions in  $x$  and try to quantize the  $k_x$  here.

In general, if you're imposing thinking of very large boxes, which is the case here, it doesn't matter much whether you impose periodic or vanishing boundary conditions or anything essentially at large number of states it makes no difference. So we quantize in  $x$ . So we want  $e^{ik_x x}$  to be periodic under  $x$  goes to  $x$  plus  $L_x$ . I'm almost done with sine. So  $k_x L_x$  will have to be equal to a multiple with  $N_x$ .

Since we know that  $Y_0$  is equal to minus  $k_x l_b$  squared, we should take  $N_x$  negative so that you're within the sample. You must be in  $y$  positive and therefore  $k_x$  should be negative,  $N_x$  should be negative. And now I have a way to count because I can take  $N_x$  negative up to some value minus  $N_x$  bar. And when  $N_x$  grows,  $k_x$  grows and  $y$  grows. So I can take the last  $N_x$  that I can use is the one in which the orbit is still in the sample up to the value  $Y_0$ .

So this number is really the degeneracy because this is how many values of  $N_x$  I can have, from minus  $N_x$  up to 0, are the number of values that are consistent with a state still in this sample. So  $Y_0$  equal to  $L_y$  should be equal to minus  $k_x$  times  $l_b$  squared. So it's minus  $2\pi N_x$  bar over  $L_x$  times  $l_b$  squared. And this gives you  $N_x$ . We can solve for  $N_x$  there.

$N_x$  is  $L_y L_x$  over  $l_b$  squared over  $1$  over  $2\pi$ . So  $N_x$  is the degeneracy. This is the degeneracy. And it's equal to the area divided by  $l_b$  squared, which is  $\hbar c$  over  $qB$  times  $1$  over  $2\pi$ . So it's equal to area times  $b$  divided by  $2\pi \hbar c$  over  $q$ .

So we're back to the kind of thing we were saying before in which the degeneracy is equal to the flux divided by the flux quantum that we figured out earlier today. So this is how much states you can put on the sample. So you're given a magnetic field, a Tesla and you have some area. You find the  $\phi$ , you divide by  $\phi_0$ , and that's the number of degenerate states of each Landau level.

So in particular, given that we have that number that  $\phi_0$  is equal to about  $2 \times 10^{-7}$  Gauss then centimeters squared. If you have a sample of  $1$  centimeter squared and you put one Gauss, the value of the flux over  $\phi_0$  would be  $1$  Gauss centimeter squared over  $2 \times 10^{-7}$  same units. So it's about  $5$  million states. That's just to give you an idea of how big the numbers are. That's the degeneracy.

So this is a classic problem. Very important in condensed matter physics. Is a first step in trying to understand quantum Hall effect on many things. And it's important to solve it and think it in several ways. And I think you will be doing that in homework and recitation.