PROFESSOR: I'm going to write this e to the ikz somewhat differently so that you appreciate more what it is. So e to the ikz, I'll write it as square root of 4 pi over k. You say, where does that k come from? We'll see in a second. Sum over l, square root of 2l plus 1, i to the l, yl0, 1 over 2i. Basically what I'm going to do is I'm expanding this jl function for large x. So I'm going to take this equation and I'm going to expand it so that the argument is large, which means r is large.

I want to describe for you those waves that are really going on here. So since the argument is kr and you have a kr here, that's the origin of the k that I pulled out in here. And then you have e to the i kr minus l pi over 2 over r, minus e to the minus i kr minus l pi over 2 over r. And this is valid for r much bigger than a. It's an approximate thing, because we approximated the Bessel function. This is exact. This equation is exact for all r.

But that equation, now, is not. That equation is showing you that you have here an outgoing wave because is e to the i kr times e to the minus i et, the energy terms time. So that's an outgoing wave. This is an ongoing wave. So this is ingoing and this is outgoing.

So I'm going to say a couple of things about this that are going to play some role later. It's a series of comments. Because this is a particular expansion and due to the fact that this is a general expansion, you could say that this is an approximate solution of the Schrodinger equation far away. But in fact, each term, each value of l is an approximate solution. This is not an approximate solution because we added over many ls and somehow they helped to create an approximate solution. This is an approximate solution because each l is an approximate solution, because to get a solution, you could have said l is equal to 50, and that's all that I'm going to use, and that's an exact solution. And therefore when I look far away it will look like that with l equal 50, and that would be an approximate solution that can be extended to a full solution.

So each l term here is actually independent. If you tell me a wave looks like that far away, I would say good, yes, that's possible and that comes from a solution. You're getting an approximate solution. And you could make it exact by turning the exponentials into Bessel functions.

Even more, we will show-- we'll discuss it a little later-- I could get a solution that is approximate by considering either this wave or that wave. Suppose we just have the outgoing wave. The fact that the Schrodinger equation works for this one approximately doesn't require the ingoing wave. The Schrodinger equation doesn't mix, really, the ingoing waves and the outgoing waves. It keeps them separate. So this is an approximate solution and this is an approximate solution as well.

Every term here, independently, every l and every ingoing and every outgoing wave is a good approximate solution of the Schrodinger equation. Now this is important to emphasize because this is called partial wave. So the term partial wave, waves, refers to the various l's that work independently.

Now to motivate what we're going to do, I'm going to go back to the case of one dimension so that you need the map to orient yourself in the argument we're going to do now. So in order to understand partial waves and how they work, let's think about the one-dimensional case for a minute. I used to do that in 804. I don't know if that has been done by other instructors, but we've done in 804 that stuff. So let me tell you about it and how it works in that case.

So this is an aside 1D case of scattering. So in scattering in one dimension, one usually puts a hard wall at x equals 0. There is some potential up to x equals a, and then there's nothing. It's a finite range potential. So even in the case of d equals 1, you look for a solution when there is no potential. This is the v of x. This is the x-axis. If v is equal to 0, you would have a solution. Phi of x would be a solution, energy eigenstate, and would be sine of kx.

This is the analog of our e to the ikz, that we call it, phi of r. Here you have a phi of x, which is the sine of kx, and it's your energy eigenstate. That's the solution if v is equal to 0. If v is equal to 0, the wave must vanish at the hard wall, at x equals 0. So that's your solution.

On the other hand-- or, well, we can also write it as 1 over 2i e to the ikx minus e to the minus ikx. And this wave is the incoming wave, and this is the outgoing wave. Very analogous to above, because this x is defined to be positive and plays the role of radius. In this problem, incoming means going down in x. Incoming in spherical dimensions means going down in r. So we're doing something very analogous. That's why the study of scattering in one dimension is a good preparation for the study of scattering in three dimensions.

So you have an incoming wave and an outgoing wave, also because, again, there's an e to the minus iet over h bar. So here it is. What we try to do is to write a scattering solution corresponding to the same physics-- so the scattering solution, psi of x, is the full solution-- is going to have the same incoming wave because that's the physics that we're trying to understand.

We put the same incoming wave and we try to see what happens. That's what do we do when we do scattering. So if we want to compare our solution to the case when v is equal to 0, we put the same incoming wave. But then solving the scattering problem means finding the outgoing wave, which now may be different. Because, sadly, when you have a potential, you have something in, that you have control what you send in, but you have no control what comes out. That's solving the scattering problem.

So the thing that you have here must be an outgoing wave, e to the ikx, with the same energy because energy is conserved. We're talking about an energy eigenstate anyway, so it must have the same energy, and it must have the same probability flux as the incoming wave. So the magnitude of this wave and the magnitude of this wave must be the same.

It almost seems that you'd have to write the same thing that I wrote here, but no. I can add one more little factor. It's conventionally written as 2i delta of k. A phase shift, delta of k, an extra phase, and that's the claim. That's the whole thing. You have to solve what's going on here. But outside-- and this is only valid for x greater than a-- outside, where the potential is zero, you must have the solution is of this form. Here, this is for 0 potential everywhere. So this was valid everywhere, but in particular is also valid for x greater than a, and here we compare these two things.

So this is the general solution for scattering here. And now we can also add another definition. We can say that psi of x is going to be equal to phi of x plus the scattered wave. Our intuition is that this wave here has the same incoming component as the reference wave, but nevertheless, it has the different outgoing component and that's the scattered wave.

It is exactly analogous to what we did here, in which we have the total wave being a reference incoming wave. Well, actually, incoming and outgoing wave. The e to the ikz has incoming and outgoing wave and needs a solution when the potential is equal to 0, just like this one, this phi. So psi is equal to 5 plus scattering wave. Here it is the scattering wave. It's purely outgoing, because the incoming part is already taken care of. So completely analogous thing.

So let's solve for the scattered wave here. The scatter wave. So in this equation, we have 1 over 2i, e to the ikx plus 2i delta k, minus e to the minus ikx is equal to 1 over 21, e to the ikx minus e to the minus ikx plus psi s of x.

Well, these two terms cancel. It is the fact that they are ingoing waves are the same. And then I can subtract this other two to find psi s is equal to 1 over 2i. I can factor the e to the ikx. That tells me it's an outgoing wave. And then I have e to the 2i delta k minus 1. That's a term there. I think I got everything there.

It's written-- I'll write it here, there's a little bit of space-- psi s as e to the ikx, e to the i delta k, sine of delta k. I factor out 1 times i delta k, so this becomes e to the i delta k minus e to the minus i delta k, the form together with a 2i in front, a sign of delta k. So this is your outgoing scattered wave. That's the shape of the outgoing scattered wave in terms of the phase shift.

OK, so now we have to redo this in the slightly more complicated case of three dimensions and get it right. So that is our task. That's what we have to figure out how to do.