**PROFESSOR:** So what do we do? We are going to sum over final states the probability to go from i to final at time t0 to first order. Since the sum of our final states is really a continuum, this is represented by the integral of the f i t0 1, multiplied by the number of states at every little interval. So this will go rho of Ef dEf. So this is what we developed about the number of states. So I'm replacing this-- I have to sum but I basically decide to call this little dN, the little number of states in here, and then I'm going to integrate this probability, so the number of states over there, and therefore the dN is replaced by rho times dEf.

So then this whole transition probability will be 4 integral, I'm writing now the integral, VfI squared, sine squared, omega f i t0 over 2 Ef minus Ei squared, rho of Ef, d of Ef. And you would say at this moment, OK, this is as far as you go, so that must be Fermi's golden rule, because we don't know rho of Ef, it's different in different cases, so we have to do that integral and we'll get our answer. But the great thing about this golden rule is that you can go far and you can do the integral.

Now I don't even know, this VfI also depends on the energy, how am I ever going to do the integral? That seems outrageous. Well, let's try to do it, and part of the idea will be that we're going to be led to the concept that we already emphasized here because of this suppression, that only a narrow band of states contribute, and in that narrow band, if the narrow band is narrow enough, in that region VfI high be approximately constant in a narrow enough region, and rho will be approximately constant.

So we'll take them out of the integral, do the rest of the integral, and see later whether the way we're doing the integral shows that this idea is justified. So I'll just-- you know, sometimes you have to do these things, of making the next step, so I'll do that. I'll take these things out, assuming they're constant enough, and then we'll get 4 VfI squared, rho of E, what should I put here? E sub i, is that right? Because if it's all evaluated at the initial energy Ei, if only a narrow band will contribute, I'll put an h squared here so that this will become omega fi, and now I will integrate over the sum range of energies the function sine squared omega f t0 over 2, over omega fi squared dEf.

So I just took the thing out of the integral and we're going to hope for some luck here. Whenever you have an integral like that it probably is a good idea to plot what you're integrating and think about it and see if you're going to get whatever you wanted. Look, I don't know how far I'm going to integrate, I probably don't want to integrate too far because then these functions that I took out of the integral are not constants, so let's see what this looks like, the integrand, this function here.

Well, sine squared of x over x squared goes to 1, you know when-- this we're plotting as a function of omega fi. Why? Time is not really what we're plotting into this thing, we're plotting-- we're integrating our energy, Ef, omega fi is Ef minus Ei, so omega is the variable you should be plotting, and when omega goes to zero, this whole interval goes like t0 squared over 4, and then sine squared of x over x squared does this thing, and the first step here is 2 pi over t0, 2 pi over t0 and so on.

And now you smile. Why? Because it's looking good, this thing. First what's going to be this area? Well, if I look at this lobe, roughly, I would say height t0 squared with 1 over t0, answer proportional to t0. This whole integral is going to be proportional to t0. The magic of the combination of the x squared growth, t0 squared and the oscillation is making into this integral being linear in t0, which is the probability the transition [INAUDIBLE] is going to grow linearly is going to be a rate, as we expected. So this is looking very good.

Then we can attempt to see that also most of the contribution here happens within this range to the integral. If you look at the integral of sine squared x over x squared, 90% of the integral comes from here. By the time you have these ones you're up to 95% of the integral. Most of the integral comes within those lobes. And look what I'm going to say, I'm going to say, look, I'm going to try to wait long enough, t0 is going to be long enough so that this narrow thing is going to be narrower and narrower and therefore most of the integral is going to come from omega fi equal to 0, which means the f equals to Ei. If I wait long enough with t0, this is very narrow, and even all the other extra bumps are already 4 pi over t0 over here is just going to do it without any problem, it's going to fit in.

So another way of thinking of this is to say, look, you could have argued that this is going to be linear in t0 if you just change variables here, absorb the t0 into the energy, change variable, and the t0 will go out of the integral in some way, but that is only true if the limits go from minus infinity to plus infinity. So I cannot really integrate from minus infinity to plus infinity in the final energies, but I don't need to because most of the integral comes from this big lobe here, and if t0 is sufficiently large, it is really within no energy with respect to the energy, Ei.

So our next step is to simply declare that a good approximation to this integral is to integrate the whole thing from minus infinity to infinity, so let me say this. Suppose here in this range omega fi is in between 2 pi over t0 and minus 2 pi over t0. What does this tell us that omega fi is in this region? Well, this is Ef minus Ei over h bar so this actually tells you Ef is in between Ei plus 2 pi h bar over t0, and Ei minus 2 pi h bar over t0.

All right, so this is the energy range and as t0 becomes larger and larger, the window for Ef is smaller and smaller, and we have energy conserving. So let's look at our integral again, the integral is I, that's for the integral, this whole thing, will be equal to integral dEf sine squared of omega fi t0 over 2 over omega fi squared. So what do we do? We call this a variable, u, equal omega fi t0 over 2, so that du is dEf t0 over 2 h bar, because omega fi is Ef minus Ei, and Ef is your variable of integration. So you must substitute the dEf here and the rest of the integron. So what do we get from the dEf and the other part? You get at the end h bar t0 over 2 integral from-- well, let's leave it, sine squared u over u squared du.

So look at this, the omega fi squared, by the time you get here omega fi goes like 1 over time, so when it's down here we'll give you a time squared, but the dE gives you 1 over time so at the end of the day we get the desired linear dependence on t0 here, only if the integral doesn't have t0 in here, and it will not have it if you extend it from minus infinity to infinity. And there's no error, really, in extending it from minus infinity to infinity because you basically know that n lobes are going to fit here and are going to be accurate, because there is little energy change if t0 is large enough. If t0 is large enough, even a 20 pi h bar and a 20 by h bar here, that still will do it. So we integrate like that, we extend it, and we get this whole integral has value pi, so we get h bar t0 pi over 2, that's our integral, I.

So our transition probability, what is it? We have it there, over there, we'll have the sum over final states, i to f of t0, first order is equal to the integral times this quantity, so that quantity is h t0 pi over 2, so it's 4, what do we have, VfI squared, rho of Ei over h squared, then h bar t0 pi over 2. So your final answer for this thing is 2 pi over h bar VfI squared, rho of Ei t0. So let's box, this is a very nice result, it's almost Fermi's golden rule by now. Let's put a time t here, t0 is a label, not to confuse our time integrals or things like that, so we could put the time, t, here, is 2 pi over h bar VfI squared rho of Ei t.

From here we have a transition rate, so a transition rate is probability of transition per unit time, so a transition rate would be defined as the probability of transition after a time t, divided by the time t that has elapsed, and happily, this has worked out so that our transition rate, w is 2 pi over h bar VfI squared rho of Ei, and this is Fermi's golden rule, a formula for the transition rate to the continuum of final states.

You see, when I see [INAUDIBLE] it almost seemed you still have to integrate, there is a rho of E and let's integrate [INAUDIBLE] but the interval has been done and it says transmission amplitude squared evaluated at the state initial and final with the same energy and final state, and the rho evaluated at the energy of the initial state. You don't have to do more with that.

So we have this formula, let's look at a couple more things. Do units work out? Yes, this is transition per unit, this is 1 over time, this is energy squared, this is 1 over energy, and this is an h bar, this will give you 1 over time, so this thing goes well. How about our assumptions? This was calculated using some time t, we used to call it t0. How large does it have to be? Well, the larger it is the more accurate the integral is, but you don't want to take it too large, either, because the larger it is, the transition probability eventually goes wrong at first order of perturbation theory. So this argument is valid if there is a time, t0, that is large enough so that within this error bars, rho and the transition matrix elements are constant so that your integral is valid. But this t0 being large enough should be small enough that the transition probability doesn't become anywhere near 1.

That will happen in general or if VfI is sufficiently small, so when VfI is sufficiently small, this will always hold, and in physical applications this happens and it's OK. So that's our presentation and derivation of Fermi's golden rule, and we will turn now to one application and we will discuss.